LINEAR PROGRAMMING IN SINGLE- & MULTIPLE-OBJECTIVE SYSTEMS

James P. Ignizio

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PREFACE

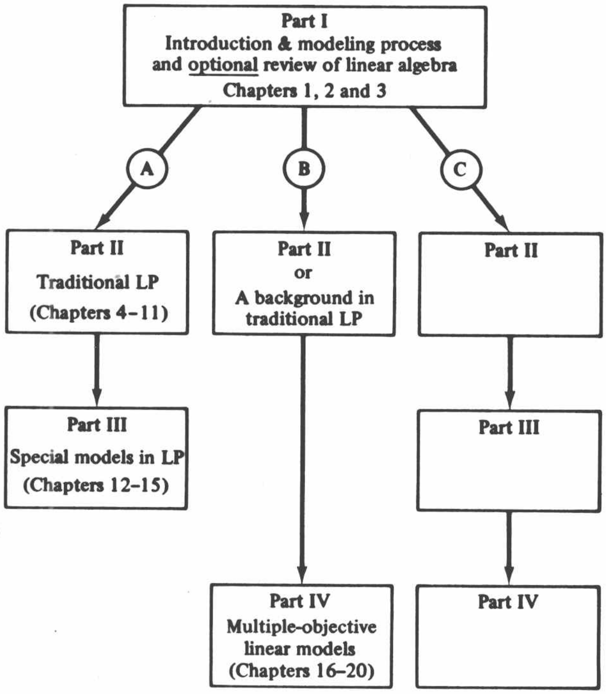
Within the very broad subject area known as "optimization," the most widely known and implemented technique for modeling and solution is, by far, the methodology denoted as "linear programming." Linear programming, in turn, deals with the optimization (in terms of a measure or measures of performance) of a system that may be modeled as a set of linear, mathematical functions.

At the time of the writing of this text, the field of linear programming was already midway into its fourth decade and, as such, it should not be surprising that a substantial number of linear programming textbooks have already been published. These other books differ from one another primarily in regard to style, length, depth of coverage, and level of mathematical sophistication. They differ little, however, in the basic subject matter: the development of the linear programming model and the method of solution. This is neither surprising nor intended as a criticism, since the traditional presentations of the modeling and solution process should not be radically changed simply for the sake of change. Consequently, the reader may well ask why the need for another textbook on linear programming, and how this book differs from the others in the field.

This textbook was written with two primary purposes in mind. First, it has been written as an introductory, yet relatively complete exposition of linear programming for the practitioner. That is, the text is intended for those individuals who wish to both understand and implement the methodology. The second purpose of the text is to incorporate, in the coverage, a *unified* presentation of both traditional (i.e., single-objective) linear programming and multiple-objective linear programming. These two purposes, and in particular the latter, are what serve to differentiate this text from most other linear programming textbooks presently in print.

To accomplish the first purpose, the author has attempted to present the mathematical background, and the modeling and solution process, in a clear and concise fashion and accompanied by illustrative examples that serve to demonstrate and reinforce the discussions. The accomplishment of the second purpose rests upon the initial presentation of a unified mathematical model, known as the baseline model. The reader is then provided with the set of assumptions necessary to either convert this baseline model to the traditional linear programming model or to the multiple-objective version and, in either case, to then solve, interpret, and evaluate the results obtained as well as the impact of assumptions used in the conversion. The inclusion of the multiple-objective model and the use of a unified approach are factors that contribute to making this linear programming textbook unique and, hopefully, more flexible then texts dealing primarily with the traditional, single-objective model.

The textbook has been written so as to support either a sequence of courses or as a text for a course either in strictly traditional linear programming or multiple-objective linear programming. For the latter course, it is assumed that the student has had a background in single-objective linear programming. The following diagram serves to indicate these options as well as prerequisite material.



In the diagram, three alternative plans for course content are given. Plan A consists of a one-term course in, primarily, traditional linear programming. Since there are a total of 15 chapters in this plan, there is more than enough material for the term, and the instructor may choose to delete some topics or give them less emphasis. Plan B is also a one-term course, in which the concentration of the course is placed on Parts One and Four, thus leading to a plan of study that emphasizes the multiple-objective models and methods in linear programming. Under such a plan, the material in Part Two (i.e., the traditional linear programming background) need not be covered if the student has a previous background in this area. The final plan of study, C, is intended as a two-term sequence of linear programming courses and covers the entire text.

Regardless of the study plan selected, the alternatives are appropriate for either senior-level or first-year graduate students having some familiarity with linear algebra. When directed specifically at the graduate student, the course may be enhanced by individual or group projects.

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CONTENTS

part one

THE UNIFIED LINEAR MODEL

INTRODUCTION

The Linear Decision Model

Despite the claims of "seat-of-the-pants" decision making, divine revelations, and woman's intuition, the human mind is simply not equipped to perform a thorough, systematic, and objective analysis of most of the large and complex decisions that we often face. Consequently, the majority of credible approaches to decision making must employ an aid: a *model* of the problem under investigation. Such models do not, as some managers fear, *make* decisions. Rather, they are, or should be, used to *complement* the judgment process, to help clarify the situation, and thus to provide for an improvement in the decisions and policies ultimately set forth. It would then seem obvious that, the better the model, the better should be the resulting decision.

In this text we restrict our attention to a single, yet exceptionally useful and important type of quantitative model. This is the *linear* mathematical model: a mathematical model consisting solely of linear functions. A linear function, in turn, is one in which all terms consist of a single variable and where each variable is raised to the power 1. Thus, functions (1.1) and (1.2) are linear functions:

$$x_1 + x_2 = 10 \quad (1.1)$$

$$3x_1 + 7x_2 - 8x_3 = 7 \quad (1.2)$$

$$2x_1^2 + 3x_2^{3.7} = 82 \quad (1.3)$$

$$4x_1 - 3x_1x_2 + 2x_2^2 = 30 \quad (1.4)$$