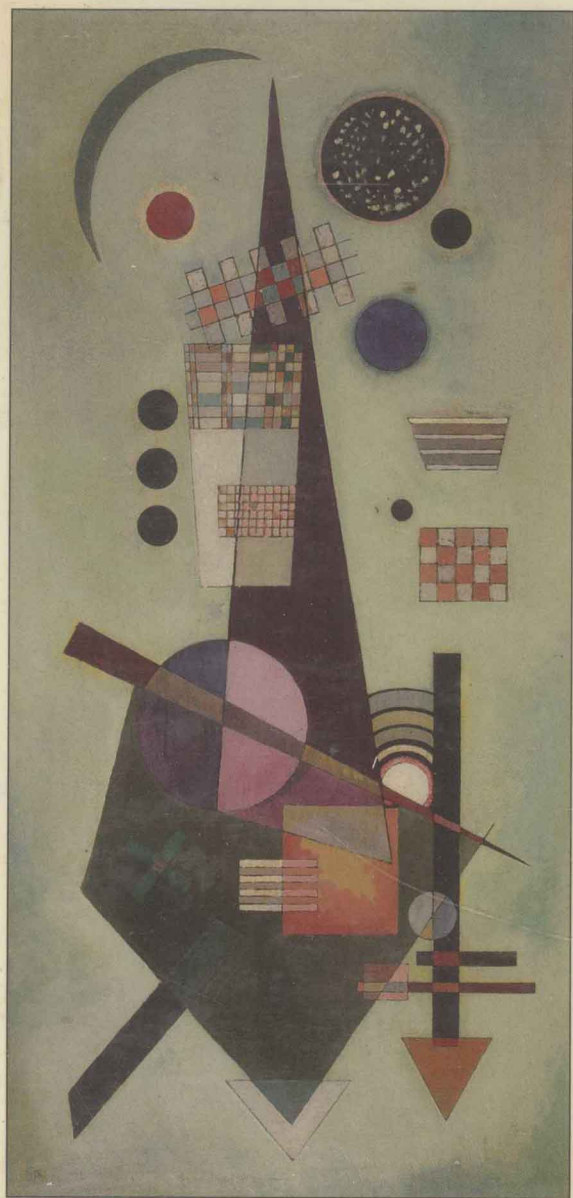


THIRD EDITION



MARGARET L. LIAL
CHARLES D. MILLER

FINITE MATHEMATICS AND CALCULUS WITH APPLICATIONS

FINITE MATHEMATICS AND CALCULUS WITH APPLICATIONS

THIRD EDITION

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SCOTT, FORESMAN AND COMPANY
Glenview, Illinois London, England

TO THE STUDENT

If you want further help with this course, you may want to obtain a copy of the *Student's Solutions Manual* that accompanies this textbook. This manual provides detailed step-by-step solutions to the odd-numbered exercises in the textbook and can help you study and understand the course material. Your college bookstore either has this manual or can order it for you.

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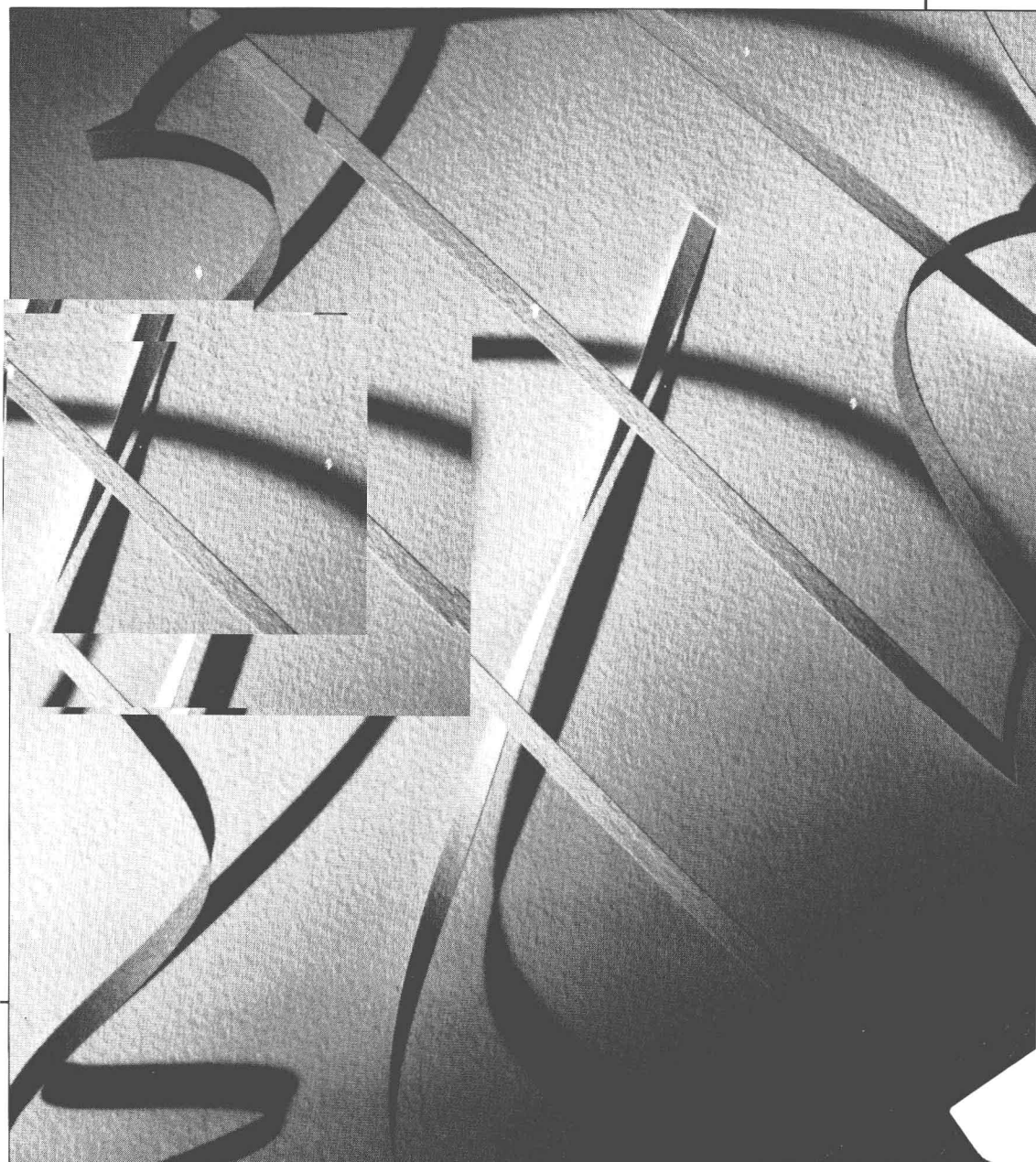
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FINITE MATHEMATICS AND CALCULUS WITH APPLICATIONS

THIRD EDITION



Finite Mathematics and Calculus with Applications, Third Edition, is a solid, applications-oriented text for students majoring in business, management, economics, or the life or social sciences. The book can accommodate a year-long course, and the prerequisite is three to four semesters of high school algebra. The text also may be used for separate courses in finite mathematics and calculus.

Explanations and examples are clear, direct, and to the point; exercises are carefully graded; and numerous examples are provided that directly correspond to the exercises. Highlights of the text are the Extended Applications and abundant applied problems that show students how mathematics is used in the careers they are exploring.

Many portions of the third edition have been extensively revised. New examples, exercises, and applications have been added, and parts of the text have been completely rewritten. In response to comments from those who have used the most recent edition of the book, greater emphasis has been given to explanations of topics from business and economics. Interesting new applications in these areas enhance the book's appeal for the broad range of students enrolled in courses using this text.

KEY FEATURES

FLEXIBILITY The text has been organized to be as flexible as possible. For those students who have not studied algebra recently, an algebra review is provided at the beginning of the book. This review may be used either in class or individually for reference. Chapter 1, on functions and graphs, can be skipped by well-prepared students. Some instructors may choose to begin the finite mathematics part of the course with Chapter 2 and proceed through linear programming, and some may choose to begin with Chapter 5 on sets and counting and continue with probability. The core ideas of calculus are covered in Chapters 10 through 13, and instructors can choose among the remaining chapters as desired.

APPLICATIONS A wide range of applications is included in examples and exercise sets to motivate student interest. An Index of Applications, grouped by discipline, includes those applications given in the exercises, examples, and Extended Applications, and is located after the table of contents.

EXAMPLES About 865 worked-out examples clearly illustrate the techniques and concepts presented and prepare students for success with the exercises. For clarity, the end of each example is indicated with the symbol ■

EXERCISES Approximately 5075 exercises, including about 4111 drill problems and 964 applications, feature a wide range of difficulty from drill to challenging problems. A new format is used for exercises in this edition. Routine exercises are given first, followed by a clearly marked applications section arranged by discipline, with a descriptive heading for each application.

EXTENDED APPLICATIONS Special lengthy applications are included at the end of most chapters. Designed to help motivate students in the study of mathematics, the Extended Applications are derived from current literature in various fields and show how the course material can actually be applied.

FORMAT Important rules, definitions, theorems, and equations are enclosed in boxes and highlighted with a title in the margin, for ease of study and review. A second color is used to annotate equations, illuminate troublesome areas, and clarify concepts and processes. A new format in the exercise sets highlights the many diverse applications included in the book.

STUDY AIDS Each chapter ends with a list of key words presented in the chapter and an extensive set of review exercises. The key words are referenced to the section in which each appeared, for extra studying help.

■ CONTENT FEATURES

The book has **two complete chapters on linear programming**, one on the graphical method and one on the simplex method. Mixed constraints are handled with the Phase I/Phase II method. This method also provides one way to solve minimizing problems. A section using the dual approach to minimizing is also included for those instructors who prefer that approach.

The **discussion of permutations and combinations has been expanded** to two sections (Sections 5.3 and 5.4) to give more emphasis to the multiplication rule and to the differences between permutations and combinations.

Mathematics of finance is a self-contained chapter following the topics on finite mathematics and preceding the chapters on the calculus, permitting coverage when desired.

Comprehensive coverage of calculus is included. Students are given a thorough introduction to this key portion of mathematics. The derivative is introduced early in the calculus material. An intuitive approach employs graphs and tables wherever possible.

New motivational material in this edition includes historical references throughout and more attention to the background and development of new topics. New topics are introduced with an appeal to past experience. See, for example, the introduction to maxima and minima in Section 11.2.

Expanded discussions of topics from business and economics are included. See, for example, marginal analysis in Section 10.4, elasticity of demand in Section 11.6, And consumers' and producers' surplus in Section 13.5.

Chapters were written for maximum flexibility of course design. Chapter interdependence is as follows:

Chapter 1, "Functions and Graphs," has no prerequisite. (The noncalculus portions of the chapter on exponential and logarithmic functions may be covered at any time after Chapter 1.)

Chapter 2, on matrix theory, has no prerequisite.

Chapters 3 and 4, on linear programming, depend on Chapters 1 and 2.

Chapter 5, "Sets and Counting," has no prerequisite.

Chapter 6, "Probability," requires Chapter 5.

Chapter 7, "Statistics and Probability Distributions," requires Chapter 6.

Chapter 8, on further applications of probability, depends on Chapter 6 and Section 7.2.

Chapter 9, "Mathematics of Finance," has no prerequisite.

Chapters 10 through 16, the calculus chapters, require Chapter 1.

SUPPLEMENTS

The **Instructor's Guide and Solutions Manual** gives a lengthy set of test questions for each chapter, organized by section, plus answers to all the test questions. It also provides complete solutions to all of the even-numbered text exercises.


The **Instructor's Answer Manual** give the answers to every text exercise, collected in one convenient location and presented in an easy-to-use format.

The **Student's Solutions Manual**, available for purchase by students, provides detailed, worked-out solutions to all of the odd-numbered text exercises, plus a chapter test for each chapter. Answers to the chapter test questions are given at the back of the book.

The **Scott, Foresman Test Generator for Mathematics** enables instructors to select questions by section or chapter or to use a ready-made test for each chapter. Instructors may generate tests in multiple-choice or open-response formats, scramble the order of questions while printing, and produce multiple versions of each test (up to 9 with Apple, up to 25 with IBM). The system features a preview option that allows instructors to view questions before printing, to regenerate variables, and to replace or skip questions if desired. A Macintosh version of the Test Generator will be available.

Computer Applications for Finite Mathematics and Calculus by Donald R. Coscia is a softbound textbook packaged with two diskettes (in Apple II and IBM-PC versions) with programs and exercises keyed to the text. The programs allow students to solve meaningful problems without the difficulties of extensive computation. This book bridges the gap between the text and the computer by providing additional explanations and exercises for solution using a micro-computer. Appropriate exercises in the textbook are identified by the heading "For the Computer."

A set of two-color **overhead transparencies** showing charts, figures, and portions of examples is available and can be used to accompany a lecture.

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