JOHN L. CASTI

ALTERNATE REALITIES

MATHEMATICAL MODELS OF NATURE AND MAN

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To the scientific ideals of the
INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
May aspirations and achievements someday coincide

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PREFACE

In experimental science, the principal focus is upon developing new instruments and techniques for measuring various aspects of natural phenomena, with all other considerations subservient to this primary task. So it is also in theoretical science, where the overarching problem is how to capture the processes of Nature and man in formal mathematical representations, with all matters of technique and choice of methods being of secondary consideration. This book addresses the interface between these complementary aspects of scientific practice.

If the natural role of the experimenter is to generate new observables by which we know the processes of Nature, and the natural role of the mathematician is to generate new formal structures by which we can represent these processes, then the system scientist finds his niche by serving as a broker between the two. In this volume my goal is to show (more by example than by precept) how to bridge the gap between experimental science, on the one hand, and the world of mathematics on the other. The objective is to provide both the student and the scientific practitioner with an attitude, or philosophy, as well as a set of tools, with which to probe the workings of both Nature and man.

I hasten to point out that this is *not* a book on systems analysis, only a volume on the theory and practice of mathematical modeling. As a result, the focus is on how to go from observing the behavior of natural and human systems to formal mathematical models of such behavior, and it will not be our concern here to enter into the pedestrian (although often very useful) issues surrounding collection of data, design of information systems, consultation with the client, selection of equipment and the myriad other nonscientific activities involved in what is sometimes termed "systems analysis." Thus the principal aim of this book is to bring modern mathematics into the service of applied system modeling, with primary emphasis upon the word "modern."

Should you have the misfortune to pick up a typical current textbook purporting to address the arcane arts of mathematical modeling, the chances are overwhelmingly high that the author will transport you back into the 1950s with an account of how to model an oscillating pendulum, freeway traffic, or dogfood mixing using the static, equilibrium-centered, linear techniques of mathematical programming, regression analysis or, perhaps, elementary functional analysis. My feeling is that the time is long overdue to bring the mathematics of the 1980s into contact with the students of the 1980s and offer courses on modeling that stress dynamics rather than statics, nonlinearity rather than linearity, and possibility rather than optimality.

The concepts noted above constitute the "fingerprints" of what has been termed elsewhere the *system-determined* sciences. In the language of Aristotelian causes, the system-determined sciences emphasize the manipulation of *formal* cause in contrast to classical physics and engineering, which concern themselves primarily with *material* and *efficient* cause. In more familiar terms, our focus is upon issues of information as opposed to a concern with matter or energy *per se.* As a result, our motivation and examples come mainly from the social, behavioral and life sciences rather than the physical sciences, an emphasis that is, in my opinion, long overdue in textbook accounts of mathematical modeling.

In 1985 and 1986 I was invited to put these radical notions to the test and gave just such a course of lectures, both at the University of Vienna and at the Technical University of Vienna. The focus of these lectures was upon system-theoretic concepts such as complexity, self-organization, adaptation, bifurcation, resilience, surprise and uncertainty, coupled with the kinds of mathematical structures needed to pin down these notions within the confines of a formal mathematical system. This book is an outgrowth of those lectures.

While it's not normal practice in a book's Preface to already start anticipating reviewer opinion and commentary, if referees' reports on the original manuscript are taken as reliable indicators, this is no normal book. Moreover, taking past experience as a guide, when one starts tip-toeing through the no-man's land between mathematics and the real world, which any book on mathematical modeling necessarily must, it's a narrow path indeed that must be tread to avoid landmines and sniper fire from both sides of the divide. So to set this volume in perspective, let me first say what it's not. It is not a treatise about the *mathematics* of modeling, nor is it an account of *case studies* in the real-world application of mathematical concepts. Neither is the book intended as a compendium or an encyclopedic survey of the entire field of theoretical system thinking. In this sense I have been remarkably self-indulgent, choosing to discuss those areas in which I have an interest and ignoring those in which I don't. With all these negatives and disclaimers, what then **is** this book? Perhaps the best way to describe what it is (in my view, at least) is to term it a book on the **theory of models**. At this point you might well ask: "What in heaven's name could this possibly mean?"

The genesis of the idea from which this book took form was my desire to put together a course of lectures on modeling for mathematically trained students in disciplines like economics, biology, computer science and so forth—but not mathematics. The underlying goal was to expose these students to a spectrum of *concepts* and *ideas* from across the landscape of modern applied mathematics, and to indicate some of the ways in which these notions had been used to represent toy versions of real-world situations. Thus the emphasis was on the ideas underlying cellular automata, chaos, catastrophe theory and the like, as well as on the *kinds* of situations where they had been used, but not on the fine-grained technical details. But since the students all had a good grounding in basic mathematics, I saw no reason to present these ideas in the language of popular science using no mathematics at all. On the other hand, the technical finery and rigor demanded by dues-paying members of the mathematical community was also inappropriate, since the students were really interested in what kinds of questions these concepts might illuminate, not in delicate webs of definitions or intracacies of proofs. The book before you represents my compromise between the Scylla of mathematical precision and the Charybdis of messy real-world detail.

No doubt there will be readers who will take umbrage at what they see as my cavalier disregard for the delicacies of their art, and perhaps rightly so. They have my apologies along with the recognition that even in a 500-page volume such as this, doing justice to the way things "really are" in so many areas is just plain impossible. My hope is that even St. Simon the Stylite will still find something of interest here, a point or concept that piques his curiosity, at least partially compensating for the book's obvious faults elsewhere. On balance, I think of the book as more of an Impressionist painting than a photographic account of the philosophy and ideas of modeling, and I'll consider my time in writing it well spent if the reader ends up saying to himself, "yes, there really is a legitimate intellectual activity here called 'modeling,' and it's not mathematics and it's not applications." Elicitation of this very sentence is the real goal of this book.

Since the Table of Contents speaks for itself, I will content myself at this point only with the admonition to the reader that each chapter's Discussion Questions and Problems and Exercises are an integral part of the book. The

Discussion Questions amplify many of the points only touched upon in the text and introduce topics for which there was no room for a more complete account elsewhere. The Problems and Exercises provide challenges ranging from simple drill to state-of-the-art research and are deliberately worded so that the reader will know what the answer should be—even if he can't see how to get it! In any case, my message in this volume will be only half-transmitted if just the chapter text is read. Although the mathematical *ideas* used in the book are quite contemporary, the actual *technique* involved in using these ideas is, for the most part, rather modest. As a result, most of the sections of each chapter should be accessible to that proverbial seeker of knowledge, "the well-trained undergraduate," with a **working** knowledge of ordinary differential equations, linear algebra, matrix theory and, perhaps, abstract algebra. Note, however, the emphasis on the term "working." To really understand the ideas presented here, it will not be sufficient just to have sat through a course in the above topics; the student will have to actually recall the material of the course and know how to apply it. With this caveat, the book should serve as a text-reference for a course in modern applied mathematical modeling for upper-division undergraduates, not to mention their presumably more mathematically sophisticated colleagues in graduate school.

It's impossible to put together a book such as this without the generous help, encouragement, and cooperation of numerous friends and colleagues. At the head of this list come Profs. Karl Sigmund of the Mathematics Institute of the U. of Vienna and Manfred Deistler, my colleague at the Institute for Econometrics, Operations Research, and System Theory of the Technical U. of Vienna. Each offered me not only a forum in which to present these slices of modern system theory, but also a group of talented students who in some cases were willing to serve as enthusiastic "guinea pigs" for the particular brand of system-theoretic medicine that I was serving up. Also occupying a prominent position on this roll call of honor is Steven Dunbar, who courageously took on the task of keeping me technically honest with a thorough review of much of the mathematical finery in the book. Naturally, and as always, I accept final responsibility for the book's inevitable *faux pas.* But Steve's hard work has insured me a far lower embarrassment level than would otherwise have been the case.

Others who contributed by providing examples, software consultations, stimulating discussions or just plain friendship include Hugh Miser, Paul Makin, Hans Troger, Robert Rosen, René Thom, Nebojsa Nakicenovic, Lucien Duckstein, Mel Shakun, George Klir, George Leitmann, Alain Bensoussan, Myron Allen, Don Saari, and Clifford Marshall. In addition, the enthusiasm and encouragement of Maria Taylor, my editor at Wiley, has been a constant source of support. Last, but far from least, kudos to my wife Vivien, who tolerated my spending more time with the vagaries of computers and

system modeling than with her for much longer than any companion should. Finally, in a work of this kind it's impossible for me not to cite the role of the International Institute for Applied Systems Analysis (IIASA), where I spent far more years than was good for me in developing and promoting much of the philosophy and some of the technique appearing in this *"magnum opus."* In particular I must acknowledge the various managements of the Institute, whose understanding and appreciation of the need for theoretical work and conceptual foundations served as a never-ending source of motivation for the preparation of this volume. Without remarkable views such as theirs on the value of system thinking as an intellectual undertaking, books like this would never have to be written.

JLC
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CONTENTS

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CHAPTER ONE

The Ways of Modelmaking: Natural Systems and Formal Mathematical Representations

1. *A Theory of Models*

What do you think of when you hear the word "model"? An elegant mannequin from the pages of *Vogue* perhaps? Or maybe a miniature version of that super-exotic Ferrari or Lamborghini that you've been drooling over? To a system scientist or applied mathematician, quite a different picture comes to mind. For such practitioners of the "black arts," a *model* means an encapsulation of some slice of the real world within the confines of the relationships constituting a formal mathematical system. Thus, a model is a mathematical representation of the modeler's reality, a way of capturing some aspects of a given reality within the framework of a mathematical apparatus that provides us with a means for exploring the properties of that reality mirrored in the model.

This book is about the ways and means of constructing "good" models of reality, the properties of such models, the means for encoding specific realities into definite formal systems and the procedures for interpreting the properties of the formal system in terms of the given real world situation. In short, we're interested in the ways of modelmaking. Before embarking upon a more detailed account of what we mean by the terms *model, encoding, formal system,* and so forth, it's necessary to examine some basic epistemological and operational issues lying at the heart of what we shall term the *theory of models.*

According to the great nineteenth-century British physicist Maxwell, "the success of any physical investigation depends upon the judicious selection of what is to be observed as of primary importance." This view suggests the notion that what constitutes one's reality depends upon one's capacity for observation. Here we adopt the position that since natural phenomena impinge upon our consciousness only through instruments of observation, then, to paraphrase Maxwell, "the success of any modeling venture depends upon a judicious selection of observables and means for encapsulating these observables within the framework of convenient formal mathematical systems."

As noted by Rosen, in dealing with the idea of a natural system, we must necessarily touch on some basic philosophical questions of both an ontological and epistemological character. This is unavoidable in any case and must be addressed at the outset of a work such as this, because our tacit assumptions in these areas determine the character of our science. It's true that many scientists find an explicit consideration of such matters irritating, just as many working mathematicians dislike discussions of the foundations of mathematics. Nevertheless, it's well to recall the remark of David Hawkins: "Philosophy may be ignored but not escaped; and those who most ignore escape least."

Our viewpoint is that *the study of natural systems begins and ends with the specification of observables belonging to such a system, and a characterization of the manner in which they are linked.* Purely theoretical issues may be pursued in the process of investigating a system, but ultimately contact with reality occurs through the observables. During the course of this book, it will be argued that the concept of a model of a natural system N is a generalization of the concept of a subsystem of N, and that the essential feature of the modeling relation is the exploration of the idea that there is a set of circumstances under which the model describes the original system to a prescribed degree of accuracy. In other words, a particular facet of system behavior remains *invariant* under the replacement of the original system by a proper subsystem.

At this point it is well to consider why one constructs models of natural phenomena in the first place. Basically, the point of making models is to be able to bring a measure of order to our experiences and observations, as well as to make specific predictions about certain aspects of our experienced world. The central question surrounding the issue of model credibility is to ask to what extent "good" predictions can be made if the best the model can do is to capture a subsystem of N. The answer is wrapped up in the way in which the natural system is characterized by observables, the procedure by which observables are selected to form the subsystem, and the manner in which the subsystem is encoded into a formal mathematical system F which *represents,* or "models," the process of interest. These notions will be made more explicit later, but before doing so, let us consider a familiar example that illustrates many of these points.

We consider an enclosed homogeneous gas for which the observables are taken to be the volume V occupied by the gas, the pressure P and the temperature T. By this choice, the abstract states of the system, i.e., the actual *physical states* comprising the volume, pressure and temperature, are encoded into a three-dimensional euclidean space in the familar manner. At equilibrium, these three observables are not independent but are linked by the relation (equation of state) $PV = T$, the *ideal gas law.* Of course, we know that such a selection of observables represents an abstraction in the sense that many other observables have been omitted that, in principle, influence the gas (external radiation, properties of the container, etc.). Experience has shown, however, that the subsystem consisting of P, V and

T, together with its encoding into the region of R^3 defined by the ideal gas law, enables us to make very accurate predictions about the *macroscopic* behavior of the system. Should we desire to make predictions about the gas at the molecular level, it would be necessary to choose an alternate set of observables. In the *microscopic* context, the positions and momenta of the $\bar{N} \cong 10^{24}$ molecules composing a mole of the gas would be a natural choice, and these observables would be encoded into the euclidean space $R^{6\bar{N}}$. Linkages between the observables in this case are specified by various conservation laws operating at the microlevel.

Our primary objective in this chapter is to provide a framework within which we can speak of fundamental issues underlying any theory of modeling. Among matters of greatest concern, we find:

- What is a model?
- What features characterize "good" models?
- How can we represent a natural process N in a formal system F?
- What is the relationship between N and F?
- When does the similarity of two natural systems N_1 and N_2 imply that their models F_1 and F_2 are similar?
- How can we compare two models of the same natural process N?
- Under what circumstances can we consider a linkage between observables as constituting a "law" of Nature?
- What procedures can we invoke to identify key observables and thereby simplify a model?
- How does a given system relate to its subsystems?
- When can two systems that behave similarly be considered as models of each other?

Such a list (and its almost infinite extension) represents issues in the philosophy of science and, in particular, the theory of models. No uniform and complete answers to these issues can ever be expected; the best we can hope for is to provide a basis for considering these matters under circumstances appropriate to a given setting.

In what follows, we sketch a formalism suitable for studying the foregoing questions and indicate by familar examples some of the advantages to be gained by looking at system modeling questions in such generality. In essence, the argument is that the detailed study of a given natural system N cannot be suitably interpreted and understood within the level of the phenomenon N itself. A more general metalevel and a metalanguage provided by a theory of models is required.