

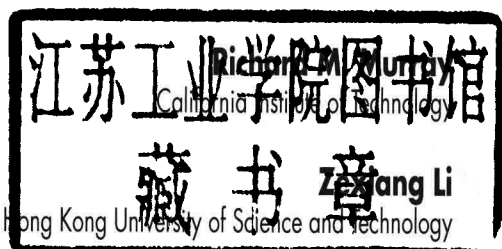
A Mathematical Introduction to



ROBOTIC MANIPULATION

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A Mathematical Introduction to ROBOTIC MANIPULATION



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***A Mathematical
Introduction to***
**ROBOTIC
MANIPULATION**

To RuthAnne (RMM)

To Jianghua (ZXL)

In memory of my father (SSS)

Preface

In the last two decades, there has been a tremendous surge of activity in robotics, both in terms of research and in terms of capturing the imagination of the general public as to its seemingly endless and diverse possibilities. This period has been accompanied by a technological maturation of robots as well, from the simple pick and place and painting and welding robots, to more sophisticated assembly robots for inserting integrated circuit chips onto printed circuit boards, to mobile carts for parts handling and delivery. Several areas of robotic automation have now become “standard” on the factory floor and, as of the writing of this book, the field is on the verge of a new explosion to areas of growth involving hazardous environments, minimally invasive surgery, and micro electro-mechanical mechanisms.

Concurrent with the growth in robotics in the last two decades has been the development of courses at most major research universities on various aspects of robotics. These courses are taught at both the undergraduate and graduate levels in computer science, electrical and mechanical engineering, and mathematics departments, with different emphases depending on the background of the students. A number of excellent textbooks have grown out of these courses, covering various topics in kinematics, dynamics, control, sensing, and planning for robot manipulators.

Given the state of maturity of the subject and the vast diversity of students who study this material, we felt the need for a book which presents a slightly more abstract (mathematical) formulation of the kinematics, dynamics, and control of robot manipulators. The current book is an attempt to provide this formulation not just for a single robot but also for multifingered robot hands, involving multiple cooperating robots. It grew from our efforts to teach a course to a hybrid audience of electrical engineers who did not know much about mechanisms, computer scientists who did not know about control theory, mechanical engineers who were suspicious of involved explanations of the kinematics and dynamics of garden variety open kinematic chains, and mathematicians who were curious, but did not have the time to build up lengthy prerequisites before beginning a study of robotics.

It is our premise that abstraction saves time in the long run, in return for an initial investment of effort and patience in learning some mathematics. The selection of topics—from kinematics and dynamics of single robots, to grasping and manipulation of objects by multifingered robot hands, to nonholonomic motion planning—represents an evolution from the more basic concepts to the frontiers of the research in the field. It represents what we have used in several versions of the course which have been taught between 1990 and 1993 at the University of California, Berkeley, the Courant Institute of Mathematical Sciences of New York University, the California Institute of Technology, and the Hong Kong University of Science and Technology (HKUST). We have also presented parts of this material in short courses at the Università di Roma, the Center for Artificial Intelligence and Robotics, Bangalore, India, and the National Taiwan University, Taipei, Taiwan.

The material collected here is suitable for advanced courses in robotics consisting of seniors or first- and second-year graduate students. At a senior level, we cover Chapters 1–4 in a twelve week period, augmenting the course with some discussion of technological and planning issues, as well as a laboratory. The laboratory consists of experiments involving on-line path planning and control of a few industrial robots, and the use of a simulation environment for off-line programming of robots. In courses stressing kinematic issues, we often replace material from Chapter 4 (Robot Dynamics) with selected topics from Chapter 5 (Multifingered Hand Kinematics). We have also covered Chapters 5–8 in a ten week period at the graduate level, in a course augmented with other advanced topics in manipulation or mobile robots.

The prerequisites that we assume are a good course in linear algebra at the undergraduate level and some familiarity with signals and systems. A course on control at the undergraduate level is helpful, but not strictly necessary for following the material. Some amount of mathematical maturity is also desirable, although the student who can master the concepts in Chapter 2 should have no difficulty with the remainder of the book.

We have provided a fair number of exercises after Chapters 2–8 to help students understand some new material and review their understanding of the chapter. A toolkit of programs written in Mathematica for solving the problems of Chapters 2 and 3 (and to some extent Chapter 5) have been developed and are described in Appendix B. We have studiously avoided numerical exercises in this book: when we have taught the course, we have adapted numerical exercises from measurements of robots or other “real” systems available in the laboratories. These vary from one time to the next and add an element of topicality to the course.

The one large topic in robotic manipulation that we have not covered in this book is the question of motion planning and collision avoidance

for robots. In our classroom presentations we have always covered some aspects of motion planning for robots for the sake of completeness. For graduate classes, we can recommend the recent book of Latombe on motion planning as a supplement in this regard. Another omission from this book is sensing for robotics. In order to do justice to this material in our respective schools, we have always had computer vision, tactile sensing, and other related topics, such as signal processing, covered in separate courses.

The contents of our book have been chosen from the point of view that they will remain foundational over the next several years in the face of many new technological innovations and new vistas in robotics. We have tried to give a snapshot of some of these vistas in Chapter 9. In reading this book, we hope that the reader will feel the same excitement that we do about the technological and social prospects for the field of robotics and the elegance of the underlying theory.

Richard Murray
Zexiang Li
Shankar Sastry

Berkeley, August 1993

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It is a great pleasure to acknowledge the people who have collaborated with one or more of us in the research contained in this book. A great deal of the material in Chapters 2 and 3 is based on the Ph.D. dissertation of Bradley Paden, now at the University of California, Santa Barbara. The research on multifingered robot hands, on which Chapters 5 and 6 are founded, was done in collaboration with Ping Hsu, now at San Jose State University; Arlene Cole, now at AT&T Bell Laboratories; John Hauser, now at the University of Colorado, Boulder; Curtis Deno, now at Intermedics, Inc. in Houston; and Kristofer Pister, now at the University of California, Los Angeles. In the area of nonholonomic motion planning, we have enjoyed collaborating with Jean-Paul Laumond of LAAS in Toulouse, France; Paul Jacobs, now at Qualcomm, Inc. in San Diego; Greg Walsh, Dawn Tilbury, and Linda Bushnell at the University of California, Berkeley; Richard Montgomery of the University of California, Santa Cruz; Leonid Gurvits of Siemens Research, Princeton; and Chris Fernandez at New York University.

The heart of the approach in Chapters 2 and 3 of this book is a derivation of robot kinematics using the product of exponentials formalism introduced by Roger Brockett of Harvard University. For this and manifold other contributions by him and his students to the topics in kinematics, rolling contact, and nonholonomic control, it is our pleasure to acknowledge his enthusiasm and encouragement by example. In a broader sense, the stamp of the approach that he has pioneered in nonlinear control theory is present throughout this book.

We fondly remember the seminar given at Berkeley in 1983 by P. S. Krishnaprasad of the University of Maryland, where he attempted to convince us of the beauty of the product of exponentials formula, and the numerous stimulating conversations with him, Jerry Marsden of Berkeley, and Tony Bloch of Ohio State University on the many beautiful connections between classical mechanics and modern mathematics and control theory. Another such seminar which stimulated our interest was one on multifingered robot hands and cooperating robots given at Berkeley in 1987 by Yoshi Nakamura, now of the University of Tokyo. We have also

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Finally, on a personal note, we would like to thank our families for their support and encouragement during this endeavor.

Contents

Contents	vii
Preface	xiii
Acknowledgements	xvii
Chapter 1 Introduction	1
1 Brief History	1
2 Multifingered Hands and Dextrous Manipulation	8
3 Outline of the Book	13
3.1 Manipulation using single robots	14
3.2 Coordinated manipulation using multifingered robot hands	15
3.3 Nonholonomic behavior in robotic systems	16
4 Bibliography	18
Chapter 2 Rigid Body Motion	19
1 Rigid Body Transformations	20
2 Rotational Motion in \mathbb{R}^3	22
2.1 Properties of rotation matrices	23
2.2 Exponential coordinates for rotation	27
2.3 Other representations	31
3 Rigid Motion in \mathbb{R}^3	34
3.1 Homogeneous representation	36
3.2 Exponential coordinates for rigid motion and twists	39
3.3 Screws: a geometric description of twists	45
4 Velocity of a Rigid Body	51
4.1 Rotational velocity	51
4.2 Rigid body velocity	54
4.3 Velocity of a screw motion	58
4.4 Coordinate transformations	59
5 Wrenches and Reciprocal Screws	61
5.1 Wrenches	61

5.2	Screw coordinates for a wrench	65
5.3	Reciprocal screws	66
6	Summary	70
7	Bibliography	72
8	Exercises	73
Chapter 3 Manipulator Kinematics		81
1	Introduction	81
2	Forward Kinematics	83
2.1	Problem statement	83
2.2	The product of exponentials formula	85
2.3	Parameterization of manipulators via twists	91
2.4	Manipulator workspace	95
3	Inverse Kinematics	97
3.1	A planar example	97
3.2	Paden-Kahan subproblems	99
3.3	Solving inverse kinematics using subproblems	104
3.4	General solutions to inverse kinematics problems	108
4	The Manipulator Jacobian	115
4.1	End-effector velocity	115
4.2	End-effector forces	121
4.3	Singularities	123
4.4	Manipulability	128
5	Redundant and Parallel Manipulators	129
5.1	Redundant manipulators	130
5.2	Parallel manipulators	132
5.3	Four-bar linkage	135
5.4	Stewart platform	139
6	Summary	144
7	Bibliography	146
8	Exercises	147
Chapter 4 Robot Dynamics and Control		155
1	Introduction	155
2	Lagrange's Equations	156
2.1	Basic formulation	157
2.2	Inertial properties of rigid bodies	160
2.3	Example: Dynamics of a two-link planar robot	164
2.4	Newton-Euler equations for a rigid body	165
3	Dynamics of Open-Chain Manipulators	168
3.1	The Lagrangian for an open-chain robot	168
3.2	Equations of motion for an open-chain manipulator	169
3.3	Robot dynamics and the product of exponentials formula	175
4	Lyapunov Stability Theory	179

4.1	Basic definitions	179
4.2	The direct method of Lyapunov	182
4.3	The indirect method of Lyapunov	184
4.4	Examples	185
4.5	Lasalle's invariance principle	188
5	Position Control and Trajectory Tracking	190
5.1	Problem description	190
5.2	Computed torque	191
5.3	PD control	193
5.4	Workspace control	196
6	Control of Constrained Manipulators	199
6.1	Dynamics of constrained systems	200
6.2	Control of constrained manipulators	202
6.3	Example: A planar manipulator moving in a slot	203
7	Summary	206
8	Bibliography	207
9	Exercises	208
Chapter 5 Multifingered Hand Kinematics		211
1	Introduction to Grasping	211
2	Grasp Statics	214
2.1	Contact models	214
2.2	The grasp map	218
3	Force-Closure	223
3.1	Formal definition	223
3.2	Constructive force-closure conditions	224
4	Grasp Planning	229
4.1	Bounds on number of required contacts	229
4.2	Constructing force-closure grasps	232
5	Grasp Constraints	234
5.1	Finger kinematics	234
5.2	Properties of a multifingered grasp	237
5.3	Example: Two SCARA fingers grasping a box	240
6	Rolling Contact Kinematics	243
6.1	Surface models	243
6.2	Contact kinematics	248
6.3	Grasp kinematics with rolling	253
7	Summary	256
8	Bibliography	257
9	Exercises	259

Chapter 6	Hand Dynamics and Control	265
1	Lagrange's Equations with Constraints	265
1.1	Pfaffian constraints	266
1.2	Lagrange multipliers	269
1.3	Lagrange-d'Alembert formulation	271
1.4	The nature of nonholonomic constraints	274
2	Robot Hand Dynamics	276
2.1	Derivation and properties	276
2.2	Internal forces	279
2.3	Other robot systems	281
3	Redundant and Nonmanipulable Robot Systems	285
3.1	Dynamics of redundant manipulators	286
3.2	Nonmanipulable grasps	290
3.3	Example: Two-fingered SCARA grasp	291
4	Kinematics and Statics of Tendon Actuation	293
4.1	Inelastic tendons	294
4.2	Elastic tendons	296
4.3	Analysis and control of tendon-driven fingers	298
5	Control of Robot Hands	300
5.1	Extending controllers	300
5.2	Hierarchical control structures	302
6	Summary	311
7	Bibliography	313
8	Exercises	314
Chapter 7	Nonholonomic Behavior in Robotic Systems	317
1	Introduction	317
2	Controllability and Frobenius' Theorem	321
2.1	Vector fields and flows	322
2.2	Lie brackets and Frobenius' theorem	323
2.3	Nonlinear Controllability	329
3	Examples of Nonholonomic Systems	332
4	Structure of Nonholonomic Systems	339
4.1	Classification of nonholonomic distributions	340
4.2	Examples of nonholonomic systems, continued	341
4.3	Philip Hall basis	344
5	Summary	346
6	Bibliography	347
7	Exercises	349
Chapter 8	Nonholonomic Motion Planning	355
1	Introduction	355
2	Steering Model Control Systems Using Sinusoids	358
2.1	First-order controllable systems: Brockett's system	358
2.2	Second-order controllable systems	362

2.3	Higher-order systems: chained form systems	363
3	General Methods for Steering	366
3.1	Fourier techniques	367
3.2	Conversion to chained form	369
3.3	Optimal steering of nonholonomic systems	371
3.4	Steering with piecewise constant inputs	375
4	Dynamic Finger Repositioning	382
4.1	Problem description	382
4.2	Steering using sinusoids	383
4.3	Geometric phase algorithm	384
5	Summary	389
6	Bibliography	390
7	Exercises	391
Chapter 9 Future Prospects		395
1	Robots in Hazardous Environments	396
2	Medical Applications for Multifingered Hands	398
3	Robots on a Small Scale: Microrobotics	399
Appendix A Lie Groups and Robot Kinematics		403
1	Differentiable Manifolds	403
1.1	Manifolds and maps	403
1.2	Tangent spaces and tangent maps	404
1.3	Cotangent spaces and cotangent maps	405
1.4	Vector fields	406
1.5	Differential forms	408
2	Lie Groups	408
2.1	Definition and examples	408
2.2	The Lie algebra associated with a Lie group	410
2.3	The exponential map	412
2.4	Canonical coordinates on a Lie group	414
2.5	Actions of Lie groups	415
3	The Geometry of the Euclidean Group	416
3.1	Basic properties	416
3.2	Metric properties of $SE(3)$	422
3.3	Volume forms on $SE(3)$	430
Appendix B A Mathematica Package for Screw Calculus		435
Bibliography		441
Index		449

Chapter 1

Introduction

In the last twenty years, our conception and use of robots has evolved from the stuff of science fiction films to the reality of computer-controlled electromechanical devices integrated into a wide variety of industrial environments. It is routine to see robot manipulators being used for welding and painting car bodies on assembly lines, stuffing printed circuit boards with IC components, inspecting and repairing structures in nuclear, undersea, and underground environments, and even picking oranges and harvesting grapes in agriculture. Although few of these manipulators are anthropomorphic, our fascination with humanoid machines has not dulled, and people still envision robots as evolving into electromechanical replicas of ourselves. While we are not likely to see this type of robot in the near future, it is fair to say that we have made a great deal of progress in introducing simple robots with crude end-effectors into a wide variety of circumstances. Further, it is important to recognize that our impatience with the pace of robotics research and our expectations of what robots can and cannot do is in large part due to our lack of appreciation of the incredible power and subtlety of our own biological motor control systems.

1 Brief History

The word *robot* was introduced in 1921 by the Czech playwright Karel Capek in his satirical play *R. U. R.* (Rossum's Universal Robots), where he depicted robots as machines which resembled people but worked tirelessly. In the play, the robots eventually turn against their creators and annihilate the human race. This play spawned a great deal of further science fiction literature and film which have contributed to our perceptions of robots as being human-like, endowed with intelligence and even personality. Thus, it is no surprise that present-day robots appear primitive