analysis and evaluation of extended surface thermal systems

allan d. kraus

ANALYSIS AND EVALUATION OF EXTENDED SURFACE THERMAL SYSTEMS

ALLAN D. KRAUS

College of Engineering
University of South Florida, Tampa

HEMISPHERE PUBLISHING CORPORATION

Washington New York London

DISTRIBUTION OUTSIDE THE UNITED STATES

McGRAW-HILL INTERNATIONAL BOOK COMPANY

Auckland Bogotá Guatemala Hamburg Johannesburg Lisbon London
Madrid Mexico Montreal New Delhi Panama Paris San Juan
São Paulo Singapore Sydney Tokyo Toronto

ANALYSIS AND EVALUATION OF EXTENDED SURFACE THERMAL SYSTEMS

1 2 3 4 5 6 7 8 9 0   B C B C   8 9 8 7 6 5 4 3 2

Library of Congress Cataloging in Publication Data

Kraus, Allan D.

Analysis and evaluation of extended surface thermal systems.

Bibliography: p.

Includes index.

1. Heat exchangers. 2. Heat—Transmission.
3. Surfaces (Technology) I. Title.

TJ263.K695   621.402'5   82-980
ISBN 0-89116-252-6   AACR2

ANALYSIS AND EVALUATION OF EXTENDED SURFACE THERMAL SYSTEMS

To

KARL GARDNER
for professional reasons

and to

RUTH KRAUS
for personal ones

PREFACE

While individual fin analysis, as evidenced by an enormously burgeoning literature, continues to receive a great deal of attention from a multitude of investigators, little attention seems to have been paid to the analysis of individual fins assembled into arrays of extended surface. This book attempts to alleviate this apparent void.

In the author's own experience, methods for analysis of arrays of extended surface would have been very useful, had they been heretofore available. These include many diverse analysis problems such as multistacked heat exchanger cores heated from one or two sides, complex finned heat sinks for electronic component heat dissipation or in waste heat applications, where fins are to be spotted in various arrangements on prime surfaces in an effort to reduce size and capital investment.

Take, for example, a rather simple array of ten individual fins and consider how the determination of the array effectiveness or efficiency is accomplished. The objective is to obtain the temperature excess equation for each fin in the array and somehow, from these equations, relate the temperature excess at the base of each fin to the temperature excess at the base of the array. Alternatively, one might determine the actual heat dissipated by each fin in the array and compare it to the heat that would flow if all fins operated throughout their entirety at the array base temperature excess. But this usually cannot be done until an expression for the temperature excess profile for each fin is obtained.

Thus, for a ten fin array one must deal with ten linear, simultaneous, second order differential equations with twenty boundary conditions for the heat flows and/or temperature excesses at the extremities of each fin. Then, by rigorous application of continuity at as few as one or as many as nine fin intersection points, the temperature excess or heat flows in each constituent fin are referred to the base of the array. The tedium involved in this procedure is discouraging and it was this tedium that led to the desire to develop the alternate methods which are presented in this book.

This book proposes that almost every common fin or spine shape possesses a simple 2 x 2 Thermal Transmission or Transmission Parameter Matrix that relates conditions of temperature excess and heat flow at the fin tip to similar conditions at the fin base. Those fins and spines that have no tip area such as the longitudinal fin of triangular profile or the conical spine possess a unique thermal transmission ratio or input admittance that relates the base heat flow to the base temperature excess. The first step is to compute these 2 x 2 matrices or input admittances for each fin in an array. Then, simple algorithms which are developed in the book may be employed to determine the overall array performance. The algorithms are matrix oriented and, in the first part of the book, require little more than the manipulation of 2 x 2 matrices to reduce the entire array to a single equivalent fin.

More complicated arrays which do not form a topological tree in the graph theoretical sense (for example, a presence of physical loops) may be treated by nodal analysis, signal flow analysis and a general topologically oriented method described in Chapter 7, 8 and 9 respectively. Chapter 10 presents several examples and operating instructions for a user oriented general purpose computer program which is listed in an appendix.

The author's purpose in providing the material in this book has not been to discourage continued work in individual analysis, but to provide computationally simple algorithms and methods for assembling the individual fins into complicated arrays. This endeavor has been assisted in great measure by the National Science Foundation and Dr. Win Aung under grant ENG-77-01297. Dr. A. D. Snider, one of the author's colleagues at the University of South Florida, has acted as a sounding board and mathematical adversary. It is his attention to detail that has resulted in a work that the author believes has the requisite mathematical rigor. The computer programs are due to the efforts of a former student, Mr. Peter Molkenthin. Finally, the author wishes to acknowledge the willingness and ability of two splendid ladies, Linda Federspiel and Janice Menendez who were able to type camera ready copy that is reproduced in the pages that follow.

Allan D. Kraus

CONTENTS

1 EXTENDED SURFACE AS A TWO PORT

INTRODUCTION

In the design and construction of various types of heat-transfer equipment, simple shapes such as cylinders, bars or plates are used to implement the flow of heat between a source and a sink. They provide heat-absorbing or heat-rejecting surfaces and each is known as a *prime* or *base surface*. When a prime surface is extended by appendages intimately connected with it, such as metal bars or discs, the additional surface is known as *extended surface*. The elements used to extend surfaces are known as *fins* or *spines*.

More recently, the demands for aircraft, aerospace, gas turbine air conditioning, cryogenic and electrical and electronic equipment auxiliaries have placed particular emphasis on *arrays* of *extended surface*. The growing availability of *compact heat-exchanger* cores as disclosed by Kays and London[1] stimulated interest in the performance of *stacked cores* in which layers of fins are *sandwiched* between metal plates called *separation plates*. In such configurations, the separation plates act as fins and the entire entity of fins and separation plates may be termed a finned array.

In the heat transfer auxiliaries often used to promote heat transfer in electronic equipment, it is not uncommon to find *finned heat sinks* composed of several individual fins connected in one way or

---

1) Kays, W. M., and A. L. London, "Compact Heat Exchangers", 2nd ed., McGraw-Hill Book Company, New York, 1964.

another to form a finned array. Moreover, the individual fins may possess a variety of profiles and the heat transfer performance of the entire array is a strong function of the manner in which the fins are interconnected.

The heat transfer characteristics of *combinations* of single extended surfaces or fins are analyzed by a procedure outlined by Kern and Kraus[1]. Briefly, the technique is as follows:

One begins by writing the differential equations relating the temperature excess and heat flow in each fin in the configuration under certain idealizing assumptions attributed to Murray[2] and Gardner[3]. Then the boundary conditions, expressing continuity at the interfaces between adjacent fins, are listed. Finally, operating temperatures or heat flow rates at the various fin extremities are appended and the overall system is treated as a rather complex, linear, boundary value problem.

When the individual differential equations can be integrated analytically, the problem ultimately becomes one of solving a system of linear algebraic equations. If analytic integrations are impossible, finite difference approximations are introduced and the equations, as discussed by Kern and Kraus, must be solved iteratively on a computer.

The present work describes a new procedure which treats each fin in the configuration as a lumped parameter (actually, a lumped

---

1) Kern, D. Q., and A. D. Kraus; Extended Surface Heat Transfer; McGraw-Hill Book Company; New York; 1972.
2) Murray, W. M.; J. Appl. Mech.; 5:A78(1938).
3) Gardner, K. A.; Trans. ASME; 67:621(1945).

parameter matrix) and combines all of the fins in the configuration via matrix operations. This algorithm improves on the existing techniques in several directions.

1) The differential equations for the individual fins are uncoupled.

2) The combining process which mathematically assembles the characteristics of the entire configuration from the characteristics of the individual fins is quite simple. It can often be done on a hand calculator and the effect of each parameter is easily isolated. This is an important advantage to the design engineer.

3) The performance of the configuration (or an individual fin) can be measured by the ratio of the total heat dissipated to the temperature excess existing at the base of the configuration. This ratio, may, in some cases, be more closely related to design specifications than the heretofore considered fin efficiency.

4) While the analysis still evaluates heat exchange operating under the Murray-Gardner idealized assumptions, some of these assumptions or limitations can be relaxed.

LIMITING ASSUMPTIONS

The Murray-Gardner assumptions that are honored in this work are as follows:

1) The heat flow in the fin and the temperature at any point on the fin remain constant with time.

2) The fin material is homogeneous, its thermal conductivity

is the same in all directions and remains constant.

3) The heat transfer coefficient between the fin and the surrounding medium is uniform and constant over the entire surface of the fin.

4) The temperature of the medium surrounding the fin is uniform.

5) The fin width is so small compared with its height that temperature gradients *across* the fin width may be neglected.

6) The temperature at the base of the fin is uniform.

7) There are no heat sources within the fin itself.

8) Heat transfer to or from the fin is proportional to the temperature excess between the fin and the surrounding medium.

The Murray-Gardner assumptions that are relaxed in this work are as follows:

9) There is no contact resistance between fins in the configuration or between the fin at the base of the configuration and the prime surface.

10) The heat transferred through the outermost edge of the fin (the fin tip) is negligible compared to that through the lateral surfaces (faces) of the fin.

FIN ANALYSIS AS A BOUNDARY VALUE PROBLEM

Consider the longitudinal fin of rectangular profile shown in Fig. 1.1 and observe the terminology and coordinate system. Assume

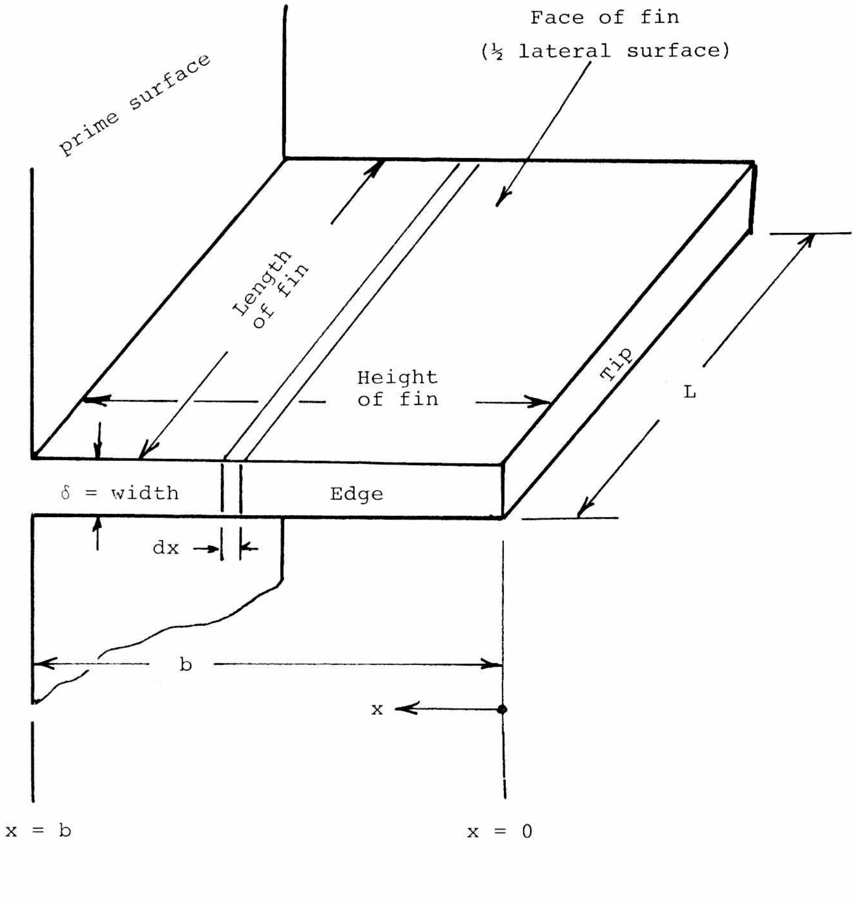

Fig. 1.1. Terminology and coordinate system, longitudinal fin of rectangular profile.

that the fin is dissipating heat to the surrounding environment at temperature, T_s. Note that x is the length coordinate with origin set at the fin tip and note that the fin cross sectional area normal to the heat flow path is $A = \delta L$.

The pedagogy adopted in the so-called standard undergraduate heat transfer texts[1] is to formulate a differential equation for temperature excess, $\theta(x) = T(x) - T_s$, and then solving the equation as a boundary value problem. With base and tip of fin designated respectively by the subscripts b and a, the boundary conditions are

$$\theta(x=b) = \theta_b \tag{1.1a}$$

and

$$q(x=0) = q_a \tag{1.1b}$$

The differential equation for temperature excess is based on a simple energy balance over the incremental height of the fin, Δx, where the temperature is T. Because there is no heat generation, in the steady state[2] the energy balance requires that the difference between the heat entering and leaving the element, Δx, must equal the

1) See, for example, Holman, J. P., "Heat Transfer", 4th ed., McGraw-Hill Book Company, New York, 1976.
2) In the steady state, there is no heat accumulation. The steady state requirement reiterates the first of the Murray-Gardner assumptions.