The Theory of Particle Interactions

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Originally published as *Theoriia vzaimodeĭstviĭ chastits* © 1986 by Nauka.

Library of Congress Cataloging-in-Publication Data

Belokurov, V. V. (Vladimir Viktorovich)
[Theoriia vzaimodeĭstviĭ chastits. English]
Theory of particle interactions/V. V. Belokurov, D. V. Shirkov.
p. cm.--(Translation series)
Translation of: Theoriia vzaimodeĭstviĭ chastits.
Includes bibliographical references.
ISBN 0-88318-715-9
1. Nuclear reactions. I. Shirkov, D. V. (Dmitriĭ Vasil'evich) II. Title.
III. Series: Translation series (New York, N.Y.).
QC794.B43413 1990
539.7'5--dc20 89-18613
CIP

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Preface to the American Edition

The theory of quantum fields, which provides the framework for a theoretical description of microparticles and their interactions, is the most advanced chapter of a modern theoretical physics.

The key element of this theory is the notion of a quantum field. From the present viewpoint, it represents the universal form of matter, underlying all its physical manifestations. One quantum field describes all particles (and antiparticles) of a given sort in the universe. The interaction of the particles is represented by the interaction of several fields of one or several species at a common space-time point.

The way quantum fields interact is determined by the symmetries existing in nature. This, the so-called gauge dynamics principle and its realizations for the known types of interactions, are explained in detail in the book. The gauge principle essentially simplifies the inner generalizations of the theory which are also discussed in the book.

The ideas and results of quantum field theory play more increasing roles in other parts of physics, such as astrophysics, cosmology, and nuclear physics. Also, many theoretical methods that had been created there were successively applied to the other diverse parts of theoretical physics. In this context, quantum field theory is a frontier part, both physically and methodologically, of the whole of theoretical physics.

This book can be recommended as the first reading for those students and physicists of various specializations to comprehend the ideas and the main methods and results of modern quantum field theory—the most abstract and mathematical branch of theoretical physics.

The authors also hope that the readers of this book will form a more complete notion of the results in this field obtained by Soviet physicists.

During the last several years, some more experimental proofs of the theories of fundamental interactions described in the book were obtained. Currently, the most significant theoretical results are associated with the superstring paradigm. However, they are far from being tested experimentally.

Preface to the First Edition

"To inflict learning gently, avoiding bloodshed as far as possible." It is this principle, formulated by a great Russian satirist, which we have attempted to follow in describing the historical development and current status of quantum field theory—the most abstract and mathematical branch of physics.

During the nearly half-century of its existence, the outward appearance of elementary particle theory has changed several times. Its history can conveniently be divided into three stages. The first 30 years were characterized by the progressive development of quantum electrodynamics, from the fundamental studies of the 1920s to the creation of the general theory of renormalization and the renormalization group method by the mid-1950s. The second period, which lasted about 15 years, was distinguished by the appearance of various theoretical schemes and approaches providing alternatives to local quantum field theory, whose development had stalled owing to difficulties encountered in applying it to the weak and strong interactions. At this time the terms "elementary particle theory" and "quantum field theory" had different meanings for most physicists. In the early 1970s the situation began to reverse quite rapidly. Quantum field theory, augmented by the dynamical principle of local gauge symmetry, confidently came to the fore, taking a firm position not only in electrodynamics, but also in the theories of the weak and strong interactions. The developmental spiral of the theory of particles and their interactions therefore came full circle, and in doing so reached a new qualitative level.

The rehabilitation of local quantum field theory on a new, gauge foundation, a triumph completed in the early 1980s, occurred relatively rapidly. This, together with the use of new, unobservable physical concepts (such as the quantum number "color," and quarks and gluons), hinders its assimilation by physicists of other specializations.

The present book originated in a brief series of lectures presented a number of times at several universities by one of the authors to a broad, but well-qualified audience of physicists. The original material of the lectures was expanded considerably in the writing of this book. This text should not be viewed as a detailed chronicle of the achievements of elementary particle theory. The authors want to highlight and at the same time comprehensibly sketch only the principle stages of its development, evaluated as such on the basis of their importance for understanding the current state of the theory. Therefore, the aim of the book is to help physicists who are not specialists in this field comprehend a new set of ideas. We have not even attempted to reflect all the contributions of the various scientists involved. We mention only a few of them by name, and give their initials only at the first mention.

In a text of this type it is impossible and, apparently, simply irrational to completely avoid the use of mathematical concepts and equations. However, the authors have tried to keep them at a minimum, and all concepts which might be unfamiliar to a substantial fraction of readers are accompanied by the necessary explanations.

At the end of the book we give a short list of relatively accessible literature recommended for further study of the topics discussed here.

We are grateful to many of our colleagues for their help and useful discussions. Particular mention should be made of A.M. Baldin, D.I. Kazakov, M.G. Meshcheryakov, V.I. Ogievetskiĭ, A.V. Radyushkin, A.A. Slavnov, and A.N. Tavkhelidze, whose advice was used in writing several sections. We would also like to thank the reviewer, L.B. Okun', for his careful reading of the manuscript and generous remarks and advice, most of which we have incorporated into the text.

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Chapter 1
Basic concepts

1. Particles and interactions

The region and objects under study

Elementary particle physics is the study of the laws governing matter at distances of the order of and smaller than the nuclear scale ($r \lesssim 10^{-13}$ cm). The phenomena occurring at such small distances, comparable to the sizes of the particles themselves, are studied by a special method: particle collisions. Here it is necessary to accelerate the colliding particles to very high energies. This field is therefore also referred to as high-energy physics.

Particle physicists usually use the system of units in which $\hbar = c = 1$. In this system energy, momentum, and mass are measured in the same units. The units used at various energy scales are based on the electron volt (eV):

$$1 \text{ MeV} = 10^6 \text{ eV}, 1 \text{ GeV} = 10^9 \text{ eV}, 1 \text{ TeV} = 10^{12} \text{ eV}$$
$$(1 \text{ GeV} = 1.602 \times 10^{-10} \text{ J} = 1.782 \times 10^{-24} \text{ g}).$$

Similarly, lengths and times are measured in identical units. In this system the units of energy and length are not independent, but are expressed in terms of each other; in particular, $1 \text{ GeV}^{-1} \approx 2 \times 10^{-14}$ cm.

At present, a very large number of particles are known.

Particles of spin 1/2 which participate only in electromagnetic, weak, and gravitational interactions (see the end of this section) fall into a special class, that of the so-called leptons. The electron, muon, τ lepton, and their three associated neutrinos are leptons.

The hadrons—particles which also participate in the strong interaction—form a large group. Nucleons (the proton and neutron), mesons, baryons, and resonances are hadrons. The current view is that hadrons are composite particles composed of more fundamental objects called quarks.

Particles such as the photon, the intermediate W and Z bosons, and the so-called gluons act as carriers of interactions.

The "elementary" nature of particles

As higher energies become attainable experimentally, i.e., as it becomes possible to study phenomena at ever smaller distances, the very concept of what is an elementary particle changes. Of the particles listed above, the photon, the leptons, and the quarks are currently considered to be elementary (structureless). Later on we shall add a few more particles to this list—the interaction carriers: the gluons and intermediate bosons. Of course, it is not impossible that further study will reveal that the particles presently assumed to be elementary also have structure. Consequently, the terms "elementary particle" and "elementary particle physics" can be ambiguous, depending on the context. Therefore, when using these terms we shall avoid the adjective "elementary" as far as possible.

Particle characteristics

The fundamental particle characteristics are the rest mass, the spin (intrinsic angular momentum), electric charge, lifetime, and a few other quantities. The fact that a particle has a spin and a definite lifetime is a manifestation of its quantum properties. We will discuss these a bit later. Let us first recall some details about masses and charges.

The mass m, energy, and momentum of a particle are related by the familiar relativistic expression $E^2 - c^2\mathbf{p}^2 = m^2c^4$. As mentioned above, the mass and momentum are usually measured as energies. For example, the electron mass is $m_e = 9.110\times10^{-28}$ g $= 0.5110$ MeV and the proton mass is $m_p = 1.673\times10^{-24}$ g $= 0.9383$ GeV. Of the particles currently known, the intermediate W and Z bosons have the largest mass (~ 100 GeV). The masses of the photon and a few other particles are zero.

Particles also possess a set of discrete characteristics—the so-called internal quantum numbers. The most familiar of these is the electric charge, which for all the observed particles is a multiple of the electron charge $e = 4.803\times10^{-10}$ esu $= 1.602\times10^{-19}$ C.

The internal quantum numbers include baryon and lepton number, parity, and also the quark flavors—characteristics like isospin, strangeness, etc., determining the type of quark (and, accordingly, the type of hadron containing that quark). Moreover, quarks of the same flavor can differ by another quantum number called color. These characteristics will be discussed in more detail in Sec. 8.

Internal quantum numbers are introduced to formalize the regularities observed experimentally in processes involving particles. For example, the absence in nature of some process can be represented as a consequence of the existence of a certain quantum number whose conservation forbids that process.

Quantum properties of particles

Spin, the intrinsic angular momentum of a particle unrelated to its motion in space, has a quantum origin.

According to quantum mechanics, particle spins can take only discrete values—half-integer multiples of Planck's constant $\hbar = 1.054\times10^{-27}$ erg·s $= 6.582\times10^{-22}$ MeV. For example, the spin of the π and K mesons is zero, the spin of the neutron, proton, electron, muon, and neutrino is 1/2 in our units, and the spin of the photon, gluons, and W and Z bosons is 1. Particles of higher spin also exist.

The nonzero spin of a particle selects a certain direction in space. The angular characteristics of any process therefore depend on the spins of the particles involved in it. In practice, this makes it possible to determine the values of particle spins by analyzing the angular characteristics of processes.

Particles are divided into two classes depending on their spin: particles with integer (in units of $\hbar$) spin and particles with half-odd-integer (1/2, 3/2,...) spin. The laws governing the behavior of systems of particles of these two classes are different. Let us consider this in more detail. The quantum mechanical description of a system of particles is based on the principle of indistinguishability, according to which only quantum states which do not change under permutations of identical particles are realized in nature. The principle of indistinguishability requires that physical characteristics be invariant under interchanges of identical particles. From this it follows that the wave function of a system of identical particles is symmetric or antisymmetric under such an interchange, i.e., either it is not changed when the particles of any pair are interchanged, or it is multiplied by -1. The former occurs for systems of particles with integer spin, and the latter for those with half-odd-integer spin.

The requirement that the wave function of a system of identical particles with half-odd-integer spin be antisymmetric is known as the Pauli principle. When interactions are neglected, the Pauli principle implies that no more than one of a given type of particle can exist in each quantum state.

The difference between the symmetry properties of wave functions is related to the fact that systems of particles with integer and half-odd-integer spins obey different laws of statistics, referred to as Bose–Einstein and Fermi–Dirac statistics, respectively. Therefore, particles with integer spin are called bosons, and those with half-odd-integer spin are called fermions.

In Bose–Einstein statistics the statistical weight of any possible state of a system of N particles is 1. In Fermi–Dirac statistics the statistical weight can have two values: 1 if no occupation number exceeds unity, and 0 otherwise. We recall that the occupation numbers are the sets of numbers $N_1,\ldots, N_s,\ldots$, specifying how many particles of a given type occupy the individual states labeled 1,..., s,...; $\Sigma N_i = N$.

The Pauli theorem gives a unique relation between the spin and the type of statistics. This theorem is fundamental, i.e., it is a direct consequence of the principles on which present-day quantum field theory is based.

Another manifestation of the quantum properties is the fact that most of the known particles live only a finite time before decaying. The term "decay," borrowed from nuclear physics, does not refer to the breakup of a particle into its constituents, but to its transformation into a set of different particles. One of the first decay processes known was that of neutron β-decay, in which the neutron is transformed into three particles: a proton, an electron, and an electron antineutrino, i.e., $n \to p + e^- + \bar{\nu}_e$.

Like any quantum process, particle decay is probabilistic in nature. Its probability depends exponentially on the time t: $w = 1 - \exp(-t/\tau)$. The parameter τ, which has the dimensions of time and characterizes the rate of this process, is referred to as the particle lifetime. Of course, like any probabilistic quantity, it is not a feature of an individual particle, but rather a certain average for a large set of particles.

Particle lifetimes differ widely. For example, for the neutron $\tau \approx 10^3$ s $\approx$ 15 min, while for the shortest-lived particles, the so-called resonances, it is 10^{-23}–10^{-24} s. A large group of unstable hadrons—mesons and baryons—have lifetimes intermediate between those of neutrons and resonances.

The difference between particle lifetimes is in part related to the strength of the interactions responsible for the decays. For example, neutron decay is caused by the weak interaction, and resonance decays by the strong interaction.

With few exceptions, almost all the known particles decay. Only the electron, proton, photon, and neutrions are absolutely stable, i.e., have practically infinite lifetime in the free state, within the limits of accuracy of the present experimental data. (We shall discuss searches for proton decay in Sec. 23.)

Interconvertibility of particles

The particle instability discussed above is a manifestation of the most important feature of the microworld—particle interconvertibility. This means that particles and sets of particles, as a rule, undergo transitions into other particles and sets of particles if such transitions are not forbidden by any conservation laws.

The interconvertibility of particles provides the basis for their interaction mechanisms. Delaying the detailed discussion of this problem until the next section, here we only note that, according to the current point of view, any particle interaction process can be represented as a set of elementary events with certain particle transformations occurring in each.

Therefore, in the microworld particle conversion processes are ordinary and necessary phenomena. Moreover, the absence of any transitions allowed by the known conservation laws is perceived as an indication that a new, as yet unknown conservation law exists.

The *conservation laws* for internal quantum numbers are, as mentioned above, a sort of formalization of experimentally observed regularities. The most familiar example is electric charge conservation. Also, all the presently existing experimental data are consistent with baryon and lepton number conservation (see Sec. 23).

At the same time, certain regularities are observed in processes of a particular type, but not in others. In such a case the conservation laws for the quantum numbers introduced are not general, but have a limited region of validity. Examples of such quantum numbers are the quark flavors, which are conserved in the strong interaction but not in the weak. Another example is parity, which is always conserved except in weak interaction processes.

As is well known, fundamental laws such as energy, momentum, and angular momentum conservation are a consequence of symmetries in space-time. By analogy, the conservation of internal quantum numbers can be associated with symmetries in certain abstract (internal) spaces. It is clear from the above discussion that most internal symmetries are approximate. Except for the symmetries associated with electric charge and baryon and lepton number conservation, and also the special quark "color" symmetry, internal symmetries are present in certain interactions and are violated in others. In the next chapter we shall discuss the role of approximate symmetries in particle physics in more detail.

Types of interactions

At the present time it is thought that all the various types of interactions between matter reduce to four types of fundamental interaction between particles: the strong, weak, electromagnetic, and gravitational interactions.

As already mentioned, the strong interaction involves a large group of particles, which are collectively referred to as hadrons. This interaction occurs at distances smaller than the characteristic nuclear scale $r_{\text{nucl}} \sim 10^{-13}$ cm, which is close to the Compton wavelength of the π meson. For example, the strong interaction is responsible for the nuclear forces binding protons and neutrons to form nuclei. This force is termed "strong" because it leads to large quantitative effects in the microworld. As a rule, the cross sections for strong interaction processes are close to the geometrical value ($\sim 10^{-26}$ cm^2) determined by the square of the interaction range and corresponding to the collision of spheres of radius 10^{-13} cm. At short distances ($\lesssim r_{\text{nucl}}$) the strong interaction is large in absolute value and considerably stronger than, for example, the electromagnetic interaction. The average binding energy of nucleons in a nucleus is of order 10 MeV per nucleon, which is one million times greater than the typical energy of the electromagnetic coupling of the outer electrons to a nucleus (~ 10 eV). Another way of comparing the interactions is to compare the dimensionless numerical characteristics describing the interaction strengths. A measure of the strength of the electromagnetic interaction is the so-called fine structure constant $\alpha = e^2/\hbar c = 1/137$. Strong interactions are traditionally characterized by the pion–nucleon interaction constant g. The numerical value of the dimensionless combination $g^2/\hbar c$ analogous to the fine structure constant is 14.7, which is several orders of magnitude greater than α. The current view is that the hadron interaction is not fundamental, but is the consequence of the elementary interaction between the quarks of which hadrons are composed. Chapter 6 is devoted to describing the theory of this quark interaction.

A typical example of the weak interaction is neutron β decay and the resulting radioactive decay of atomic nuclei. Another example is muon decay ($\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$). Both hadrons and leptons participate in weak interactions. At energies $\ll 100$ GeV the interaction strength depends significantly on the characteristic energy of the process and increases as a power of the energy. At fairly low energies the interaction effects are actually quite weak. For example, at energies on the order of the nucleon mass (~ 1 GeV) the dimensionless quantity $G_F M_N^2$ characterizing the interaction strength is

10^{-5} (here G_F is the Fermi constant). However, at ~ 100 GeV weak interactions are comparable in strength to electromagnetic interactions. We shall consider these questions in more detail in Chap. 5 (Sec. 17), where we discuss the current theory of the weak and electromagnetic interactions.

Electrically charged particles participate directly in electromagnetic interactions. These interactions are responsible for the binding of atomic electrons to nuclei and the binding of atoms to form molecules.

The gravitational interaction is universal. It occurs between all forms of matter and between all particles. However, at the distances corresponding to the energies attained in particle interactions gravitational effects are negligible compared to the effects of the other interactions. In particular, the gravitational constant G_N (Newton's constant) in units where $\hbar = c = 1$ is 33 orders of magnitude smaller than the Fermi weak interaction constant G_F. Another illustration of the weakness of the gravitational force is the fact that the gravitational attraction between two protons is about 10^{37} times smaller than the strength of their electromagnetic repulsion. However, gravitational interactions dominate for macroscopic objects which overall are electrically neutral.

Both the gravitational and the electromagnetic interactions are long-range. Their strength falls off slowly at large distances ($\sim r^{-1}$), so it is these interactions which play the dominant role in macroscopic physics. Even on the atomic scale ($\sim 10^{-8}$ cm) the electromagnetic force is considerably stronger than the nuclear force.

2. Quantum fields

A powerful theoretical apparatus—quantum field theory—has been created for describing particle dynamics. The fundamental physical object in this theory is the quantum field, the unique synthesis of the concepts of the classical field as in electromagnetism and the probability field of nonrelativistic quantum mechanics. It is currently believed that this field is the most fundamental and universal form of matter on which all the physical manifestations of matter are based.

The classical field

This concept encompasses all physical quantities $u(x)$ depending on the spatial coordinates and time [x is a point in spacetime: $x = (\mathbf{x},t)$]. A special role is played by the wave fields involved in the transmission of interactions between particles. A typical example is the Faraday–Maxwell electromagnetic field describing the interaction of electrically charged particles.

We note that the field concept arises naturally when one attempts to avoid the idea of instantaneous action-at-a-distance between particles—an idea which is inconsistent with special relativity. Assuming that the space between particles is filled by a field, we assign to this field the function of transmitting a perturbation from one particle to another with finite velocity. Thus, the introduction of the classical field in physics is dictated by considerations of relativistic invariance.

Viewing the field as a mechanical system with an infinite number of degrees of freedom, we can construct the Lagrangian for this system and, using the variational principle of stationarity of the action, obtain the equation for the field function $u(x)$. Here dynamical invariants such as energy, momentum, and angular momentum are formed like the corresponding quantities in classical mechanics and are expressed in terms of $u(x)$ and its derivatives.

From the methodological point of view, instead of a continuous distribution it is useful to first consider a discrete mechanical system having a countable number of degrees of freedom, and then go to the continuum limit. This is conveniently done using the discrete momentum representation, which is obtained by confining the field to a spatial cube of volume $V = L^3$ and imposing the condition of periodicity with period L on each spatial coordinate. Then, as is well known, the Fourier integrals are replaced by series and the field will be determined by a denumerably infinite set of quantities $\varphi(\mathbf{k})$, where $\mathbf{k} = (2\pi/L)\,n_1, (2\pi/L)\,n_2, (2\pi/L)\,n_3$ and n_1, n_2, and n_3 take integer values from $-\infty$ to $+\infty$. In the case of free fields, this can be regarded as an expansion of the field in an infinite sum of independent harmonic oscillators, each characterized by its frequency $\omega(\mathbf{k})$ and amplitude $\varphi(\mathbf{k})$. The continuum limit is reached by allowing L to go to infinity.

Quantization

The quantization of a field is essentially the quantization of each of these oscillators. As is well known from quantum mechanics, the oscillator energy can take a discrete set of values:

$$E_n(\mathbf{k}) = \hbar\omega(\mathbf{k})\cdot(n + 1/2), \quad n = 0,1,\dots .$$

The uniform spacing of the energy levels, i.e., the linear dependence of the energy on the state number, provides the basis for the corpuscular treatment of the theory. An oscillator with frequency $\omega(\mathbf{k})$ in the nth excited state can be represented as a set of n indistinguishable particles—excitation quanta, each of energy $\hbar\omega(\mathbf{k})$.

The number of particlelike excitations can obviously be arbitrary, which makes it possible to construct a quantum description of a system with a variable number of particles. Here the field functions become operators which change the number of particles in the system.

More precisely, each quantum field $u_i(x)$ is written as a linear combination of creation operators $u_i^+(k)$, describing the creation of a particle of mass m_i and 4-momentum k, and annihilation operators $u_i^-(k)$, corresponding to the annihilation of such a particle. The creation operator $u^+(k)$ acting on a state with energy–momentum p containing N particles transforms it into a state with $N+1$ particles and energy–momentum $p + k$. The annihilation operator $u^-(k)$ decreases the energy–momentum and particle number of the state in the analogous manner.

Therefore, as a result of quantization the field functions become operators (in the quantum mechanical sense). These operators act on a wave function Φ common to all the fields. In conformity with ordinary quantum mechanics, the wave function Φ completely characterizes the physical state of a system described by quantum wave fields. Like the usual function Ψ, the function Φ can be viewed as a vector in some linear space. It is therefore referred to as a state vector. The expectation values of dynamical quantities and transition probabilities are expressed in terms of quadratic forms of Φ, so Φ is also referred to as the amplitude of the state. The vacuum state plays a special role in this formalism. Its amplitude is denoted by the symbol Φ_0. By definition, there are no particles in this state, it has zero momentum, the minimum energy, and the annihilation operator of any field applied to it gives zero:

$$u^-(k)\Phi_0 = 0.$$

The creation operator $u^+(k)$ acting on Φ_0 gives a one-particle state. A state containing N particles can be constructed by allowing N creation operators to act on the vacuum state:

$$\Phi_N = u^+(k_1)...u^+(k_N)\Phi_0.$$

When the number of particles is not fixed, the amplitude of the state is written as a linear superposition of the amplitudes Φ_N with certain weight functions:

$$\Phi = \sum_{N=0}^{\infty} \chi_N \Phi_N.$$

This is the Fock representation of the state amplitude.

Quantum fields

We have shown how the quantum field arises as a result of quantization of the classical field. For example, upon quantization the potentials $\mathbf{A}$ and φ of the classical electromagnetic field and the field strengths $\mathbf{E}$ and $\mathbf{H}$ become quantum operators which operate on the quantum mechanical wave function of the physical system. Linear combinations of these field operators are the creation operators for quanta of the electromagnetic field—photons. At the same time, the operator potentials and field strengths satisfy the usual equations of motion of electrodynamics—Maxwell's equations. In this manner we arrive at the quantum electromagnetic field, which combines the wave and corpuscular features of electromagnetism.

The concept of the quantum field can also be reached starting from particles. Quantum mechanics gives a probabilistic description of the behavior of a particle, for example, an electron, by means of a wave function satisfying the Dirac equation (see Sec. 4). The extension of these ideas to systems containing several particles of a given type (for example, electrons) is conveniently formalized by the introduction of an operator wave field written in the occupation number representation by means of creation and annihilation operators. This operator field is completely analogous to the quantum electromagnetic field. The procedure of transforming to operators of the occupation number representation is usually referred to as second quantization. It should be clear that no "second quantization" is actually involved here, so this term is not only inappropriate, but also confusing.

Therefore, the concept of quantum field contains the organic synthesis of the wave and corpuscular properties of matter. The quantum field should be viewed as a single fundamental object replacing the fields and particles of classic physics. One such field specified at each point in four-dimensional spacetime describes all the particles of a given type in the universe.

The interaction of quantum fields

In order to obtain linear equations of motion (the usual type of equation for free fields), we must use a Lagrangian, i.e., density of the Lagrange function, which is bilinear in the field functions and their derivatives. When interactions are present the equations are no longer linear, and the Lagrangian contains products of three and more fields. In a local quantum field theory it is assumed that the field functions involved in such products are evaluated at a single point in spacetime.

Since the field functions are expressed in terms of creation and annihilation operators, the elementary event in any interaction is the creation of certain particles and the annihilation of others at the same point. This appears as a local and instantaneous transformation of certain particles into others.

At this point we must make the following important observation. In classical physics a free particle can neither emit nor absorb another particle while remaining unchanged (for example, in classical physics a free electron can neither emit nor absorb a photon), since the laws of energy and momentum conservation will not be simultaneously satisfied in such processes. The situation is different in quantum theory. According to the uncertainty principle, the time interval Δt and spatial interval Δx in which a particle is located are related to the spreads in the possible energies ΔE and momenta Δp_x by the inequalities $\Delta t \cdot \Delta E \geqslant \hbar$ and $\Delta x \cdot \Delta p_x \geqslant \hbar$. Therefore, if the emitted particle exists for a small time interval, the usual relativistic relation between its energy, momentum, and mass will not be satisfied. Such particles are encountered only in intermediate states and are called virtual particles. It is easily verified that their emission or absorption is consistent with the laws of energy and momentum conservation. In particular, a free electron can emit a virtual photon whose energy and momentum satisfy the relation $E^2 - \mathbf{q}^2 < 0$.

We note that the uncertainty relations have another important consequence. Although the interaction is assumed to be local, it is impossible to specify the actual point in spacetime at which an elementary event occurs,