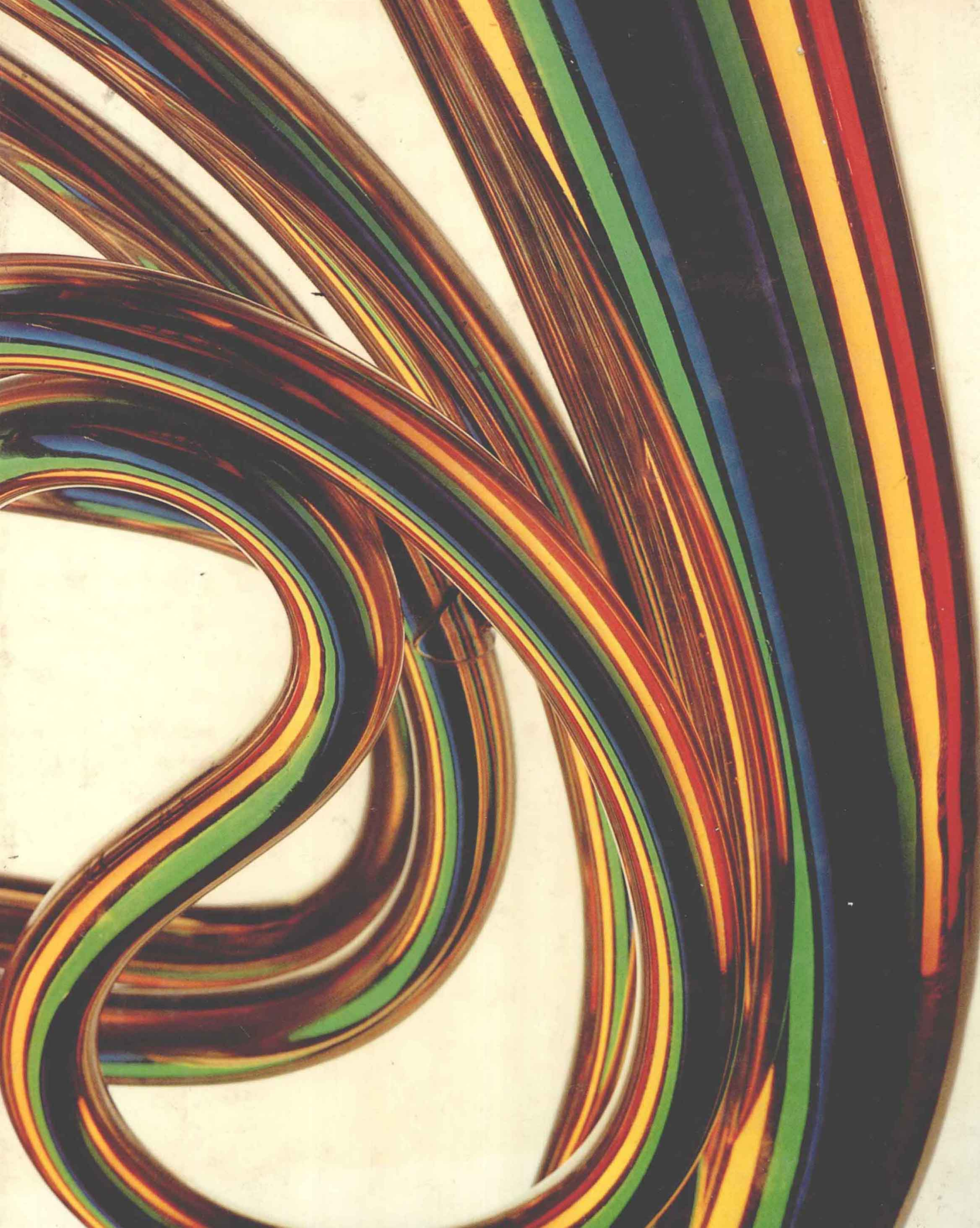

Calculus

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Calculus

organized into two successive chapters, with the simpler notions of related rates and extrema on closed bounded intervals (Chapter 4) followed by the more general discussions on increasing and decreasing functions, relative extrema, asymptotes, and graphing techniques (Chapter 5). The applications of the definite integral are split into two chapters as well (Chapters 7, 8), although the rationale for this is simply to break up what otherwise would be a long and less than coherent chapter. The unit on infinite series (Chapters 12–14) may be postponed without consequence.

ANTIDERIVATIVES, INTEGRALS, AND DIFFERENTIAL EQUATIONS: The organization of Chapters 5 and 6 reflects a concern I have held for some time, that the definite integral is often presented to students almost simultaneously with the notion of an antiderivative and the statement of the Fundamental Theorem of Calculus. As a result, students can leave their first calculus course thinking of a definite integral only as a difference between two values of an antiderivative that, coincidentally, might also be identified with the area of a certain planar region. I have addressed this concern in two ways. First, antiderivatives are introduced as one of the applications of the derivative, followed by a section on separable differential equations. This serves to introduce and apply the notion of antidifferentiation as a topic that is independent of the definite integral. Second, the definite integral is introduced as the limit of approximating sums, which result from a discussion of approximating planar regions by rectangles. This approach should help the student to grasp more quickly the notion of approximate integration (including the role computers can play), as well as making the development of the various applications of the integral more accessible. Further topics on differential equations appear in optional sections at the ends of chapters throughout the text, rather than in a single final chapter, in the hope that they will be included at points in the development where they occur naturally.

EXERCISES: More than 6,000 exercises are included, ranging from drill to challenging in type, and including many applied exercises from a broad range of disciplines. The extensive review exercises at the end of each chapter reflect the range of topics included in that chapter.

While many theorems, particularly those with instructive proofs, can and should be presented and proved, the time available to most of us for this task is not sufficient to allow a careful treatment of the least upper bound axiom, uniform continuity, differentiability of power series, or several other topics where statements of fact must simply be made. Honesty about these omissions, together with the right picture here and a good heuristic discussion there, can result in a presentation that is both factual and effective, and one that allows us to succeed in sharing with students the excitement of the triumphs of this classic subject.

ACKNOWLEDGMENTS: Many individuals played instrumental roles in the development of this text. It is my pleasure to acknowledge some of these here.

Twenty-seven teachers of the calculus scrutinized one or more drafts of the manuscript, both on matters of content and to identify errors. These were

Paul Baum, Brown University
David Bellamy, University of Delaware
George Blakley, Texas A&M University
Jan List Boal, Georgia State University

PREFACE

This text is intended for use in a traditional three-semester or four-quarter sequence of courses on the calculus, populated principally by mathematics, science and engineering students. It reflects my own philosophy that freshman calculus should be taught so as to produce skilled practitioners who have some real feeling for the mathematical issues underlying the techniques they have acquired. It was written in order to provide students of widely varying interests and abilities with a highly readable exposition of the principal results, including ample motivation, numerous well-articulated examples, a rich discussion of applications, and a nontrivial description and use of numerical techniques. In particular, I have tried to indicate, in an unobtrusive manner, how computers can be used both to illustrate the theory and to provide approximate solutions to problems for which more elegant techniques break down.

LEVEL AND RIGOR: Nearly all topics in the traditional calculus curriculum are treated. However, the discussion is informal, and geometric arguments are used wherever possible. I do not prove that a continuous function assumes its extreme values on a closed bounded interval. Limits are first presented quite informally in the context of the tangent line problem. The more formal δ-ϵ definition appears briefly at the end of Chapter 2, along with formal proofs of several limit theorems. When the topic of limits appears later in the chapters on infinite series, it is treated more formally, reflecting the increased maturity of the reader. The notion of differentiability for a function of several variables is discussed following the development of partial differentiation and the gradient, and the theorems of Green and Stokes, as well as the Divergence Theorem, are fully discussed.

STRATEGY/SOLUTION FORMAT IN EXAMPLES: An unusually large number of examples (more than 700) has been included. In many of these, especially in the early parts of the text, the solutions have been written in a two-column format, with one of the columns labelled ''strategy.'' In this column the student will find, in very abbreviated form, a description of the principal steps involved in the fuller solution. Here I have attempted to help students identify the more general aspects of the particular solution, and to develop problem-solving strategies of their own.

ORGANIZATION: The order of topics is consistent with those of most popular texts. The trigonometric functions are introduced as part of the review material in Chapter 1, and are used frequently throughout the text. Applications of the derivative are

Alan Candiotti, Drew University
George Feissner, SUNY at Cortland
Herbert Gindler, San Diego State University
Arthur Goodman, Queen's College, CUNY
Douglas Hall, Michigan State University
Peter Herron, Suffolk Community College
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William Wheeler, Indiana University
Robert Zink, Purdue University

Professor Paul Baum checked both the final manuscript and the exercise solutions for accuracy, and prepared the solutions manual. Professors Barry Granoff, of Boston University, and Richard Porter, of Northeastern University, read the entire text in galley form, rechecking all examples, exercises, and mathematical content. Professor Granoff also reviewed the artwork as it came from the artist, and scrutinized all page proofs. Each of these individuals worked meticulously to ensure the accuracy of the text. Whatever errors might remain are, of course, the sole responsibility of the author. Any comments on correcting or improving the text will be gratefully acknowledged.

At Boston University I am indebted to Tom Orowan for typing near-perfect drafts of the manuscript, and to Lisa Doherty for managing the flow of materials between Boston and Philadelphia. The BASIC programs included in the appendix were used by students on the University's IBM 3081 based time-sharing system, as well as on the author's personal computer. The computer-generated graphs of quadric surfaces were produced by the Graphics Laboratory of the University's Academic Computing Center.

Forewarned about hazards in dealings with publishers, I was delighted to experience a warm, professional, and highly supportive relationship with each of several key individuals at Saunders College Publishing: Mathematics Editor Leslie Hawke, Developmental Editor Jay Freedman, Project Editor Sally Kusch, and Publisher Don Jackson. Their commitment to excellence was a strong guiding force throughout the development of this text.

The historical notes, which provide an important human contrast, were written by Professor Duane Deal of Ball State University.

On a more general and personal level, the writing of this text was supported by three very special groups of people. First, my students at Boston University, who

have encouraged this project and helped sharpen my thinking about teaching and the calculus for the past decade. Second, my colleagues in the faculty and administration of Boston University, who really do believe in the importance of effective teaching. And, most importantly, my family, who understood my need to write this book and shared fully and willingly in the sacrifices that were required. To all I am truly grateful.

Dennis D. Berkey

Boston, Massachusetts


Isaac Newton


Gottfried Leibniz

UNIT 1

PRELIMINARY NOTIONS

1
Review of Precalculus Concepts

2
Limits of Functions

Newton and Leibniz—The Unification of Calculus

More than once in history, a significant mathematical development has been independently discovered by two mathematicians widely separated by geography and having no contact with one another. Perhaps the greatest of these discoveries was that of the calculus. Isaac Newton (1642–1727) in England, and Gottfried Wilhelm Leibniz (1646–1716) in Germany, were completely unaware of each other's work. Newton actually developed his version of calculus some ten years before Leibniz, but Newton was generally reluctant to publish his results, and Leibniz presented his own version to the world about twenty years before Newton's first publication. These time differences ultimately resulted in an extended controversy between English and Continental mathematicians concerning priority and the possibility of plagiarism. To their credit, however, Newton and Leibniz did not attack one another.

Isaac Newton was still an undergraduate at Cambridge University when he essentially created differential calculus, or *fluxions* as he named derivatives. When bubonic plague swept over England in 1665–66, the university was closed for nearly two years, and Newton spent the time at his home. In this fruitful period, he developed differential and integral calculus, made the first observations that culminated in his theory of gravitation, made his first experiments in optics and the theory of color, and stated the binomial theorem for general exponents.

Although the binomial theorem for positive integral exponents had long been known, no one had thought to apply it to negative and fractional exponents before Newton. His discoveries concerning the infinite series generated by such exponents greatly intrigued him, and led to his using infinite series as a basis for expressing functions for his calculus. From this time on, the use of infinite processes in mathematics came to be considered legitimate. Newton himself thought infinite series and rates of change to be inextricably linked, and referred to them together as "my method."

Newton's ideas came from considering a curve as the result of a continuously moving point. The changing quantity he called a *fluent,* and its rate of change he named a *fluxion* (from the Latin *fluere,* to flow). The fluxion was thus what we shall refer to as a derivative, denoted $\dot{y}$ if y were the original fluent. Similarly, Newton saw that the original fluent y could be thought of as the fluxion of another function, designated $\overset{\square}{y}$ or $\overset{|}{y}$. The theory of the integral, which we shall develop in Unit III, explains Newton's claim that the fluent is the integral of the fluxion.

For some twenty years Newton made significant discoveries in many fields, but then the light of creative genius flickered. He suffered a long illness that affected his ability and, while he continued to work, the results were not of the quality of his earlier period. He served in Parliament and was appointed Warden (and later Master) of the Mint, in which administrative capacity he spent his last quarter century checking the quality of the metal in British coins.

Gottfried Leibniz was the son of a university professor who died when the boy was only six years old. Young Leibniz had access to his father's extensive library and read broadly. A brilliant scholar, he received his bachelor's degree at age 17, and his doctorate (in law) at 20. He then became a diplomat by profession, but soon became interested in mathematics as a consuming avocation. By the time Leibniz was 30 he had invented his calculus, which is in many ways *our* calculus. About 1673 he came to the realization that areas can be calculated by *summing* "infinitely thin" rectangles, and that tangents to curves involve *differences* in the x and y coordinates. This led him to suspect that the two processes are inverses of one another. In doing these calculations, he soon found himself immersed in infinite series representing functions, as had Newton some years before. Realizing the power of his discoveries, Leibniz set to work developing terminology and notation. He selected the integral sign $\int$ as representing the Latin word *summa,* or sum, and the derivative notation $\dfrac{dy}{dx}$.

Leibniz's, and the world's, first paper on differential calculus was published in 1684 with the title (translated) *A New Method for Maxima and Minima, and also for Tangents, which is not Obstructed by Irrational Quantities*. He gave formulas for differentials of powers, products, and quotients of functions—the same formulas we learn today. Two years later he published his integral calculus, in which quadratures, or the calculation of areas, are shown to be the inverse operation of finding tangents.

Newton and Leibniz both came to realize that the calculus is a much more general tool than merely a method for finding tangent lines or areas under curves. It was this generalization that made them such important people in the development of mathematics. Others had differentiated before them, had found areas, and had even realized to some extent the relation between the two operations. But these two, independently, unified the calculus and made it applicable to many fields, and to a much broader range of functions.

Newton was respected and revered in his old age. He was reelected President of the Royal Society of London for 25 years, and was knighted by Queen Anne in 1705. He was buried in Westminster Abbey with great honor. Leibniz, on the other hand, was neglected and became embittered before his death. Only one person, his secretary, was in attendance at his funeral.

Newton and Leibniz were both creative mathematicians of the first rank. The genius of neither is diminished by that of the other.

Pronunciation Guide

Isaac Newton (Eye′zak New′ton)

Gottfried Wilhelm Leibniz (Got′freed Vil′helm Libe′nitz)

(Photographs from the David Eugene Smith Papers, Rare Book and Manuscript Library, Columbia University.)

CONTENTS OVERVIEW

CONTENTS