Lectures on Current Algebra and Its Applications

Sam B. Treiman
Roman Jackiw
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FOREWORD

The three sections of this book comprise three lectures which we delivered at the Brookhaven Summer School in Theoretical Physics during the summer of 1970. "Current Algebra and PCAC" summarizes the "classical" results of the theory. The general methodology is introduced and then applied to derivations of various sum rules and low-energy theorems. In "Field Theoretic Investigations in Current Algebra" calculations are presented which have exhibited unexpected dynamical dependence of the predictions of current algebra. In models many predictions fail due to anomalies which are present as a consequence of the divergences of local quantum field theory, at least in perturbation theory. Finally "High-Energy Behavior of Weak and Electromagnetic Processes" concerns itself with the recent uses of current algebra for high-energy processes, especially the "deep inelastic" ones. Much of this development is motivated by the MIT-SLAC experiments on electron-nucleon inelastic scattering and represents the new direction for the theory.

For permission to reprint several figures that appear in the third section, we thank R. Diebold (Figure 11); F. Gilman (Figure 12); C. E. Dick and J. W. Mock (Figure 15, which originally appeared in Physics Review 171, 75 [1968]); E. Bloom et al (Figures 16 - 21, which originally appeared in Physics Review Letters 23, 935 [1969]); and I. Bugadov et al. (Figure 22, which originally appeared in Physics Letters 30B, 364 [1969]).

We wish to thank Dr. Ronald F. Peierls for giving us the opportunity of presenting these lectures at the Summer School. Mrs. Patricia Towey, and her secretarial staff, carried out the initial production of the manuscript with admirable efficiency for which we are most grateful.

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CONTENTS

CURRENT ALGEBRA AND PCAC by Sam B. Treiman

FIELD THEORETIC INVESTIGATIONS IN CURRENT ALGEBRA by Roman Jackiw

THE HIGH ENERGY BEHAVIOR OF WEAK AND ELECTROMAGNETIC PROCESSES by David J. Gross

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CURRENT ALGEBRA AND PCAC

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I. INTRODUCTION

The basic ideas for the subject of current algebra were introduced by Gell-Mann[1] as long ago as 1961. But the development proceeded slowly for several years, until 1965, when Fubini and Furlan[2] suggested the appropriate techniques for practical applications and Adler[3] and Weisberger[4] derived their remarkable formula relating β decay parameters to pion-nucleon scattering quantities. This inaugurated the golden age and the literature soon reflected what always happens when a good idea is perceived. In 1967 Renner[5] counted about 500 papers, and the number may well have doubled by now. Of course the number of really distinct advances in understanding is somewhat smaller than may be suggested by the counting of publications. Indeed, the major theoretical outlines have not changed much since 1967, by which time most of the "classical" applications had been worked out. During the last few years numerous variations on the earlier themes have inevitably appeared. But there is also developing an increasingly sophisticated concern about the validity of some of the formal manipulations that people have indulged in to get results. These interesting matters will be discussed by Professor Jackiw in his lectures at the Summer School.

On the experimental side the situation is still unsettled for a number of processes that may well be decisive for the subjects under discussion here. This is the case with $K_{\ell 3}$ decay, where there are lots of data but not all of it consistent; and with $K_{\ell 4}$ decay, where the data is still

The lecture notes were prepared in cooperation with Mrs. Glennys Farrar.

limited. As for the interesting and perhaps crucial applications to $\pi-\pi$ scattering and to high energy neutrino reactions, relevant experimental information is altogether lacking at present. The neutrino reactions in particular have been attracting a great deal of attention in recent times, and not only from the point of view of what current algebra has to say about them. This subject will be reviewed in the lectures to be given by Professor Gross.

The primary ingredients of current algebra are a set of equal-time commutation relations conjectured by Gell-Mann for the currents that arise in the electromagnetic and weak interactions of hadrons. As will be described and qualified more fully below, the Gell-Mann scheme may be taken to refer to commutators involving the time components of the currents. Various conjectures for the space-space commutators have also been suggested subsequently. It is especially in connection with the latter that possibly dangerous manipulations come into play in the applications. The dangers and the applications will however be left to the other lecturers. In any event, commutation relations imply sum rules on the matrix elements of the operators which are involved. But the matrix elements which arise are usually not physically accessible in a practical way. It therefore takes some inventiveness to extract physically useful results; and inevitably this requires approximations or extra assumptions. Tastes can differ here! But one might well wonder what is being tested when, say, an infinite sum is truncated arbitrarily at one or two terms chosen more or less for pure convenience. On the other hand one is likely to be more charitable to the rather discrete demand that a certain dispersion integral shall merely converge, where this doesn't contradict known facts or principles. Even at the very best, however, a high order of rigor is not to be expected. Still, some topics are cleaner than others; and, even though this encroaches on Professor Gross' subject, I shall want later on to discuss the Adler sum rule for neutrino reactions, as exemplifying an especially model insensitive test of the Gell-Mann conjectures.

It will be noticed in any listing of current algebra applications that processes involving pions make a disproportionate appearance. This comes about because the commutation relation hypotheses are nicely matched to another and independent set of ideas about the weak interaction currents. These form the so-called PCAC notion of pion pole dominance for the divergence of the strangeness conserving axial vector current.[6] This too has its independent tests, i.e., independent of the ideas of current algebra, and I will want to take these up. But it is in combination that the two sets of ideas, current algebra and PCAC, display their most impressive predictive powers. A contrast must however be noted. The Gell-Mann conjectures seem to be clearly and consistently posed, so that for them the question is whether they happen to be true for the real world. The PCAC notion, on the other hand, has so far not been sharply stated; so for it the question is not only whether it's true, but—*what is it?*

These lectures will be focused mainly on concrete applications of current algebra and PCAC, with examples taken from the "classical" portion of these subjects. The question, what is PCAC, will no doubt be unduly belabored. But in general, the fare will be standard. It is addressed to people who have not yet had occasion to make themselves experts, and it is intended to serve also as introduction for the more up-to-date material which will appear in the other lecture series here. Luckily, an outstanding published review is provided in the book by Adler and Dashen: *Current Algebras* (W. A. Benjamin Publ., New York, 1968). For more general matters concerning the weak interactions, reference may be made to the book by Marshak, Riazuddin, and Ryan: *Theory of Weak Interactions In Particle Physics* (Wiley-Interscience, New York, 1969). For SU_3 matters, see Gell-Mann and Ne'eman *The Eightfold Way* (W. A. Benjamin Publ., New York, 1964).

Owing to the existence of these excellent works, the present notes will be sparing in references.

II. THE PHYSICAL CURRENTS

The Adler-Weisberger formula makes an improbable connection between strong interaction quantities and a weak interaction parameter. By generalizing the formula to cover pion scattering on various hadron targets, one can eliminate the weak interaction parameter and obtain connections between purely strong interaction quantities. This remarkable circumstance, and others like it, arises from the PCAC hypothesis concerning a certain physical weak interaction current. In retrospect it would be possible to formulate matters in such a way that no reference is made to the weak interactions: the current in question could be introduced as a purely mathematical object. However, that's not how things happened; and anyway the physical currents that constitute our subject are interesting in their own right. So we begin with a brief review of the way in which these currents arise in the present day description of the weak and electromagnetic interactions of hadrons. The Heisenberg picture is employed throughout for quantum mechanical states and operators. Our metric corresponds to

$$a.b = \vec{a}.\vec{b} - a_0 b_0 .$$

(i) Electromagnetic Hadron Currents: –

Of all the currents to be dealt with, the electromagnetic is no doubt the most familiar. The coupling of hadrons to the electromagnetic field aperator a_λ is described, to lowest order in the unit of electric charge e, by an interaction Hamiltonian density

(2.1) $$\mathcal{H}^{em} = e j_\lambda^{em} a_\lambda ,$$

where j_λ^{em} is the hadron electromagnetic current. It is a vector operator formed out of hadron fields in a way whose details must await a decision about fundamental matters concerning the nature of hadrons. It is in line with the trend of contemporary hadron physics to put off such a decision and instead concentrate on symmetry and other forms of characterization which are supposed to transcend dynamical details. Thus, reflecting conservation of electric charge, we assert that the charge operator

$$Q^{em} = \int d^3x j_0 (\vec{x}, x_0)$$

is independent of time x_0 and that the current j_λ^{em} is conserved:

$$\partial j_\lambda^{em} / \partial x_\lambda = 0.$$

It is customarily assumed that j_λ^{em} is odd under the charge conjugation symmetry transformation defined by the strong interactions. In connection with the discovery of CP violation it has however been suggested that the electromagnetic current may also have a piece which is *even* under charge conjugation; but for present purposes we shall overlook this still unconfirmed possibility. Electromagnetic interactions of course conserve baryon number (N) and strangeness (S)—or equivalently, hypercharge $Y = N + S$; and they conserve the third component I_3 of isotopic spin. According to the familiar formula

$$Q^{em} = I_3 + Y/2$$

it is generally supposed that j_λ^{em} contains a part which transforms like a scalar $(I = 0)$ under isotopic spin rotations and a part which transforms like the third component of an isovector ($I = 1, \Delta I_3 = 0$):

(2.2) $$j_\lambda^{em} = j_\lambda^{em}\,(I = 0) + j_\lambda^{em}\,(I = 1, \Delta I_3 = 0)\ .$$

The two pieces are separately conserved, reflecting conservation of hypercharge and third component of isotopic spin; and the corresponding "charges" measure these quantities:

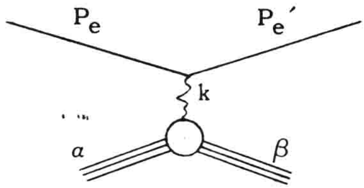
$$\frac{Y}{2} = \int j_0^{em}\,(I = 0)\, d^3x$$

(2.3) $$I_3 = \int j_0^{em}\,(I = 1)\, d^3x\ .$$

Consider now an electromagnetic process of the sort $\alpha \to \beta + \gamma$, where γ is a real photon of momentum k and α and β are systems of one or more hadrons. To lowest electromagnetic order the transition amplitude is given by

(2.4) $$e < \beta | j_\lambda^{em} | \alpha > \epsilon_\lambda\ ,$$

where ϵ_λ is the photon polarization vector and the states $|\alpha>$ and $|\beta>$ are determined purely by the strong interactions, with electromagnetism switched off. The photon process probes the structure of these states via the current operator j_λ^{em}. Here of course $k = P_\alpha - P_\beta$, $k^2 = 0$, $k \cdot \epsilon = 0$. Off mass shell ($k^2 \neq 0$) electromagnetic probes are provided in processes involving the interactions of electrons or muons with hadrons. For example, to lowest relevant order the process $e + \alpha \to e + \beta$ is described by the Feynman diagram shown below, where P_e and $P_e{}'$ are the initial and final electron momenta and the virtual photon has momentum $k = P_e - P_e{}'$. The amplitude is given by



(2.5) $$e^2 < \beta | j_\lambda^{em} | \alpha > \frac{1}{k^2} \bar{u} (P_e') \gamma_\lambda u(P_e) \quad .$$

The matrix element of the electron part of the overall electromagnetic current comes out here as a trivially known factor. All the complexities of the strong interactions are again isolated in the matrix element of the hadron current j_λ^{em}; but now, in general, $k^2 = (P_\beta - P_\alpha)^2$ is not zero. Other related variations have to do with processes such as $\alpha \to \beta + e^+ + e^-$ and, for colliding beam experiments, $e^+ + e^- \to$ hadrons. In all cases it is the hadronic matrix element that is of interest; and current conservation implies the relation

$$k_\lambda < \beta | j_\lambda^{em} | \alpha > = 0 \ , \quad k = P_\beta - P_\alpha \ .$$

(ii) Weak Lepton Currents: –

So much for electromagnetism. Concerning the weak interactions, you recall that they group themselves phenomenologically into three classes. (1) Purely leptonic processes, of which muon decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ and the conjugate μ^+ decay are the sole observed examples. (2) Semi-leptonic processes, i.e., those which involve hadrons and a lepton pair $(e\nu_e)$ or $(\mu\nu_\mu)$. There are many observed examples, e.g., $n \to p + e^- + \bar{\nu}_e$, $\bar{\nu}_\mu + p \to n + \mu^+$, $K \to e + \nu + \pi$, etc. (3) Non leptonic weak processes, e.g., $\Lambda \to p + \pi^-$, $K^+ \to \pi^+ + \pi^0$, etc.

The purely leptonic muon decay reaction is well described phenomenologically to lowest order in an interaction Hamiltonian which couples an $(e\nu_e)$ "current" with a $(\mu\nu_\mu)$ "current":

(2.6) $$\mathcal{H}^{\text{leptonic}} = \frac{G}{(2)^{1/2}} \mathcal{L}_\lambda(e\nu)\mathcal{L}_\lambda^\dagger(\mu\nu) + \text{h.c.}$$

where the current operators are given by

$$\mathcal{L}_\lambda(e\nu) = i\bar{\Psi}_{\nu_e}\gamma_\lambda(1+\gamma_5)\Psi_e$$

$$\mathcal{L}_\lambda(\mu\nu) = i\bar{\Psi}_{\nu_\mu}\gamma_\lambda(1+\gamma_5)\Psi_\mu$$

The factor $(1+\gamma_5)$ expresses the presumed 2-component nature of neutrinos and gives to each current both a vector and axial vector part. The coupling constant G has the dimension $(\text{mass})^{-2}$ and can be taken generally as a characteristic (but dimensional) measure of the strength of all classes of weak interactions. From the fit to the muon decay rate, one finds that $Gm^2 \simeq 10^{-5}$, where m is the nucleon mass.

(iii) Weak Hadron Currents:–

The observed semi leptonic reactions seem to be well described to lowest order in an interaction Hamiltonian which effectively couples the lepton currents to a weak hadron current j_λ^{weak} which, in fact, contains both vector and axial vector parts:

(2.7) $$\mathcal{H}^{\text{semi leptonic}} = \frac{G}{(2)^{1/2}} j_\lambda^{\text{weak}} \cdot \mathcal{L}_\lambda^\dagger + \text{h.c.}$$

where $$\mathcal{L}_\lambda = \mathcal{L}_\lambda(e\nu) + \mathcal{L}_\lambda(\mu\nu).$$

Notice that $\mathcal{L}_\lambda(e\nu)$ and $\mathcal{L}_\lambda(\mu\nu)$ couple equally to a common hadron current, reflecting the present belief in $e-\mu$ universality. In the subsequent discussion we will let the symbol ℓ denote either e or μ; and the symbol ν will stand for the appropriate neutrino ν_e or ν_μ.