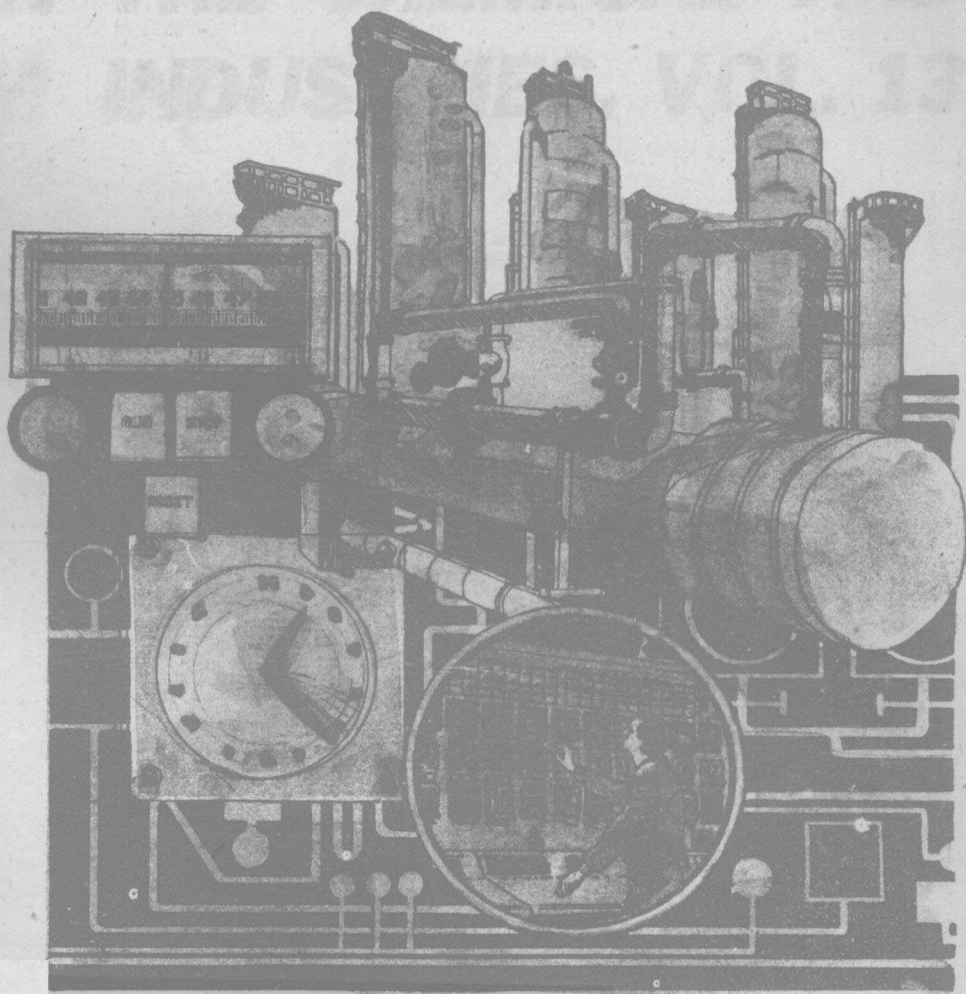


**INSTRUMENTATION  
IN THE CHEMICAL AND  
PETROLEUM INDUSTRIES**

**VOL. 13**

**1977**

# **INSTRUMENTATION IN THE CHEMICAL AND PETROLEUM INDUSTRIES, VOL. 13**



*Programmed by:*  
**ISA's Chemical and Petroleum Industries Division**



**Proceedings from the 1977  
Spring Industry Oriented Conference  
Anaheim, California**



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Phone: (412) - 281-3171

**Library of Congress Catalog Card Number 64-7505**

**ISBN 87664-363-2**

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400 Stanwix Street  
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Printed in U.S.A.

# **INSTRUMENTATION IN THE CHEMICAL AND PETROLEUM INDUSTRIES, VOL. 13**

**Proceedings of the 14th Annual ISA Chemical  
and Petroleum Instrumentation Symposium**

**May 2-5, 1977, Anaheim, California**

**Instrumenting and Controlling  
Centrifugal Compressors**

*Edited by*

**R. C. Waggoner**

**University of Missouri—Rolla  
Rolla, Missouri**

ISBN 0-894-363-2

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## DIRECTOR'S FOREWORD

1977 is the eighteenth (18th) consecutive year that the Chemical and Petroleum Industries Division of ISA has presented a spring symposium. The proceedings volume from this CHEMPID symposium has been assembled in keeping with the current interests of the chemical and petroleum industries to serve as a permanent record and a guide to today's process systems technology. It is intended to benefit those who continue to lead the way in the conception, design, and implementation of process control.

The program coordinator for these CHEMPID sessions has been Dr. Raymond C. Waggoner, University of Missouri—Rolla. Dr. Waggoner, our Education Chairman, has worked diligently to put together an informative group of papers.

K. L. Hopkins  
Chempid Director

## PROGRAM CHAIRMAN

The papers in this volume were programmed by the Chemical and Petroleum Industries Division and were presented at the ISA/77 Conference, May 2-5, in Anaheim, California. These papers implement the conference theme "Leading the Way in Control Systems" by presenting applications of advanced control techniques in the chemical and petroleum industry. Processes in these industries are inherently multi-variable and non-linear. Advanced control techniques are requisite if the most economical and effective operating levels are to be sustained. However, the complexities of the control algorithms virtually require digital computer implementation.

The fundamental concepts of advanced control techniques are presented to provide a background for the following papers. These advanced concepts, including feedforward control, multivariable control, optimal control, and digital control algorithms are developed and papers describe their application to actual processes, pilot plant scale equipment, or calibrated process simulations. A group of papers is specifically directed to chemical reactor control. The economic aspects of advanced control are then shown in the papers taken from the final two sessions.

This volume is presented to illustrate that advanced control concepts can be made practical in chemical and petroleum processes and to aid the reader in their implementation.

R. C. Waggoner  
Program Coordinator

## DIRECTOR'S FOREWORD

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IMPROVED CONTROL BY APPLICATION  
OF ADVANCED CONTROL TECHNIQUES

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ABSTRACT

Sophisticated computer-aided design procedures leading to specified optimal operating conditions for process units, are now in use in many design groups. Accompanying this change in design techniques is an increased awareness of the necessity of developing control system strategies, based on advanced control techniques, to achieve the desired optimal operation.

Of the advanced techniques leading to improved control performance, feedforward control has gained the most acceptance by industry. Combined feedforward-feedback control will be briefly discussed. Further improvements in control system design have resulted by considering the multivariable nature of the process system to be controlled. Although the subject of much attention, design based on linear state space models has not been too successful due to the difficulty of characterizing process systems by such a model. Consequently the discussion will be restricted to multivariable frequency domain design methods. Non-interacting/decoupling control and design by the characteristic loci technique will be considered in this review.

INTRODUCTION

Rapid advances in computer technology have now made computer-aided design techniques standard practice in many engineering departments. As a consequence of this development, frequently the designer will specify a unit on some optimal design basis that is very dependent on operating conditions. In order to derive maximum economic benefit from the improved design, it logically follows that this can only be achieved by improved control of the system. In conjunction with the use of computers for design has come the utilization of computers for process control. Although, initially the control algorithms were simply replacements for conventional single variable feedback PID controllers, it was soon realized that to achieve the specified operating conditions that more advanced control techniques would be required. Probably the first advanced control technique to gain acceptance in the process industries was that of feedforward control, and then not until the early 1960's. However as Buckley (1) correctly points out, the idea of feedforward control was not a new control concept at all. Feedforward control had already been used for many years for manipulating feedwater rate to improve level control in boiler drums. The 1960

decade also saw the development of what came to be known as modern control theory, namely the body of control theory predicated on describing system behaviour in the time domain by means of a linear state space model. A typical model formulation would be of the following form:

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u} + \underline{D}\underline{d} \quad (1)$$

$$\underline{y} = \underline{C}\underline{x} \quad (2)$$

where  $\underline{x}$  = vector of "n" state variables  
 $\underline{u}$  = vector of "m" control inputs  
 $\underline{d}$  = vector of "p" disturbance inputs  
 $\underline{y}$  = vector of "q" output variables  
 $\underline{A}, \underline{B}, \underline{D}, \underline{C}$  = matrices of appropriate order

Hundreds of articles have appeared which develop the theory necessary for establishing control laws for specific types of models using a wide variety of performance indices. However, much of the work is not directly applicable to the regulatory control problems of the process industries. The reader interested in gaining an appreciation of the design approach involved when a system is characterized by a state space model should consult the case study of Fisher and Seborg (2). The authors present experimental results from applying several different multivariable control system strategies to a pilot plant evaporator that could adequately be characterized by a model of the form expressed by Equations (1) and (2).

Unfortunately, the nonlinear characteristics of most process systems are such that it is very difficult to describe their behaviour by such a linear model. As noted by Rijnsdorp and Seborg (3), few applications of control schemes based on such a model have been adopted by the process industries. In contrast, the authors noted that non-interacting or decoupling control schemes, based on a transfer function representation, had gained wide acceptance by industry. Such a control scheme is but one approach to designing a control system using multivariable frequency domain techniques pioneered by Rosenbrock, MacFarlane and co-workers at the University of Manchester Institute of Science and Technology in England. Research has demonstrated that the single variable concepts of Bode and Nyquist can be extended to developing control strategies for multivariable systems. The design techniques that have been developed are known as the inverse Nyquist array (4,5,6); characteristic loci (7,8,9,10); direct



Nyquist array (11,12,13) and commutative controller (14,15,16). Another class of design techniques are those based on the sequential return difference method originally developed by Mayne (17,18,19) and subsequently modified by Owens (20,21). A recent evaluation of the characteristic loci and both the inverse and direct Nyquist array design techniques has been performed by Kuon (22).

In this review, the accepted technique of combined feedforward-feedback control will first be discussed. This material is included to emphasize that implementation of feedforward action is simple and worthy of consideration for improving the control of any process system. The remainder of the discussion will be directed to reviewing the non-interacting/decoupling control and characteristic loci multivariable frequency domain design techniques. It is hoped that by drawing attention to such multivariable techniques that designers of control systems will not be reluctant (23) to consider such techniques.

#### COMBINED FEEDFORWARD-FEEDBACK CONTROL

Applications of feedforward control action combined with conventional feedback control did not become routine until the installation of process control computers. With the advent of the process control computer, implementation no longer meant that a special pneumatic or electronic unit (24,25) had to be constructed. Despite the ease with which a feedforward control loop can now be implemented, using standard electronic feedforward control units or by computer, it is surprising that more combined feedforward-feedback systems are not utilized. Implementation of a feedforward control loop can lead to a substantial cost saving (26) for a small capital investment.

The block diagram of a combined feedforward-feedback control system is shown in Figure 1. The control variable can be expressed in terms of the disturbance,  $D(s)$  and the set point,  $R(s)$  as:

$$C(s) = \left\{ \frac{G_c G_v G_p}{1 + G_{OL}} \right\} R(s) + \left\{ \frac{G_L}{1 + G_{OL}} \right\} D(s) \quad (3)$$

+  $\left\{ \frac{G_{MD} G_{FF} G_v G_p}{1 + G_{OL}} \right\} D(s)$   
where  $G_{OL} = G_c G_v G_p G_{MC}$ . Writing the expression for  $C(s)$  in this form, serves to emphasize that the stability of the control system is unchanged by the addition of feedforward control action and furthermore if there is no feedforward control,  $G_{FF} = 0$ , the expression is that for a conventional feedback control system. Considering the case of regulatory control ( $R(s) \equiv 0$ ), allows Equation (1) to be written as

$$C(s) = \left\{ \frac{G_L + G_{MD} G_{FF} G_v G_p}{1 + G_{OL}} \right\} D(s) \quad (4)$$

Since the control objective is no change in the control variable for load upsets, it follows from

Equation (4) that to satisfy this condition it is necessary that  $G_L + G_{MD} G_{FF} G_v G_p = 0$ , or rearranging

$$G_{FF} = - \frac{G_L}{G_{MD} G_v G_p} \quad (5)$$

Equation (5) is the specified form of the feedforward controller,  $G_{FF}$ . It is particularly interesting to note the form that the feedforward controller takes under certain circumstances. If the load and process transfer functions,  $G_L$  and  $G_p$  respectively, have the same dynamics and the valve,  $G_v$  and measurement device,  $G_{MD}$  dynamics can be neglected then Equation (5) becomes

$$G_{FF} = - \frac{K_L}{K_{MD} K_v K_p} \quad (6)$$

In this particular case, the required feedforward controller is simply a gain device, and readily implemented by cheap conventional instrumentation or even by the measurement device itself. For the case where the dynamics of the load and process transfer functions are the same, the dynamics of the valve represented by  $K_v/(\tau_v s + 1)$  and the dynamics of the measurement device neglected then Equation (5) can be written as

$$G_{FF} = - \frac{K_L (\tau_v s + 1)}{K_{MD} K_v K_p} \quad (7)$$

This form of the feedforward controller is simply a PD controller! More typically, the load and process transfer function dynamics are not the same and the dynamics of the valve and measurement device can be neglected giving the required form of the controller as

$$G_{FF} = - \frac{G_L(s)}{K_{MD} K_v G_p(s)} \quad (8)$$

Obviously, it would be a simple matter to implement the required controller action by means of a digital computer. However in actual practice despite the fact that  $G_L(s)$  and  $G_p(s)$  may be of high order, experience has shown (26) that it is generally adequate to consider the feedforward controller as

$$G_{FF} = - \frac{K(1 + \tau_1 s)}{(\tau_2 s + 1)} \quad (9)$$

and employ on-line tuning of the parameters. Although this type of lead-lag unit, proposed several years ago by Shinskey (27), has been installed for many applications, experience has shown that satisfactory control can be achieved without employing lead action. This experience is substantiated by tests on a pilot scale distillation column by Wood and Pacey (28) who found that satisfactory control could be achieved using only a tuned first order lag or time delay element. It is important to realize that the choice of the form of feedforward controller can significantly influence system control behaviour. For instance in the paper

by Davis and Smith (29), in this symposium, they considered only gain feedforward action. As can be seen in their Figure 5, without dynamic compensation, the controlled variable was initially driven below the set point. It would be interesting to compare the control behaviour of the combined feedforward-feedback control scheme, using a dynamic feedforward controller, with that of the dual control loop scheme (in which the effect of the disturbance is minimized by the temperature loop).

#### MULTIVARIABLE FREQUENCY DOMAIN DESIGN

Frequently the design of a process control system will involve the control of more than a single output variable by manipulating more than one input variable. Such a system is multivariable, so a control system designed using conventional single variable theory may not yield satisfactory performance. The control performance using multiple single variable feedback loops will depend on the extent of interaction between the input and output variables. In order to establish whether such an approach will be satisfactory the degree of interaction can be checked using the procedure suggested by Bristol (30). Since the development of control strategies based on a state space model have had very limited success, the discussion here will focus on two design techniques for systems characterized by a transfer function representation.

##### A) NON-INTERACTING/DECOUPLING DESIGN TECHNIQUE

This design technique, directed at reducing the multivariable feedback design problem to one of conventional single loop design, was first proposed by Boksenbom and Hood (31). In this approach, decoupling controllers that render the system completely non-interactive are first designed, and then conventional single variable design methods are employed to design feedback controllers. Despite the possible difficulties (32,33) with such a procedure, it has been successfully employed to pilot scale and industrial process units (3,34,35, 36). Although in theory the concept could be extended to high order dynamic systems, reported applications have been concerned only with control of two variables by manipulation of two input variables.

The design procedure will now be outlined for the case of a  $2 \times 2$  plant transfer function matrix,  $G_p(s)$ . The block diagram of such a system is shown in Figure 2 where  $U_1(s)$ ,  $U_2(s)$  are the input variables;  $C_1(s)$ ,  $C_2(s)$  the output (controlled) variables and  $D(s)$  the load disturbance. In vector-matrix form this may be expressed as

$$\underline{C}(s) = \underline{G}_p(s) \underline{U}(s) + \underline{G}_L(s) \underline{D}(s) \quad (10)$$

where

$$\underline{C}(s) = \begin{bmatrix} C_1(s) \\ C_2(s) \end{bmatrix} \quad \underline{U}(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \quad \underline{G}_L(s) = \begin{bmatrix} G_{L1} \\ G_{L2} \end{bmatrix}$$

$$\underline{G}_p(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

Now, if a feedback controller and measurement device are included for each of the controlled variables, that is  $G_{C11}$ ,  $H_1$ ,  $G_{C22}$ ,  $H_2$  as well as two additional controllers,  $G_{C12}$ ,  $G_{C21}$ , then the system can be represented as shown by the block diagram in Figure 3. (Note: The control valve transfer functions are considered to be included in the plant transfer function matrix). The two controllers,  $G_{C12}$  and  $G_{C21}$  are generally designated as decoupling controllers or compensators. Defining

$$\underline{R}(s) = \begin{bmatrix} R_1(s) \\ R_2(s) \end{bmatrix} \quad \underline{H}(s) = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \quad \underline{G}_C(s) = \begin{bmatrix} G_{C11} & G_{C12} \\ G_{C21} & G_{C22} \end{bmatrix}$$

allows the block diagram to be redrawn in general multivariable feedback form as shown in Figure 4. With the block diagram arranged to the same form as that of a conventional single variable feedback control system, the following expression logically follows:

$$\underline{C}(s) = [\underline{I} + \underline{G}_p(s) \underline{G}_C(s) \underline{H}(s)]^{-1} \{ \underline{G}_C(s) \underline{G}_p(s) \underline{R}(s) + \underline{G}_L(s) \underline{D}(s) \} \quad (11)$$

where  $\underline{I}$  = identity matrix. Defining  $\underline{Q}(s) = \underline{G}_p(s) \underline{G}_C(s)$ , as the open loop transfer function matrix allows Equation (11) to be expressed as

$$\underline{C}(s) = [\underline{I} + \underline{Q}(s) \underline{H}(s)]^{-1} \{ \underline{Q}(s) \underline{R}(s) + \underline{G}_L(s) \underline{D}(s) \} \quad (12)$$

Defining the closed loop transfer function matrix as

$$\underline{P}(s) = [\underline{I} + \underline{Q}(s) \underline{H}(s)]^{-1} \underline{Q}(s)$$

allows Equation (12) to be written as

$$\underline{C}(s) = \underline{P}(s) \underline{R}(s) + \underline{P}(s) \underline{Q}(s)^{-1} \underline{G}_L(s) \underline{D}(s) \quad (13)$$

It now follows that if  $\underline{Q}(s) \underline{H}(s)$  can be diagonalized, then  $[\underline{I} + \underline{Q}(s) \underline{H}(s)]^{-1}$  will be diagonal since  $\underline{H}(s)$  is already diagonal. Therefore the closed loop transfer function matrix,  $\underline{P}(s)$  will be diagonal which means that there will be no interaction between  $\underline{C}(s)$  and  $\underline{R}(s)$  (or  $\underline{D}(s)$ ). From Figure 4, it can be shown that

$$\underline{Q}(s) \underline{H}(s) = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

where



$$\alpha_{11} = G_{11}G_{C11}H_1 \left[ 1 + \frac{G_{12}G_{C21}}{G_{11}G_{C11}} \right] \quad (14)$$

$$\alpha_{12} = \{G_{11}G_{C12} + G_{12}G_{C22}\} H_2 \quad (15)$$

$$\alpha_{21} = \{G_{21}G_{C11} + G_{22}G_{C21}\} H_1 \quad (16)$$

$$\alpha_{22} = G_{22}G_{C22}H_2 \left[ 1 + \frac{G_{21}G_{C12}}{G_{22}G_{C22}} \right] \quad (17)$$

The requirement for diagonalization means that the off-diagonal terms must be zero, that is  $\alpha_{12} = 0$ ,  $\alpha_{21} = 0$ . Setting the off-diagonal entries to zero gives the required form of the decoupling controllers as

$$G_{C12} = -\frac{G_{12}G_{C22}}{G_{11}} \quad (18)$$

$$G_{C21} = -\frac{G_{21}G_{C11}}{G_{22}} \quad (19)$$

Substituting for the decoupling controllers, as given by Equations (18) and (19), in Equations (14) and (17) gives

$$\alpha_{11} = G_{C11}G_{11}H_1 \left[ 1 - \frac{G_{12}G_{21}}{G_{11}G_{22}} \right] = G_{C11}\hat{G}_{11} \quad (20)$$

$$\alpha_{22} = G_{C22}G_{22}H_2 \left[ 1 - \frac{G_{12}G_{21}}{G_{11}G_{22}} \right] = G_{C22}\hat{G}_{22} \quad (21)$$

which in turn allows the closed loop transfer function matrix,  $\underline{P}(s)$  to be written as

$$\underline{P}(s) = \begin{bmatrix} \frac{\alpha_{11}H_1^{-1}}{1 + \alpha_{11}} & 0 \\ 0 & \frac{\alpha_{22}H_2^{-1}}{1 + \alpha_{22}} \end{bmatrix}$$

Expressing  $\alpha_{11}$  and  $\alpha_{12}$  in Equations (20) and (21) in terms of  $G_{11}$  and  $G_{22}$  is done to emphasize that these groups are calculated from plant data. All that remains is selection of the controllers  $G_{C11}$  and  $G_{C22}$  which can proceed on the basis of two separate single variable systems since the interaction has been eliminated. Once the desired feedback controllers have been determined the required form of the decoupling controllers can be calculated from Equations (18) and (19).

## B) CHARACTERISTIC LOCI DESIGN TECHNIQUE

An alternate approach to the design of a completely non-interacting/decoupled control system is simply to employ a design procedure that minimizes but not totally eliminates interaction. Design techniques that employ such an approach strive to achieve diagonal dominance. A control system designed on this basis may actually provide better control than is possible with the non-interacting system. This will depend upon the reliability/variability of the transfer function parameters that determine the parameters of the decoupling controllers.

A comprehensive survey of the existing multivariable frequency domain procedures has been presented in the excellent five part review of MacFarlane (37). Since the characteristic locus method as well as the non-interacting design approach has been employed in the paper of Schwanke et al. (38), to be presented in this symposium, this discussion will deal only with this technique.

In order to restrict the length of this review, some fundamental definitions, concepts, conditions and/or requirements will simply be stated without proof. Most of the fundamental theoretical concepts have been presented by Belletrutti (8), MacFarlane (9) and MacFarlane and Belletrutti (7,10). Since  $s$  is a complex variable, then for every specific value of  $s$  (over the domain of definition,  $\mathcal{D}$ ) it follows that an  $m \times m$  matrix function of a complex variable  $\underline{G}(s)$  is a matrix with complex entries. Thus it has a set of eigenvalues  $\{g_i(s) : i = 1, 2, \dots, m\}$  such that

$$g_i(s) \in \mathcal{C} \quad i = 1, 2, \dots, m$$

and corresponding sets of eigenvectors

$$d_i(s) \in \mathcal{C}^n \quad i = 1, 2, \dots, m$$

(Note: A vector function of a complex variable, say  $\gamma(s)$ , is a mapping  $\gamma(s) : \mathcal{D} \rightarrow \mathcal{C}^n$  from the set of complex numbers  $\mathcal{C}$  to the set of complex vectors  $\mathcal{C}^n$ ). This notation means that the eigenvalues of a matrix function of a complex variable are functions of a complex variable, and the corresponding eigenvectors are vector functions of a complex variable. The eigenvalues,  $g_i(s)$  of  $\underline{G}(s)$  are designated as characteristic transfer functions while the corresponding eigenvector,  $d_i(s)$  is called the characteristic direction vector.

Also the set of loci in the complex plane obtained by evaluating a characteristic transfer function,  $\underline{G}(s)$  along the standard Nyquist contour is known as the set of system characteristic loci and is denoted as  $\{g_i(j\omega)\}$ . The design considerations involve consideration of the open-loop transfer function matrix,  $\underline{Q}(s)$  expressed in dyadic form as

$$\underline{Q}(s) = \sum_{i=1}^m q_i(s) w_i(s) v_i^T(s) \quad (22)$$

where  $q_i(s)$ ,  $w_i(s)$  and  $v_i(s)$  are the characteristic transfer functions, characteristic direction vectors and reciprocal characteristic direction vectors of  $\underline{Q}(s)$ . The corresponding dyadic expansion for the

closed-loop system can be written as

$$\underline{R}(s) = \sum_{i=1}^m \left| \frac{q_i(s)}{1 + q_i(s)} \right| w_i(s) v_i^T(s) \quad (23)$$

The design procedure based upon these and related theoretical concepts, is a generalization of the classical frequency domain approach, involving the conflicting objectives of stability, integrity, non-interaction and accuracy. This is accomplished by attaining required closed-loop stability and performance specifications by appropriate manipulations of sets of open loop characteristic loci and characteristic directions. Simplification of the task is achieved by letting the feedback matrix  $\underline{F}(s) = \underline{I}$ , since it can then be shown that if  $\{q_i(j\omega)\}$  is the set of characteristic loci of the open loop system  $\underline{Q}(s)$ , then the set of characteristic loci belonging to the closed loop system  $\underline{R}(s)$  is simply

$$\left\{ \frac{q_i(j\omega)}{1 + q_i(j\omega)} \right\}$$

Furthermore the set of characteristic directions for both the open-loop and closed-loop systems are the same, namely  $\{w_i(j\omega)\}$ . Thus, the design effort is concerned with synthesizing the controller  $\underline{G}_{c1}(s)$  (cf. Figure 4), which is considered to be square as is the plant,  $\underline{G}_p(s)$ .

Design in the vector frequency response approach requires that the controller

- i) modify the phases of appropriate sets of characteristic loci in order to achieve acceptable stability and integrity results.
- ii) align the characteristic directions at high frequencies and balance the gains of the characteristic loci at low frequencies in order to achieve acceptable interaction.
- iii) inject gain to improve overall performance.

Clearly, the controller must satisfy many objectives simultaneously. Thus a controller structure formed as a cascaded combination of several sub-controllers,  $\underline{G}_{c1}(s)$  so that

$$\underline{G}_c(s) = \prod_{i=1}^p \underline{G}_{c1}(s)$$

is employed in which each of the  $\underline{G}_{c1}(s)$  achieves only part of the overall design objectives. Obviously each of the  $\underline{G}_{c1}(s)$  must be simple and if possible only contain constant factors. Specific restrictions on the  $\underline{G}_{c1}(s)$  are:

- i) all dynamical elements must be rational functions in "s".
- ii)  $\det \underline{G}_{c1}(s)$  must be identically non-singular.
- iii) poles of  $\underline{G}_{c1}(s)$  must lie in the open left-half plane.
- iv)  $\det \underline{G}_{c1}(s)$  must not have any right-half plane zeros (to prevent non-minimum phase difficulties).

Many different types of sub-controllers which provide for certain manipulations of system characteristic loci and characteristic directions have been developed. Some of the more useful (7,10) types of sub-controllers are those that provide for

#### a) Elementary transformation

$$i) \underline{G}_{c1}(s) = \text{diag}[1, 1, \dots, g_{jj}(s), \dots, 1]$$

$$ii) \underline{G}_{c1}(s) = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & g_{jk}(s) & \dots & 0 & 0 \\ \vdots & \vdots & 0 & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 1 & \vdots \\ 0 & 0 & 0 & \dots & \dots & \dots & 1 \end{bmatrix}$$

Such controllers are suitable for improving integrity when only subsystems require modification and in reducing interaction by diminishing the magnitudes of the off-diagonal elements of the plant  $\underline{G}_p(s)$ .

#### b) Scalar

$$\underline{G}_{c1}(s) = k(s) \underline{I}$$

This controller multiplies the plant characteristic loci (each eigenvalue) by the scalar,  $k(s)$  while leaving the characteristic directions unchanged.

#### c) Permutation

$$\underline{G}_{c1}(s) = [e_1 \dots e_{q-1} e_p e_{q+1} \dots e_{p-1} e_q e_{p+1} \dots e_m]$$

where  $p > q$  and  $e_j$  is column  $j$  of  $\underline{I}$ . A controller of this form interchanges columns  $p$  and  $q$  of the plant matrix,  $\underline{G}_p(s)$  which may be helpful in improving integrity.

#### d) Proportional plus Integral Action

$$\underline{G}_{c1}(s) = \underline{K}_1 \underline{D}_1 + \underline{G}_p^{-1}(0) \underline{D}_2 / s$$

The matrix  $\underline{K}_1$  tends to render  $\underline{G}_p(s)$  diagonal as  $|s| \rightarrow \infty$ , so tends to align the characteristic directions of  $\underline{G}_p(s)$  with the standard basis vectors. Matrices  $\underline{D}_1$  and  $\underline{D}_2$ , which are diagonal, can be used to adjust the weighting between zero and infinite frequencies in each column of  $\underline{Q}(s) = \underline{G}_{c1}(s) \underline{G}_p(s)$ . Consequently the controller eliminates steady-state error by ensuring that at low frequencies, the moduli of all characteristic loci are large as  $|s| \rightarrow 0$ . High frequency interaction is also reduced by use of a sub-controller of this form.

The very nature of the iterative characteristic loci design procedure, as is the case for other multi-variable frequency domain design techniques, may in



fact limit its use. This is because to effectively utilize the technique requires the use of a digital computer with a visual display unit. Not to mention the vast number of man-hours required to develop the necessary software for implementation. Notwithstanding this limitation, to gain some appreciation of the actual procedure involved in designing the series of sub-controllers the various phases that are involved will be summarized:

- a) Stability phase - This involves determination of the right-half plane zeros in the open loop characteristic polynomial. Closed loop stability is then assessed for a gain,  $k$ , applied to each loop. This is done by inspecting a display of the loci of  $G_p(j\omega)$  in the form of a Nyquist plot relative to the critical point  $(-1/k, 0)$  for a finite number of frequencies.
- b) Integrity phase - The same procedure as employed in the stability phase except that the encirclement theorem (stability check) is applied to the characteristic loci of the principal submatrices of  $Q(j\omega)$ . Specifically when applied to the diagonal element  $q_{ii}(j\omega)$  of  $Q(j\omega)$ , the theorem establishes the stability margin when all loops except loop "i" are open. This analysis plus that of the stability phase provides an excellent indication of the stable operating regions for all possible combinations of loop gains  $k_i$ ,  $i = 1, 2, \dots, m$ . Should integrity be poor, a controller factor  $k_i(s)$  is synthesized and the stability phase is then repeated.
- c) Interaction phase - Once the stability and integrity requirements have been satisfied the amount of interaction can be assessed from plots of  $|q_i(j\omega)|$  versus  $\omega$  and  $\theta_i(j\omega)$  versus  $\omega$  for  $i = 1, 2, \dots, m$  where the set  $\{q_i(j\omega)\}$  represents the characteristic loci of  $Q(j\omega)$  and  $\theta_i(j\omega)$  is the minimum angle of misalignment between the standard basis vector  $e_i$  and the characteristic directions,  $w_j(j\omega)$  of  $Q(j\omega)$  for all  $j$ . Interaction can be suppressed by insuring that either  $|q_i(j\omega)| \gg 1$  or  $\theta_i(j\omega) \approx 0$  for  $i = 1, 2, \dots, m$ . If the degree of interaction is not acceptable, compensation by means of a controller factor  $G_{ci}(s)$  is designed and the stability phase is repeated.
- d) Performance phase - Finally when the major design considerations of stability, integrity, and interaction have been satisfied, compensation by applying single loop techniques to the diagonal elements of  $Q(s)$  can be undertaken. It is in this phase that the final loop gain values are tuned to give the controller factor  $G_{ci}(s) = \text{diag } G_{ci}$ .

As noted previously such a design procedure involves man-computer interaction by means of a visual display unit. Consequently, the experience of the control engineer will significantly influence the detailed steps and types of sub-controllers employed in achieving a final control system design. The reader interested in gaining a further appreciation of this technique should study the examples presented by Belletrutti (8) and

Belletrutti and MacFarlane (7,10). Unfortunately because of the large amount of software development that is necessary to employ this, or for that matter any of the other multivariable frequency domain design techniques, the adoption of such design procedures is likely to be slow.

## CONCLUSION

The improved control behaviour that can be achieved using combined feedforward-feedback control is well known so consequently the specification of such a control strategy is becoming commonplace in most organizations. However, this is not so for the multivariable frequency domain design techniques for developing control schemes. The non-interacting/decoupling design approach has found some application to industrial control problems but not the characteristic loci, or for that matter either of the Nyquist array techniques. This was the finding of Rijnsdorp and Seborg (3) in a survey prepared for the Engineering Foundation Conference on Chemical Process Control held in January, 1976. This lack of acceptance is in part due to the investment of time and resources required to implement the software to effectively utilize these design procedures. Furthermore, with the present lack of experimental evaluations of these techniques, such as the study of Kuon (22), the reluctance of designers to employ such techniques is understandable.

With the availability of process control computers has come the development of robust on-line adaptive techniques which involve estimation and control. Typical of these single variable techniques are the self-tuning regulator developed by Åström and co-workers (39) at the Lund Institute of Technology in Sweden and the self-tuning controller of Clarke and Gawthrop (40). Successful industrial applications of the self-tuning regulator approach have been reported in the mineral (41) and pulp and paper (42) industries. Availability of process control computers has also seen the development of multivariable control strategies based on steady state models. Using the data acquisition capability of the computer, static model calculations are made at frequent intervals (e.g. a few seconds) and the set points of several control loops adjusted. Also employed are on-line estimation procedures for periodic updating of model parameters.

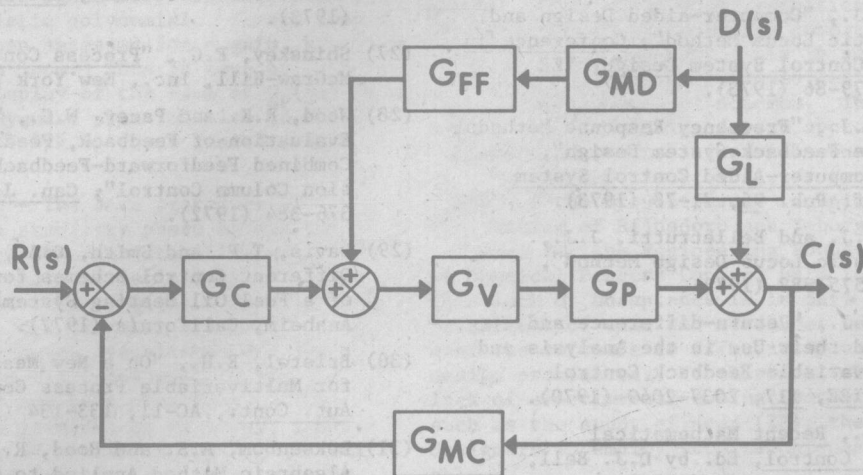
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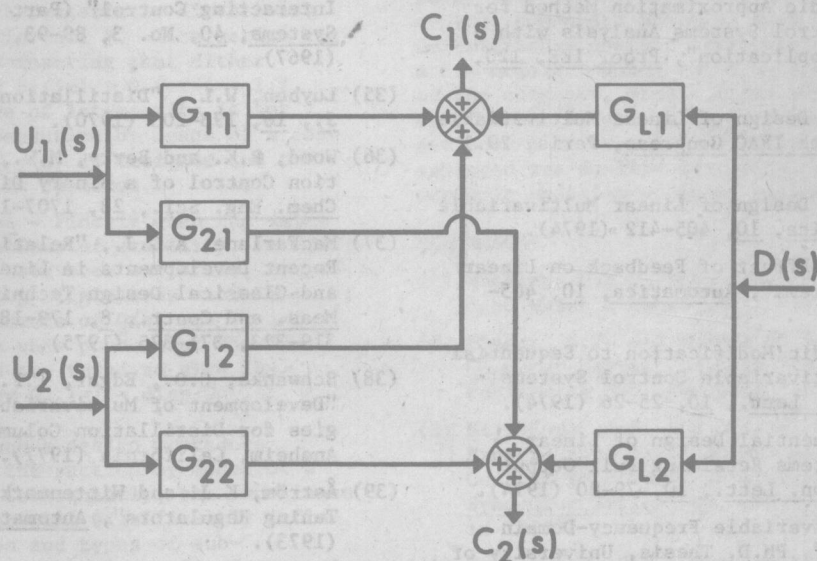
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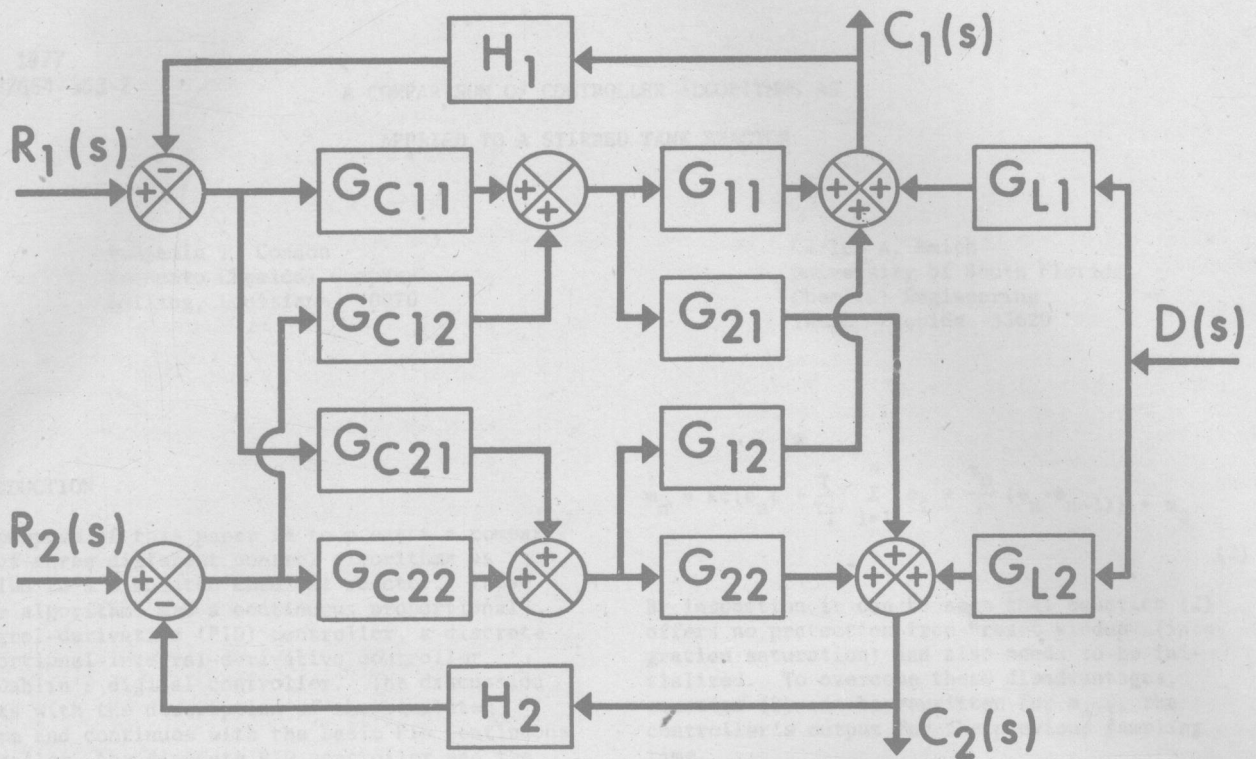
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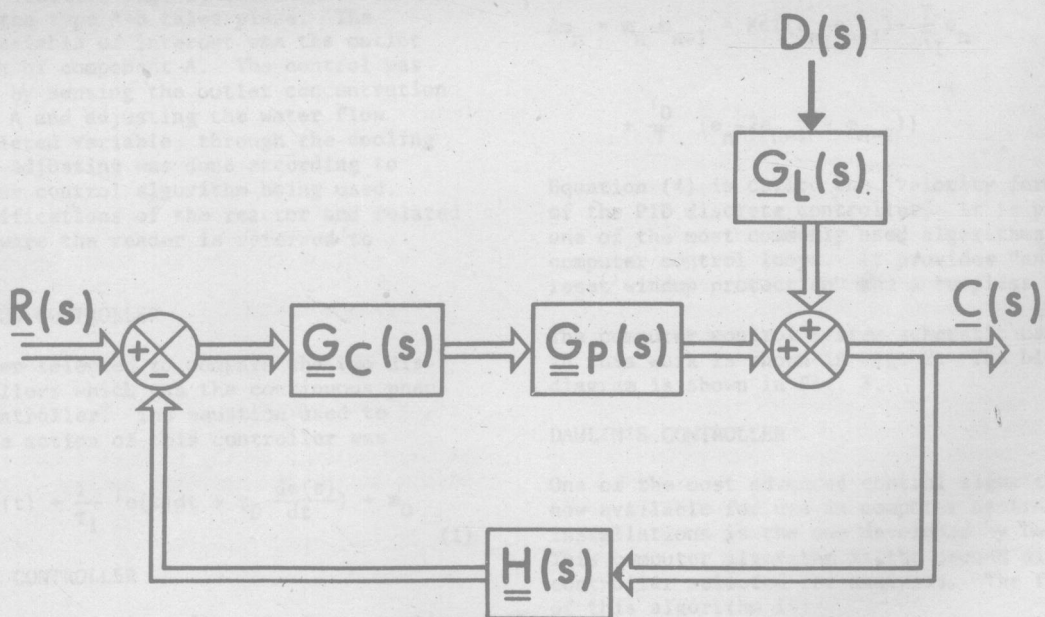
**Figure 1. Block Diagram of Combined Feedforward-Feedback Control System**



**Figure 2. Block Diagram of Multivariable System**



**Figure 3. Block Diagram of Non-interacting Control System**



**Figure 4. General Block Diagram for Multivariable Feedback System**



