LANNING
PRODUCTION,
INVENTORIES,
and WORK FORCE



# Planning Production, Inventories, and Work Force

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# Planning Production, Inventories, and Work Force



#### PRENTICE-HALL INTERNATIONAL SERIES IN MANAGEMENT

BAUMOL Economic Theory and Operations Analysis

CHURCHMAN Prediction and Optimal Decision

Holt, Modigliani, Muth, and Simon Planning Production, Inventories,

and Work Force

MILLER AND STARR Executive Decisions and Operations Research

PFIFFNER AND SHERWOOD Administrative Organization

### Preface

In the last ten years, rapid progress has been made in the development and application of mathematical techniques to business decision-making. These techniques are improving the control of operations and reducing the cost of making routine decisions, as well as providing managers more information and gaining them more time for long-range planning and for solving non-routine problems.

This book undertakes to describe some of these methods and how they may be applied to managerial decisions in the operation of a factory-warehouse system. Because they affect all the major business functions, these decisions are of consuming interest to line managers and staff specialists in nearly every department of a business: manufacturing, warehousing, sales, and shipping.

Although the methods reported here were developed in the context of a factory supplying a warehouse system, they are applicable to the corresponding decision problems in the operation of military and other governmental organizations. Moreover, the basic mathematical tools can be adapted to use in other fields of business.

We have organized the book to make it as useful as possible to those executives and specialists who share in formulation of policy and administrative decision-making in warehouse and factory management. Company executives concerned with broad policy formulation will be particularly interested in the summarization given in Chapter 1, and in the overviews beginning each part and chapter. Some chapters set forth techniques that will enable managers and staff to apply the methods on a day-to-day basis. Other chapters provide the necessary technical and mathematical analysis for those who need a thorough understanding of the methods in order to modify and apply them to new problems.

The book is based primarily on research conducted as part of the "Planning and Control of Industrial Operations" project conducted at the Graduate School of Industrial Administration, Carnegie Institute of Technology, Pittsburgh, Pennsylvania. This work was done under contract with the Logistics Branch of the Office of Naval Research, Contract Nonr-76001, Project Nr 047001. The authors gratefully acknowledge the generous support of the Office of Naval Research. The whole industrial community owes the O.N.R. a vote of thanks for its farsighted support of research on decision problems.

Many people have participated in this research program, directly

and indirectly; academic colleagues, business managers, and graduate students. We gratefully acknowledge their indispensable contributions. We wish to mention in particular Sigurd L. Andersen, Edward H. Bowman, Robert F. Byrne, Frank J. Carr, A. Charnes, Thomas R. Cockerline, William W. Copper, James C. Emery, the late Charles R. Fay, Wesley Folsom, William S. Gere, Myron J. Gordon, Thomas V. Griffin, Henry J. Hart, William A. Hartigan, William J. Hollis, William Holter, Alfred A. Kuehn, T. T. Kwo, Toshiro Makibuchi, Takehiko Matsuda, Robert E. McGarrah, Sharu S. Rangnekar, the late Clyde E. Roberson, Robert Schlaifer, L. A. Schroeder, Ellis F. Slack, Phillips Whidden, and Bayard E. Wynn. We wish to thank the Pittsburgh Plate Glass Company, the Westinghouse Electric Corporation, and numerous other companies who gave us access to data on their production and inventory problems, and who made operating tests of the new methods. Data from actual operations have, of course, been disguised to preserve commercial security.

The contributions of three colleagues require special mention. Charles P. Bonini is the author of the paper on which Chapter 11 is based. Chapters 13 and 14 are based on papers by Peter R. Winters. We are deeply indebted to both of them for permitting us to incorporate their work in this volume. The paint company application of Chapter 1 was supervised mainly by R. W. Culbertson.

We are grateful to Mrs. Joan Andersen, Mrs. Sandra K. Hanson, and Miss Dolores Miller who so competently and forbearingly typed the manuscript.

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F.M.

J.M.

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# PART A

# **Overview for Managers**

THE PURPOSE of this book is to describe how mathematical and statistical methods may be used to reduce costs through improved production planning and inventory control systems. We shall be concerned not only with making decisions about aggregate production rates, work force sizes, and shipments over time, but also about order quantities, time of ordering, producing, and shipping of individual products.

In Chapter 1 we characterize the problems in this area and describe, in a general way, analytical approaches that may be used. In order to show that the methods are in fact applicable to live problems in production and distribution management, we also present several cases in which these or related approaches have been used. Throughout the chapter, we emphasize the role of the executive in the installation and effective use of quantitative methods in planning and control systems.

# Chapter 1

#### Problems, methods, and results

#### I-I The problem

This book deals with quantitative methods for making decisions. Although the mathematical methods themselves are general and can be applied in many different fields, they are presented here only as they are relevant to the problems of managing warehouse systems for distributing products and factories for manufacturing them. Since no book of this size can treat all such problems, we start by enumerating those which will receive attention.

#### Distribution warehouse decisions.

The manager of a distribution warehouse is concerned with ordering products to stock his warehouse so that they will be available when demanded by customers. We shall assume that the warehouse manager has relatively little control over the timing of orders; he responds to the orders as they are received and tries to anticipate future orders.

The amount of inventory kept on hand at the warehouse influences such costs as obsolescence, insurance, handling, damage, and interest on the investment. These inventory holding costs can be decreased by holding less inventory, but only at an increased risk of exhausting individual product inventories and consequent poor service to customers. Not only is there a danger that single sales will be lost when the warehouse cannot fill orders promptly, but service deficiencies may influence customers to continue to take their business elsewhere. The critical decision problem here is one of setting

a level of inventory that will give adequate customer service without excessive inventory holding costs.

Inventories cannot be adjusted instantly. A manager must place orders well in advance of the time when he will receive shipment. He has no alternative but to forecast the orders that will be received during this lead time and to reflect the forecast in his orders. Such forecasts are never perfect.

To complicate the matter further, the level of inventories depends on the quantity ordered. In general terms, the larger the shipment the larger the average inventory level and the higher the inventory holding costs. Shipping costs can be decreased by ordering full truckloads or carloads, but price discounts for large purchases must be weighed against the increased inventory that results from such purchases.

Complication arises when each shipment includes several different products. When the inventory is low for some products but high for others in common shipments, the warehouse manager must decide whether to continue with low inventories of some items or to order a shipment that will increase the inventory of all of the products.

All of these decisions must also take into account outside considerations such as working capital needs, and, where the warehouse is supplied by the company's factory, the requirement of maintaining an orderly pattern of production and distribution.

#### Factory warehouse decisions.

The manager of a *factory* warehouse faces similar decision problems, but with a few significant differences. If the factory warehouse supplies distribution warehouses, all under common ownership, factory failure to make a shipment promptly in response to an order may cause a distribution warehouse to lose sales, or may force a costly cross-shipment from another distribution warehouse. Here, too, the costs for each product of holding inventory must be balanced against the costs of having too little inventory and suffering stockouts. But operations of the factory warehouse must be coordinated not only with distribution warehouse operations but with manufacturing as well.

#### • Factory production and employment decisions.

When the total orders received by the factory fluctuate widely, a choice must be made between fluctuations in production and factory inventory levels. Wide swings in inventories may require rental of additional storage space when inventories are high, and cause stockouts when they are low. Wide swings in production require either overtime when production is high and idle time when production is low, or hiring when production is increased and layoffs or transfers when production is decreased. This choice occurs