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FOREWORD

Vibration level is one of the indicators of quality and reliability in various machines and devices In order to protect objects against dynamic disturbances, the vibration protecting systems (VPS) placed between a vibration source and an isolated object are widely applied. The most important characteristics of VPS are its natural frequency and load bearing capacity (response to static load) The vibration isolation quality is considerably enhanced by reducing the natural frequency of VPS In this way, however, load bearing capacity in linear VPS cannot be maintained without an increase in overall dimensions.

This dichotomy between load bearing capacity and natural frequency is eliminated in VPS with quasi-zero stiffness. In these systems, any desired stiffness may be ensured at a given load bearing capacity. Many papers and reports have been published on the research and development of vibration protecting systems with quasi-zero stiffness (VPSZS). Several theses are devoted to their design solutions.

This book makes an attempt to systematically set forth theoretical and practical problems of VPSZS. It gives a review of their design features, theory, technical design methods, and ways of their application. The small volume of the book has not allowed to thoroughly elucidate all the problems. The authors, however, believe that this book will help in making a proper judgement of merits and demerits of VPSZS and will encourage their introduction to science and engineering.

CONTENTS

CHAPTER

ONE

SOME FACTS OF THE THEORY OF VIBRATION PROTECTING SYSTEMS

Vibration is basically controlled by reducing it at its source. Sometimes this approach is successful. In hand-operated impact machines, for example, vibration has been brought to the sanitary standard levels.

In most cases, however, the range of external excitations from the power unit and environment is too wide to consider all vibration sources. Then vibration isolators appear to be the best means of vibration protection.

1.1 FUNDAMENTALS OF VIBRATION ISOLATION THEORY

In order to reduce vibration, various devices 2 placed between a vibration source and an isolated object are used (Fig. 1.1). The vibration source may be either the object *3* (dynamic excitation, Fig. 1.1*a*) or the base *1* (kinematic excitation, Fig. 1.1*b*). The equations of motion of the object *3* for these cases are, respectively,

$$m\ddot{x} + F = Q\,(t)$$

$$m\ddot{x} + F = -\,m\ddot{\eta}\,(t) \tag{1.1}$$

Here, m and x are the mass of object *3* and its coordinate relative to the base *1*, counted off from the static equilibrium position, respectively; $Q\,(t)$ is the external force acting upon object *3*; $\eta\,(t)$ is the time history of motion of base *1*; F is the dynamic reaction of device 2. From the mathematic point of view both equations are identical, therefore we shall refer to the first one.

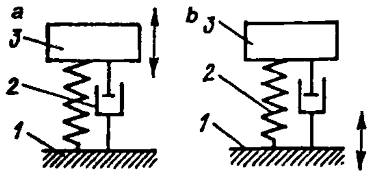

Figure 1.1 Schematic diagram of a system with one degree of freedom

Device 2 may be represented by any elastic element: metal spring, rubber, plastics, pneumatic spring, cable, etc. In most cases dynamic reaction of elastic elements may be represented as

$$F = F(x, \dot{x}) \tag{1.2}$$

Here x and $\dot{x}$ are coordinates (deformation) of device 2 and its time derivative, respectively. The reaction of elastic elements may be measured statically, too; then we get

$$F = F(x) \tag{1.3}$$

This correlation is known as a static force characteristic of device 2 or, simply, a force characteristic. Depending on the form of Function (1.3) force characteristics are subdivided into linear and nonlinear. For the linear system

$$F = cx \tag{1.4}$$

where c is the stiffness of element 2.

The condition of the static equilibrium yields

$$F_\lambda = c\lambda = mg \tag{1.5}$$

Here, λ is the static deformation of the elastic element; g is the acceleration of the gravitational force; F_λ corresponds to load bearing capacity of VPS. For a linear case, load bearing capacity of VPS is proportional to stiffness of an elastic device and its static deformation.

We assume that dynamic reaction (1.2) is representable in the form

$$F = cx + b\dot{x} \tag{1.6}$$

where b is the coefficient specifying damping properties of device 2. Let this system be subjected to the harmonic force only. Then the equation of motion is written as

$$m\ddot{x} + b\dot{x} + cx = H \cos \omega t \tag{1.7}$$

Here, H and ω are amplitude and frequency of the exciting force. For steady-state vibrations we have

$$x = \frac{H}{m\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}} \cos(\omega t - \theta) \tag{1.8}$$

where

$$\omega_0 = \sqrt{c/m}, \quad \beta = b/(2m) \tag{1.9}$$

$$\operatorname{tg} \theta = 2\beta\omega/(\omega_0^2 - \omega^2) \tag{1.10}$$

As is well known, the angle θ defines the phase shift between the exciting force and vibrations of object *3* (Fig. 1.1*a*). Substituting Eq. (1.8) into (1.6) we find dynamic reaction for the linear VPS

$$F = kH \cos[\omega t - (\theta - \varepsilon)] \tag{1.11}$$

where

$$k = \sqrt{\frac{\omega_0^4 + 4\beta^2\omega^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}} \tag{1.12}$$

$$\operatorname{tg} \varepsilon = 2\beta\omega/\omega_0^2 \tag{1.13}$$

Here $(\theta - \varepsilon)$ is the phase shift between the force acting upon the base and the exciting force. The force F defined by Exp. (1.11) acts upon base *1* (Fig. 1.1*a*). Evidently, the less k, the better the vibration protecting properties of the system. Therefore, k is called a vibration isolation coefficient of VPS. If $k < 1$, VPS application is effective; if $k \geq 1$, it is wasteful.

As it is seen from the expressions for a vibration isolation coefficient, if $\omega/\omega_0 = \omega_c > \sqrt{2}$, VPS is effective and with increasing ω_c vibration isolation is improved. The analysis of Exp. (1.12) indicates that the lower the natural frequency of VPS, the more advantageous its application is. In other words, the elastic elements have to be very soft. Damping should be introduced if $\omega_c < \sqrt{2}$. If $\omega_c \geq \sqrt{2}$, damping properties have a negative influence because damping increases the vibration source effect on a vibration isolated object.* It is of some use when near the resonance.

Without dwelling on the other properties of elastic elements having linear force characteristics we shall note that in order to improve the vibration isolation an elastic element with the lowest stiffness should be used. However, reduction in stiffness leads to increasing of overall dimensions of the elastic element (since its static deformation is increasing). When choosing the correct stiffness it should be remembered that in most cases the overall dimensions of a vibration isolator are limited by the operating conditions of the protected object.

These shortcomings together with others inherent to linear elastic systems prompted the VPS designers to turn to nonlinear elastic systems.

1.2 PASSIVE AND ACTIVE VIBRATION PROTECTING SYSTEMS. MERITS OF VIBRATION PROTECTING SYSTEMS WITH QUASI-ZERO STIFFNESS

The above-described vibration protecting method is a representative of passive systems. To protect objects against low frequency vibrations where conventional passive systems are not very effective, or to protect them against a varying

* This statement is correct only for viscous damping. Many common types of damping (such as structural, hysteretic, etc.) do not have such an adverse effect at $\omega_c \geq \sqrt{2}$ (Editor of translation).

vibration spectrum, the active VPSs have lately been used. The active VPS has an additional power source and some control devices. A diagram of one active VPS [31] is given in Fig. 1.2. Here mass m is installed on elastic elements c and on magnetoelectric converters *1* and *3*. The displacement transducer *2* sends a signal $x(t)$ to the amplifier *6*. The amplified signal is differentiated in device *5* and after another amplification in device *4* is sent to the magnetoelectric converters *1* and *3* thus executing the feed-back and changing the parameters of the whole VPS.

The main advantage of active vibration protecting devices is their high load bearing capacity at a low dynamic stiffness. It is achieved, however, by complicating the VPS structure. It should be noted that a given load bearing capacity at a low stiffness may be provided by combining elastic elements linked in a definite manner. The schematic diagram of one of VPSZS [11] is given in Fig. 1.3. It contains a vibration isolated object of the mass m which may perform only vertical vibrations and is connected to the base by means of both a vertical load bearing spring of stiffness c_1 and two identical side (correcting) springs of stiffness c_2.

Let the reference point x coincide with the static equilibrium position, correcting springs being horizontal when $x = 0$. In order to deflect the vibration isolated object from the equilibrium position it is to be subjected to the force F the value of which is equal to the resultant restoring force, i.e., to the spring reaction.

Applying the principle of virtual displacements we find

$$(F - mg - F_1)\,\delta x - 2F_2\,\delta L = 0 \tag{1.14}$$

Here F_1 and F_2 are reactions of vertical and horizontal springs, respectively,

$$F_1 = c_1\,(x - \Delta b),\ F_2 = c_2\,(L - \delta_0) \tag{1.15}$$

where L is the length of a correcting spring in an arbitrary position; $L_{01} = b + \Delta b$ and $L_{02} = a + \delta_0$ are lengths of undeformed vertical and horizontal springs, respectively.

Taking into account the correlation

$$L = \sqrt{x^2 + a^2}$$

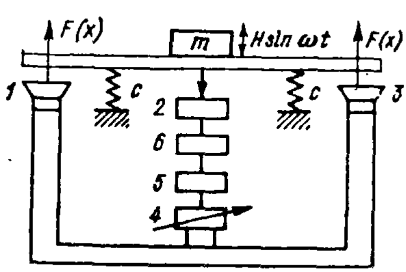

Figure 1.2 Active vibration protecting system.

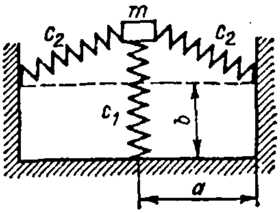

Figure 1.3 Structural model of vibration protecting system with quasi-zero stiffness.

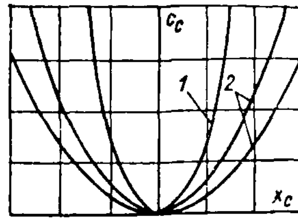

Figure 1.4 Stiffness characteristics of vibration protecting system with quasi-zero stiffness: *1*) $c_{2c} = 4.0$; $a_c = 0.125$; 2) $c_{2c} = 2.0$; $a_c = 0.250$.

and equalities (1.15) , we derive from Eq. (1.14) the expression for the system reaction

$$F = c_1x + 2c_2x\left(1 - \frac{a + \delta_0}{\sqrt{x^2 + a^2}}\right) \tag{1.16}$$

Here the static equilibrium condition is taken into account. Let's find the range of the parameter values in which the elastic reactions are optimal. By differentiating expression (1.16) with respect to x, the system stiffness referred to the stiffness of a vertical spring can be found as

$$c_c = 1 + 2c_{2c}\left[1 - \frac{1 + a_c}{(1 + x_c^2)^{3/2}}\right] \tag{1.17}$$

where $c_{2c} = c_2/c_1$; $x_c = x/a$; $a_c = \delta_0/a$.

In the static equilibrium position we have

$$c_c = 1 - 2c_{2c}a_c \tag{1.18}$$

This expression indicates that in the static equilibrium position stiffness of the system may assume any values depending on parameters c_{2c} and a_c. The zero stiffness condition in this position is

$$2a_cc_{2c} = 1 \tag{1.19}$$

Figure 1.4 presents plots of stiffness characteristics of VPSZS for different values of c_{2c} and a_c satisfying condition (1.19).

The largest displacement from the equilibrium position at which stiffness of VPSZS is less than (or equal to) any given value c can be found from expression (1.17)

$$s_1 = \sqrt{\left(\frac{1 + 2c_{2c}}{1 + 2c_{2c} - c'}\right)^{2/3} - 1}$$

It follows from the aforesaid that VPSZS provides a given load bearing capacity with as low as desired stiffness of the system. The load bearing capacity of an elastic system is determined in VPSZS in the same way as in the linear system by the static equilibrium condition, i.e.,

$$F_{\lambda} = mg = c_1 \Delta b \tag{1.20}$$

Consequently, the load bearing capacity in the VPSZS depends only on stiffness of a basic spring and its initial deformation. Moreover, the structure of this system is simple and its overall dimensions are the same as those of a linear one. In this way, the main shortcoming of linear VPS is essentially eliminated.

CHAPTER

TWO

STRUCTURAL FEATURES OF VIBRATION PROTECTING SYSTEMS WITH QUASI-ZERO STIFFNESS

2.1 CLASSIFICATION OF VIBRATION PROTECTING SYSTEMS WITH QUASI-ZERO STIFFNESS

Elastic systems of quasi-zero stiffness have numerous applications. They are used in vibration measuring instruments for suspending sensitive elements [6, 26], technical and biological objects [10, 17], in hand-held tools [7, 12, 21, 28], in vehicle seats [4, 27], etc.

All VPSZSs may be classified into four main groups. The first group is the largest and contains load bearing elastic elements with constant positive stiffness as well as devices with negative stiffness[1]. The first group of systems (Fig. 2.1) were analyzed in Refs. [3, 9, 11]. Their application to vibration isolation of operators's seats in vehicles is described in Ref. [27], to impact action hand-held machines in Refs. [21, 8, 28], to railway car suspensions in Refs. [5, 33].

The second group of elastic systems, (Fig. 2.2) has no structurally designated load bearing elastic elements[2]. It consists of the systems used in the first

[1] Invention Certificate 343038 (USSR), I.C. 203556 (USSR), I. C. 192132 (USSR).
[2] I.C. 514765 (USSR), I. C. 272176 (USSR).

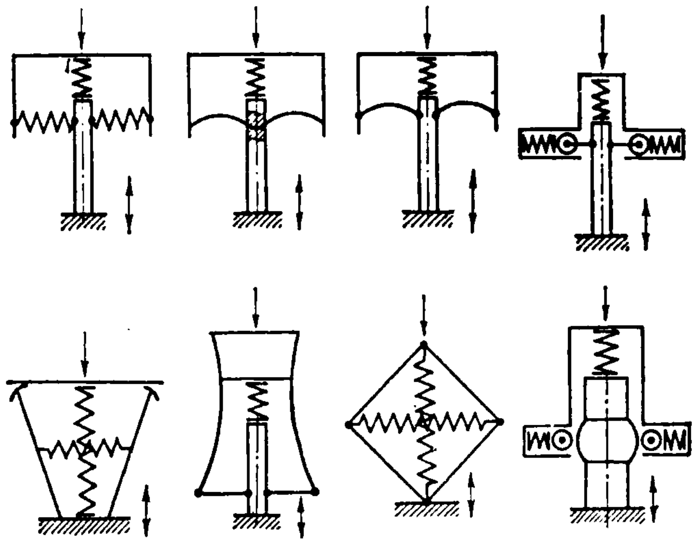

Figure 2.1 Schematic diagrams of VPSZS of the first group.

group as devices with negative stiffness. This group was analyzed in Refs. [5, 18, 25, 32].

The third group of systems is characterized by the fact that during their displacements there are intervals in which the stiffness is reduced.[1] These systems contain elastic elements and levers of a variable structure (Fig. 2.3).

[1] I.C. 259942 (USSR), I.C. 331951 (USSR).

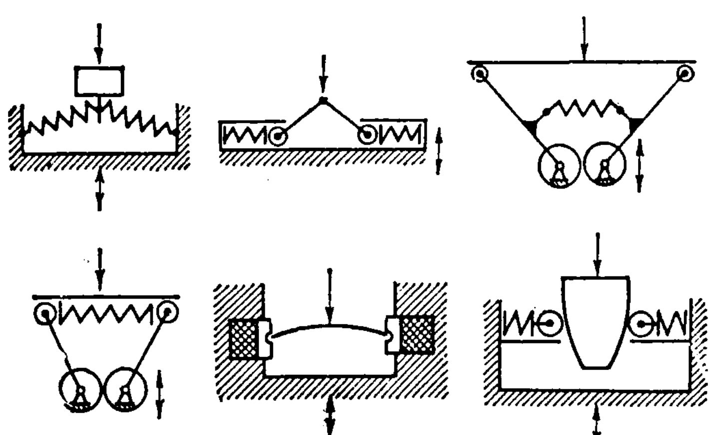

Figure 2.2 Schematic diagrams of VPSZS of the second group.

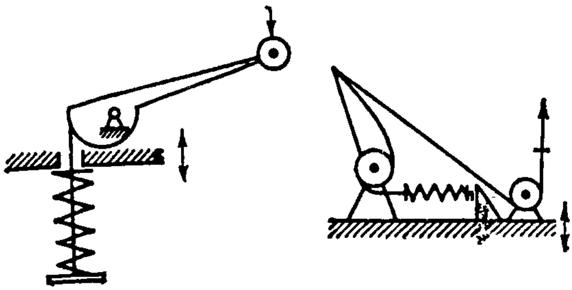

Figure 2.3 Schematic diagrams of VPSZS of the third group.

The fourth group comprises cable vibration isolators in which the displacement intervals with reduced stiffness are developed due to buckling of vibration isolators under the axial deformation.[1] In recent years cable vibration isolators are being intensively developed [19, 20].

This book deals with the analysis of elastic systems of the first and second groups.

2.2 MECHANICAL SYSTEMS CHARACTERIZED BY NEGATIVE STIFFNESS DISPLACEMENT INTERVALS

If stiffness of a load bearing structure (load bearing elastic element) is reduced by a sequential attachment to it of additional elastic elements, it results in an undesirable increase in overall dimensions of the system. When the elastic elements are connected in parallel stiffness the load bearing element may be reduced only by using elastic systems or elements possessing displacement intervals which are characterized by negative stiffness.

The devices for changing stiffness of load bearing elastic elements are known under different names: system with a jump, flapping system (diaphragm), stiffness compensator, stiffness corrector, v-shaped stiffness corrector, etc. We will use "stiffness corrector."

Stiffness correction consists of reducing the stiffness of the load bearing elastic element by connecting in parallel to it the devices-correctors having negative stiffness.

Examples of mechanical systems having displacement intervals with negative stiffness include systems which have at least one position of unstable equilibrium.

In spite of the structural variety of elastic stiffness correctors containing linear elastic elements they may be represented by three basic diagrams. Fig. 2.4 illustrates a type *A* stiffness corrector. It consists of an elastic element 2, one end of which is hinged to the base *3*, the other to the rod *1* placed on the guide

[1] I.C. 259162 (USSR), I.C. 172258 (USSR).