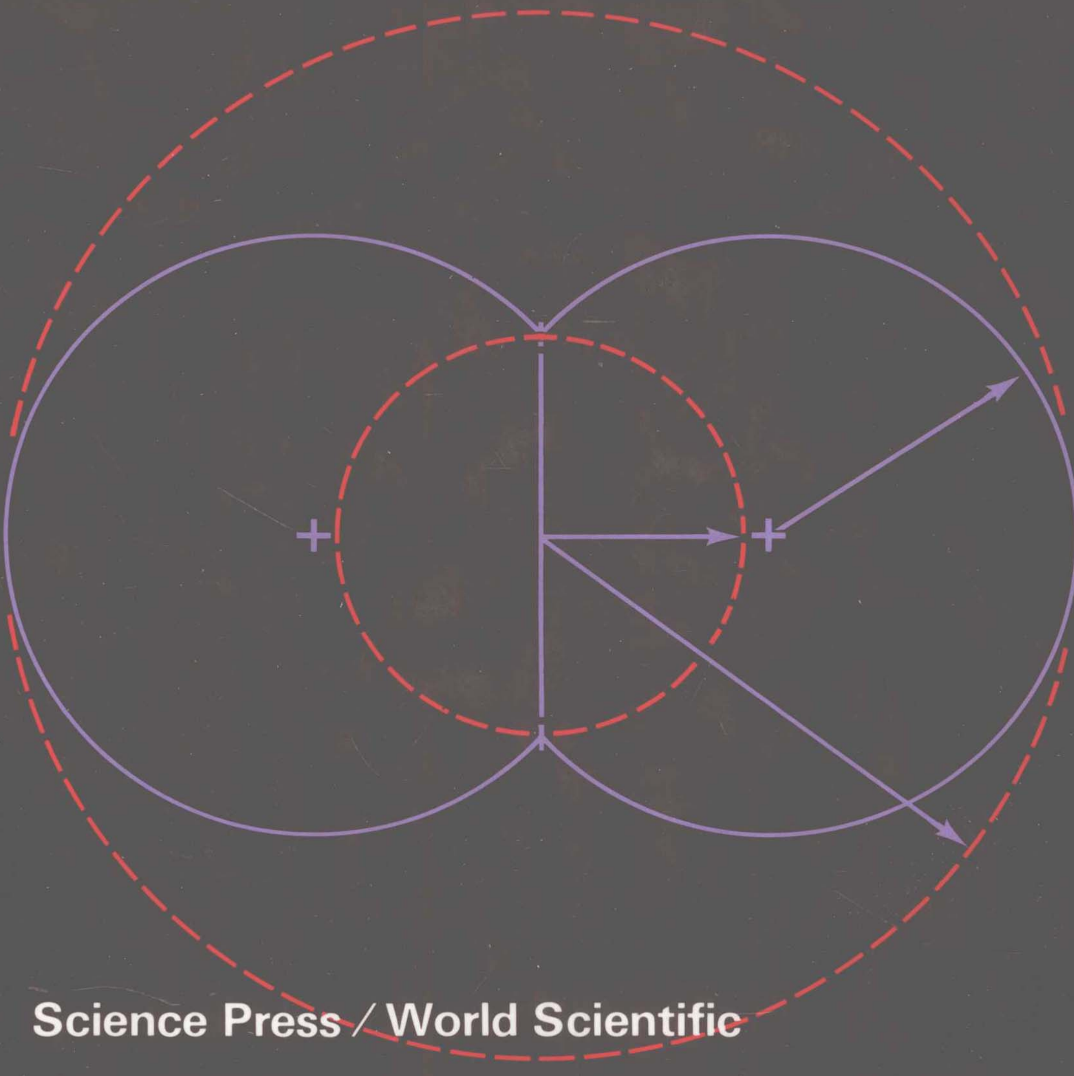
PHYSICS OF DENSE MATTER

Y. C. Leung



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PHYSICS OF DENSE MATTER

This book provides an introduction to contemporary research in the physics of dense matter, which refers to matter in the density range 500–10^{17} g/cm^3. The book concentrates on the investigation into the structure of matter with an aim in deriving the equation of state for matter at such densities. It is divided into four chapters each referring to a specific density range, which is determined according to the nature of physics involved. Presentation is intended for the application of dense matter physics to astrophysical problems, such as those manifest in the supernova process and in the structure of the neutron star. This book may be used as a text book for graduate study in astrophysics as well as a reference source for research scientists in this field.

Chapter 1 deals with matter found normally in white dwarf stars. The dominant physics for matter at these densities is fairly well known. This chapter serves to introduce the nature of the physical problem and the general method of attack. Many-body techniques like the Hartree and Hartree-Fock methods at zero and finite temperatures are presented.

Chapter 2 reviews current methods employed in the study of matter at subnuclear densities. At these densities matter consists of heavy nuclei. Nuclear physics and nuclear models are needed to obtain an understanding of its structure. We take as nearly as possible the phenomenological approach so as to maintain close contact with reality.

Chapter 3 studies matter at nuclear and transnuclear densities. A great deal of efforts

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had been spent in the study of nuclear matter in the past and several well-developed theories had evolved over the years. We present here the derivations of these theories and work out the mathematical details involved in the computations so as to provide a general background for readers to understand many of the research articles published in the field of dense matter physics employing these theories.

Chapter 4 is an extension of the theoretical methods presented before to deal with matter at ultradense matter. The relevant topics are related to the formation of baryonic matter, pion condensation and quark matter. They are presented in a concise way.

The development of modern physics led to a major advancement in the understanding of the structure of matter. Conversely, a study of the structure of matter provides a comprehensive review of modern physics. The present volume serves thus the dual purposes of being an investigation into the structure of dense matter and at the same time a review in modern physics. As it turns out to be the case that the complexity of the problems in dense matter physics increases with increasing density. Hence, by ordering the text according to ascending densities the book develops its theoretical tools progressively, and may therefore be used as a text book in astrophysics. Exercises are provided for individual sections in the book. We have tried to make all derivations explicitly clear so that the book may be useful for self-learning or be called upon as a reference to clear up obscure derivations in journal articles. It is also intended as a monograph containing sufficient reference materials as to be useful to an active researcher in the field.

To My Parents

PREFACE

This book grew from lecture notes prepared for a ten-week summer short-course on dense matter physics that I gave to a group of students and physicists at the Beijing Normal University (China) in 1979. My aim was to introduce the field of dense matter physics which saw rapid development in the past ten to fifteen years to an audience who was not yet active in the field. I wanted to make the subject matter as simple and yet as complete as I could by concentrating on only a few well-developed topics, and I tried to spell out all details as clearly as possible. My hope was to provide enough background information on these topics that the participants of the short-course might be prepared to comprehend the current research articles. In reality, I was far from reaching my goal, and so in 1981 when I was approached by the Science Press to publish the lecture notes, I decided to rewrite them. The final result comprises the present book, which contains fewer topics but more detailed and complete description of each.

This book is intended to prepare the readers for theoretical studies in dense matter physics. It may also be used pedagogically as an introduction to the many-body theory with emphasis on illustrative examples. Many mathematical operations needed for this field are carried out in details, usually not in the most elegant way but in a practitioner's way, so that readers who are handicapped in background training can still follow. Hopefully this book will enable more newcomers to enter into the study of dense matter.

I wish to thank my colleagues at the Beijing Normal University for comments and helpful suggestions on the contents of this book. Among them I wish to mention the following persons: Liang Shao-rong, Li Zong-wei, Gao Shang-hui, Ge Yun-zao and Shi Tian-yi, whom I had the pleasure to work with during the preparation of this book. Finally, I wish to thank Chen Chung-kuang of the Graduate School of the Academy of Sciences for arranging with the Science Press for me to publish this book.

Y.C. Leung

N. Dartmouth, Massachusetts

May 4, 1983

CONTENTS

INTRODUCTION

In this book an attempt is made to provide a theoretical background for the study of the structure of very dense matter. The densities under investigation are those between 10^4 to 10^{16} g/cm^3 found in inert stellar objects like the white dwarfs and neutron stars. Even though dense matter physics is also related closely to contemporary investigations in heavy ion collisions and the Early Universe, we shall make little reference to these issues. Instead the emphasis is placed on the stellar objectes with the final aim in deriving the appropriate equation of state for matter forming them. Thus, not all aspects of dense matter physics are dealt with but just the structure of dense matter.

All macroscopic solids on earth have average densities low compared with the range of densities mentioned above. Their structures are of electrostatic in origin. We shall say very little about them. We begin our study at densities above 10^4 g/cm^3, for which electrostatic effects play minor roles. At these densities quantum effects and nuclear interactions occupy the central roles. These are the topics under review here.

The average density of our Sun is very low only of the order of 1.5 g/cm^3. This is because energy coming from thermal nuclear reaction prevents the collapse of matter to high density. Only the helium core at the center of the Sun is reaching high density, but the core is still small due to the youthfulness of our Sun.

As a star ages its helium core grows in size. The support of the helium core is due to degenerate electrons, which in a dense state can resist considerable pressure. The relationship between density and pressure for a substance is called its equation of state. Once the equation of state is known the mass and radius of the core can be determined. The maximum stable size of such a core is known to be given by the Chandrasekhar limit, which is about 1.2 solar mass. Chapter 1 deals with topics which are related to the inert core below the Chandrasekhar limit. Matter in the core is composed of nuclei whose atomic numbers are below that of iron. We refer to such substance as subferrous matter. In Chapter 1 general many-body techniques like the Hartree and Hartree-Fock methods are introduced. Relevant concepts in dealing with degenerate fermions, particle interactions and thermal effects are reviewed for the familiar case of the electron gas. It serves to establish the notations and general method of approach.

What then is the fate of a star whose helium core exceeds the Chandrasekhar limit? A sequence of events would have followed leading eventually to a possible supernova process. There are considerable astrophysical interests in knowing the detailed nature of the supernova process for it will lead to an understanding of the chemical composition of our Solar System. Although some of the basic mechanisms of the supernova process are known, many more fine details are needed to complete the picture

Chapter 2 makes an attempt to understand the physical properties of matter during the process of a gravitational collapse. The results shown in Chapter 2 are far from complete and undoubtedly a great deal of progress in understanding matter at such densities will be made in the near future. Chapter 2 serves to introduce the methods presently employed to tackle such problems. In presenting these methods we have not try to cover all possible approachs but concentrate on a few which are phenomenological and show promise for future development. We try to indicate their strength and weakness whenever we can. Chapter 2 deals with matter at densities above those of the white dwarf stars but below that found in atomic nuclei. It is comparable to a gas of nuclei whose atomic numbers are above that of iron and is referred to as subnuclear matter. The transformation in matter's composition with density requires careful analysis. A definitive knowledge of the process will be crucial to an understanding of the atomic abundance of the Solar System.

Matter at subnuclear densities is quite compressible. It will not halt a gravitational collapse once it is initiated. Stellar objects involved in a collapse must necessarily pass through a stage where their core densities are as high as matter densities found inside atomic nuclei. Densities in atomic nuclei are quite similar and are usually specified by a single density called the nuclear density. Gravitational collapse will compress matter density well beyond the nuclear density. The halting of a gravitational collapse is due to the structure of matter at transnuclear densities. Consequently, knowledge of matter at such densities is crucial to the understanding of the supernova process. In Chapter 3 we introduce techniques in nuclear physics appropriate to the study of matter at approximately the nuclear density. The purpose is to provide the general background needed to understand many of the research articles published in this field. We try to work out all the mathematical details involved in the theory, so that newcomers to this field will not be hindered by obscure derivations. We leave out however philosophical comments on such theory since they can be found in severalgood review articles quoted at the end of each section. The historical sketch on the development of the theory is also left out. In Section 14, we introduce relativistic models which allows the extension of nuclear physics techniques to deal with matter at densities above nuclear density. Chapter 3 covers a density range terminating at about 10^{16} g/cm^3.

Matter at densities above 10^{16} g/cm^3 is called ultradense matter. Physical processes not commonly found in nuclear matter will appear in ultradense matter. The discussion of these processes composes the subject matter of Chapter 4. They are listed under the headings of baryonic matter, pion condensation and quark matter. Recognition of these processes is largely theoretical. We try to present these issues in a unified way without having to introduce drastically new devices. Since the tone of this book is highly phenomenological, many of the sophisticated diagrammatic techniques have been neglected, and therefore it is impossible to give a full treatment of some of the topics.

There are three appendices to this book. Appendix A compiles the physical constants and astronomical data which will be useful for the subject matter. Appendix B presents the mathematical functions employed in the text. Appendix C is a compendium of the equation of state for matter at various densities. It represents our present day knowledge of such matter. Exercises are provided for each section and are given at the end of the book.

CHAPTER 1

Regime I: $500 < \rho < 10^8$ g/cm^3. Subferrous Matter.

1. Variational Methods

The study of the physical properties of matter by means of theoretical methods comes under a formulation generally referred to as the many-body problem, in which matter is assumed to consist of a number of basic constituents called particles obeying known physical laws. All physical properties of matter are to be deduced from the dynamics of these constituents. The quantum mechanical formulation of the many-body problem is based on a generalization of the rather successful Schrodinger equation formulation of the one- and two-body problems. Thus, we write for a N-body system the following equation:

$$H \Psi = E \Psi \qquad (1.1)$$

where Ψ is a function of the 3N spatial coordinates $\vec{x}_i$, and the Hamiltonian H is given by:

$$H = \sum_{i=1}^{N} - \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i=1}^{N} V(\vec{x}_i) + \sum_{\substack{i,j=1 \\ i<j}}^{N} v(\vec{x}_i - \vec{x}_j) , \qquad (1.2)$$

which consists of the kinetic energy terms and the interaction terms. The particles are assumed to interact with the background through $V(\vec{x}_i)$ and pairwise through $v(\vec{x}_i-\vec{x}_j)$. In a quantum mechanical formulation of particle dynamics, each particle is provided with its own spatial coordinates, and hence there are 3N coordinates. Interactions between two particles are prescribed by means of $v(\vec{x}_i-\vec{x}_j)$, which couples their coordinates. Without $v(\vec{x}_i-\vec{x}_j)$, (1.1) will be separable into N sets of independent coordinates and all particles are then free from each other. We have in mind here a system of identical particles and there will be the same potential functions $V(\vec{x}_i)$ and $v(\vec{x}_i-\vec{x}_j)$ for all particles. For the general case additional subscripts for V and v will be needed. These subscripts are suppressed here for clarity.

It is clearly a very ambitious task to solve for Ψ from (1.1). Furthermore, when the system represents a macroscopic system, the determination of Ψ is not only impossible but not necessary. For such a system, the process of a physical measurement samples some average characteristics of the system, and this is far from a determination of the many-body wave function Ψ. However, the concept of a wave function is still useful as a vehicle in establishing relations among physical parameters. Henceforth, we shall attack the problem in a manner consistent with an approximate solution of the many-body problem (1.1).

One common strategy in finding the ground state energy, or the lowest eigenvalue of (1.1), is by means of the variational method. The variational principle tells us that the expectation value of the Hamiltonian with an arbitrary function must be at least as great as the lowest expectation value of the Hamiltonian. One therefore specifies a class of trial wave functions which are presumed variable functionally from one to the other. The variational method then provides the necessary equations for the selection of the one trial wave function which gives the lowest expectation value of the Hamiltonian within the class. Such a trial wave function may be a useful approximation to the true ground state wave function if the class is properly chosen, and its expectation value would be a close upper bound of the ground state energy of the system. Different classes of trial wave functions may also be appraised by comparing their lowest expectation values obtained. We shall make use of three classes of trial wave functions. They shall be designated as (1) Hartree, (2) Hartree-Fock, and (3) Jastrow trial wave functions.

After the approximate ground state wave function is found for the variational method. The excited state wave functions may be established successively. The first excited state wave function must be orthogonal to the ground state wave function and at the same time yielding the next lowest expectation value of the Hamiltonian. The second excited state wave function would again be orthogonal to the first two with the next lowest eigenvalue, and so forth. We shall be dealing mainly with the ground state of a system.

A N-particle Hartree trial wave function consists of a product of N single-particle wave functions:

$$\Psi = \phi_1(\vec{x}_1) \cdot \phi_2(\vec{x}_2) \; \ldots\ldots\ldots \; \phi_N(\vec{x}_N) \; . \tag{1.3}$$

The single-particle wave functions ϕ_i are individually normalized:

$$\int d^3x \;\; \phi_i^*(\vec{x}) \; \phi_i(\vec{x}) \;\; = \;\; 1, \tag{1.4}$$

so that,

$$\int d^Nv \;\; \Psi^* \; \Psi \;\; = \;\; 1 \; , \tag{1.5}$$

where

$$d^Nv \;\; = \;\; d^3x_1 \; d^3x_2 \; \ldots\ldots\ldots \; d^3x_N \; . \tag{1.6}$$

Let us denote the expectation value of the Hamiltonian by:

$$< H > \;\; = \;\; \int d^Nv \;\;\; \Psi^* \; H \; \Psi \; . \tag{1.7}$$

A variation in the trial wave function Ψ: $\Psi \to \Psi + \delta\Psi$ gives rise to a variation in $<H>$: $<H> \to <H> + \delta<H>$. Since Ψ is complex, Ψ and Ψ^* may be varied independently. The variation should however preserve the normalization conditions (1.5). This can be accomplished by means of the method of Lagrange multiplier. The variational condition to be satisfied by the trial wave function having the lowest expectation value of the Hamiltonian is:

$$\delta \; \{ \; < H > + \;\; \lambda \int d^Nv \; \Psi^* \; \Psi \; \} \;\; = \;\; 0 \; , \tag{1.8}$$

where λ is the Lagrange multiplier.

Since the single-particle wave functions are normalized, $<H>$ is given by:

$$\begin{aligned} < H > \; &= \sum_i \int d^3x \; \phi_i^*(\vec{x}) \; \{ - \frac{\hbar^2}{2m} \nabla^2 + V_i(\vec{x}) \} \; \phi_i(\vec{x}) \; + \\ &+ \sum_{i<j} \int d^3x \; d^3y \; \phi_i^*(\vec{x}) \; \phi_j^*(\vec{y}) \; v(\vec{x}-\vec{y}) \; \phi_i(\vec{x}) \; \phi_j(\vec{y}) \; . \end{aligned} \tag{1.9}$$

Let us consider variations of Ψ^*: $\Psi^* \to \Psi^* + \delta\Psi^*$ by varying just one of the single-particle wave functions at a time, such as:

$$\delta\Psi^* \;\; = \;\; \delta\phi_1^*(\vec{x}_1) \cdot \phi_2^*(\vec{x}_2) \; \ldots\ldots\ldots \; \phi_N^*(\vec{x}_N) \; . \tag{1.10}$$

The variational condition,

$$\int d^Nv \; \{ \; \delta\Psi^* \; (H + \lambda) \;\;\; \Psi \; \} \; = \;\; 0 \tag{1.11}$$

simplifies to:

$$\int d^3x\, \delta\phi_1^*(\vec{x}) \{ -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x}) + \sum_{j=2}^{N} \int d^3y\, \phi_j^*(\vec{y})\, v(\vec{x}-\vec{y})\, \phi_j(\vec{y}) + $$

$$+ (\lambda + \sum_{j=2}^{N} b_j + \sum_{1<\ell<j} c_{\ell j})\} \phi_1(\vec{x}) = 0 , \qquad (1.12)$$

where,

$$b_j = \int d^3x\, \phi_j^*(\vec{x}) \{-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})\} \phi_j(\vec{x}) , \qquad (1.13)$$

and,

$$c_{\ell j} = \int d^3x\, d^3y\, \phi_\ell^*(\vec{x})\, \phi_j^*(\vec{y})\, v(\vec{x}-\vec{y})\, \phi_\ell(\vec{x})\, \phi_j(\vec{y}) . \qquad (1.14)$$

since $\delta\phi_1^*$ is an arbitrary complex function, the vanishing of the integral can only be satisfied by the vanishing of the integrand. Thus, we obtain a Hartree equation for the single-particle wave function labelled by subscript 1. This may be generalized for arbitrary particle label i. The equations obtained are the Hartree equations, one for each particle wave function:

$$\{-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x}) + \sum_{\substack{j=1\\(j\neq i)}}^{N} \int d^3y\, \phi_j^*(\vec{y}) v(\vec{x}-\vec{y}) \phi_j(\vec{y})\} \phi_i(\vec{x}) = \varepsilon_i^H \phi_i(\vec{x}) , \qquad (1.15)$$

where,

$$\varepsilon_i^H = -(\lambda + \sum_{\substack{j=1\\(j\neq i)}}^{N} b_j + \sum_{\substack{\ell,j=1\\(\ell<j)\\(\ell,j\neq i)}}^{N} c_{\ell j})$$

are the Hartree single-particle energies. The approximate ground state energy of the system, denoted again by E, is given by the expectation value of the Hamiltonian:

$$E = \langle H \rangle = \sum_{i=1}^{N} \int d^3x\, \phi_i^*(\vec{x}) \{-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x})\} \phi_i(\vec{x}) + $$

$$+ \sum_{i<j}^{N} \int d^3x\, d^3y\, \phi_i^*(\vec{x})\, \phi_j^*(\vec{y})\, v(\vec{x}-\vec{y})\, \phi_i(\vec{x})\, \phi_j(\vec{y})$$

$$= \sum_{i=1}^{N} \varepsilon_i^H - \sum_{i<j} \int d^3x\, d^3y\, \phi_i^*(\vec{x})\, \phi_j^*(\vec{y})\, v(\vec{x}-\vec{y})\, \phi_i(\vec{x})\, \phi_j(\vec{y}) . \qquad (1.16)$$

Here we see that the total energy of the system is given by the sum of the Hartree single-particle energies minus the interaction energies, which have been doubly counted in the summation of the Hartree