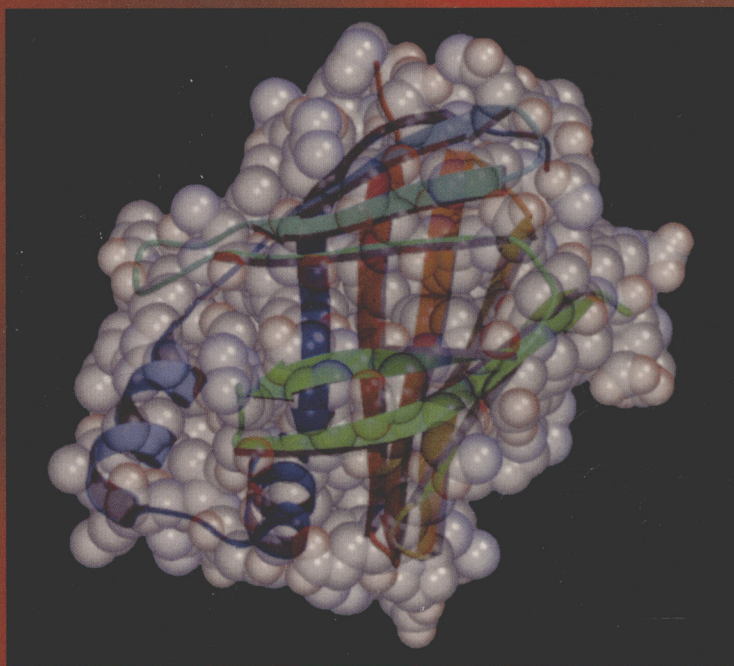


Fundamentals of Crystallography

Second Edition

C. GIACOVAZZO, H. L. MONACO
G. ARTIOLI, D. VITERBO
G. FERRARIS, G. GILLI
G. ZANOTTI, M. CATTI

Edited by C. GIACOVAZZO



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Edited by

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*Professor of Crystallography,
University of Bari, Italy*

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Preface to Second Edition

The volume *Fundamentals of Crystallography* has been published in 1992. The impressive progresses in this last decade obliged us to enrich the treatment of various branches of the book and to add new topics. Most of the material in the first edition has been preserved, important new topics have been added and the presentation of the old ones has been improved. The new Chapter 4, *Beyond ideal crystals*, by C. Giacovazzo, joins topics spread out in the old book with new material, to offer a comprehensive view of the crystallography of the real crystals.

New authors entered the team: the Chapter 5, *Experimental methods in X-ray and neutron crystallography*, has been jointly written by H. L. Monaco and G. Artioli; the Chapter 7, *Mineral and inorganic crystals*, by G. Ferraris.

Learning crystallography may not be easy, and teaching crystallography may be difficult. Students can have quite different curricula, and may be scarcely prepared to easily understand mathematical tools like Fourier transform, convolutions, etc. or to reconstruct in their minds the three (or higher) dimensional spaces in which crystallography operates. In order to overcome the above difficulties, a computer-based tool is associated with this new edition, aiming at helping students to learn, and teachers to be more effective. The acronym of the tool is ABC (*An interactive book on general crystallography*): it has been written by G. Cascarano, C. Giacovazzo, and G. Polidori. ABC devotes special attention to the general crystallography, which constitutes a sort of common background for all people learning crystallography.

I thank the new authors for entering our team and all the authors for their enthusiastic participation in the project. The possible success of this new edition will be entirely due to the efforts and to the creative cooperation of all the authors.

Carmelo Giacovazzo

Preface to First Edition

Crystallography, the science concerned with the study of crystals, is a very old subject. However, only in this century has it developed into a modern science, after the discovery of X-rays and their diffraction by crystals. In recent years crystallography has assumed an increasingly important role in the modern sciences because of its interdisciplinary nature, which has acted as a bridge between, and often as a stimulus for, various rapidly evolving disciplines. Indeed, Chemistry, Physics, Earth Sciences, Biology, Mathematics, and Materials Science have all provided stimuli to the development of new crystallographic interests and techniques. In turn, crystallography has significantly contributed to the advancement of these sciences. Thus, while on the one hand crystallography has been enriched, on the other hand writing a textbook describing all of its aspects has been made more difficult.

Recently, the demand for a compact book that gives a comprehensive account of the modern crystallographic subjects has increased. This volume should therefore be a useful and handy textbook for university courses that cover crystallography, fully or only partially. It should also be useful at the more advanced level required for doctorate studies as well as for experienced researchers.

It was with these ideas in mind that I first set out to co-ordinate the publication, in 1985, of a textbook in Italian (*Introduzione alla cristallografia moderna*, Edizioni Fratelli Laterza, Bari) of which *Fundamentals of crystallography* is not only an English translation, but a completely revised and updated version with a new chapter on crystal physics. It was clear to me that (a) the book had to be written by several authors in order to take advantage of their specific expertise; (b) the different chapters had to be carefully harmonized in order to conform them to a unified plan.

It seems to me that these two requirements are even more valid today and their achievement is entirely due to the creative co-operation of the co-authors of this book.

Two of the co-authors of the Italian textbook, M. Bolognesi and A. Coda, were unable to carry out the translation and revision of their chapters. I wish to express my thanks for their valuable contribution to the previous edition. In this book their topics are treated by G. Zanotti and H. L. Monaco. An additional chapter on crystal physics has been written by M. Catti. I thank the three new authors for entering our team and all the authors for their enthusiastic participation in this project.

Bari
Italy
August 1991

Carmelo Giacovazzo

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Second Edition

Thanks are due:

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First Edition

We have the pleasant task of expressing our gratitude to Dr C. K. Prout and to Dr D. J. Watkin for encouraging us to write this book and for their useful advice. We have also to thank many friends and colleagues who have commented on the manuscript or given technical support. The cover picture was kindly provided by Professor A. Zecchina and Dr S. Bordiga, who used the Chem-X software of Chemical Design Ltd, Oxford, UK. Thanks are also due from C. Giacobazzo to Mr D. Trione for his accurate and rapid typing of the manuscript, to Mr G. Pellegrino for his aid with the drawings and to Ms C. Chiarella for her technical support; from H. L. Monaco to Mr M. Tognolin for the figures, to Mr R. Pavan for the photographs, and to Mrs A. Migliazza for typing the manuscript; from F. Scordari to Professor F. Liebau for the critical reading of the silicate section and for helpful suggestions, Mr Trione is also acknowledged for the typing of the manuscript; from G. Gilli to Professor V. Bertolasi, Dr V. Ferretti, and Dr P. Gilli for their help in the critical reading of the manuscript; from M. Catti to Mr M. Bandera for his aid with the drawings.

Lumen propagatur seu diffunditur non solum Directe, Refracte,
ac Reflexe, sed etiam alio Quarto modo, DIFFRACTE.

Francesco Maria Grimaldi

Physico-Mathesis de Lumine, Coloribus, et Iride

Bologna, 1665

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