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TRENDS AND DEVELOPMENTS IN MECHANISMS, MACHINES, AND ROBOTICS — 1988

Volume Three

Dynamics • Design of Robots and Vehicles • Robot Workspace • Kinematics • Geometry • Task Planning and Control

edited by
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Dynamics
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FOREWORD

The nearly two hundred papers presented at the 20th Biennial ASME Mechanisms Conference, held in Kissimmee, Florida during September 25-28, 1988, are contained in these three volumes entitled "Trends and Developments in Mechanisms, Machines and Robotics." The conference has been organized by the Mechanisms Committee of the Design Engineering Division, ASME, under the auspices of the Design Technology Conferences.

Volume one contains such topical areas as global motion properties of mechanisms, expert systems, structural design, and analysis and synthesis of mechanisms. The contents of volume two include theoretical kinematics, automated design, simulation of mechanical systems, machine systems and elements, and vibration and compliant systems. Finally, in volume three are contained dynamics of mechanisms and manipulators, mechanical design aspects related to robots and vehicles, and workspace, kinematics, geometry, task planning and control of robots.

Professor Crossley's plenary presentation, and the opening paper in volume one under "Milestones," is entitled "Recollections from Forty Years of Teaching Mechanisms." This excellent personal and historical narration should help us understand our origins as a professional community, our present health, and the bright future that lies ahead. Approprliately so, in the latter vein, this article is followed by one enlightening us to the research needs and opportunities in machine dynamics, by Professor Soni et al. This important and timely study is a result of sponsorship by the National Science Foundation and ASME.

A significant increase in participation, as measured by the number of papers herein, has been in good part due to many a young researcher on the scene, meeting the challenges of the day. Classical concepts are integrating with newer notions, as is exemplified by the union of structural synthesis and expert systems. Increased attention is being given to automation in design through more efficient analytical, algorithmic and computational efforts, as well as use of geometric data bases. More sophisticated modeling and prediction techniques are now being implemented in the consideration of compliance and vibration in machine systems. Finally, as is evident in volume three, its entirety is devoted to the various issues confronting us in the area of robotics. This surge of research activity is evidence of our community's responsiveness to societal needs.

These proceedings, however, would have been impossible without the fine contributions, and the selfless reviewing and counseling activities of numerous colleagues,from far and near. I owe a great debt of gratitude to members of the Mechanisms Committee: Joseph Duffy, Chairman, E. Roland Maki, Secretary, and Stephen Derby, Papers Review Chairman, Outside North America, for their strong support throughout. I am grateful to Steven Dubowsky and Kenneth Waldron, the past and present Technical Editors of the *Journal of Mechanisms, Transmissions, and Automation in Design,* for making the interaction at the journal end a most pleasurable one. I gratefully acknowledge the staunch support in many a form over the years from members of the School of Mechanical Engineering at Purdue University: Dr. Winfred Phillips, former Head, members of the Design Faculty, and Patricia Booth and Kathey Freeman. Last but not least, the ASME staff members in New York are thankfully acknowledged for making this momentous event possible.

Ashok Midha
Papers Review Chairman
Mechanisms Committee
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CONTENTS

ROBOT TASK PLANNING AND CONTROL

MULTI-RIGID-BODY DYNAMICS: A d'ALEMBERT'S METHOD USING DUAL VECTORS

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Abstract

This paper proposes an algorithm based on d'Alembert's principle to generate dynamic equations of motion for multibody systems using dual vectors. This algorithm has been shown to be the dual equivalent of Kane's formulation. An example from both holonomic and non-holonomic systems is worked out using the method to demonstrate the general applicability in the analysis of mechanical systems. A feature of a class of holonomic systems made up solely of cylindrical joints is pointed out which makes the use of duals in dynamic analysis particularly advantageous.

Introduction

A mechanical system may be modelled as a collection of interconnected rigid bodies with joints which provide relative motion between them. Dynamics of such multibody systems has been the subject of extensive research in the past two decades and algorithms exist today to compute the equations of motion both in numeric and symbolic forms [3,4,5,7,8,10,12]. Broadly, these algorithms are based on either of the three forms: Newton-Euler's, Lagrangian or d'Alembert's. These three forms have their relative merits when analysing large multibody systems.

In Newton-Euler form of analysis using free body diagram approach, the non-contributing forces of constraints are explicitly introduced and then eliminated. In Lagrangian form of analysis, one needs to evaluate the partial derivatives of kinetic and potential energy functions with respect to generalized coordinates. In analysis using d'Alembert's principle, the inertia force and torque vectors for each body as functions of generalized coordinates and the derivatives is evaluated. The equations of motion are obtained by equilibrating the inertial and external torques/forces. The forces of constraint, if considered ideal are non-contributing to the overall dynamic equations and need not be brought explicitly in the analysis. However, one needs to do acceleration analysis for the entire mechanism. In the method proposed by Kane, systems subjected to non-holonomic constraints are analysed without explicit introduction of Lagrange multipliers.

In analysis of spatial mechanisms where successive bodies have general position and orientation and are connected by joints having multiple degrees of freedom of motion, a concise representation of the geometry and kinematics is obtained by use of dual vectors and dual transformation matrices. Certain results in spatial kinematics can be obtained by dualizing the analog results for spherical kinematics [1]. In a series of papers [9,13,14,15], a method was proposed to analyse dynamics of rigid body systems using dual vectors. The idea of dual momentum (a compact representation for linear momentum of the rigid body and angular momentum about a point) was introduced which on appropriate differentiation led to the dual inertia force vector (inertia force and torque vectors for the rigid body). The approach was based on Newton-Euler's form which needs explicit introduction of the forces of constraints and becomes unwieldy in analysis of large multibody systems.

One is led to wonder at this point, if there are dual analogs to the other forms of analysis outlined in earlier paragraphs so that one could retain their advantages and supplement with those obtained during analysis with dual quantities. In this paper, an algorithm based on d'Alembert's principle is proposed and is shown to be the dual equivalent of one proposed by Kane [7]. The examples worked out are from references [7] and [9]. These examples demonstrate the general applicability of the method to any mechanical system and reiterate the fact that an explicit dynamic model for non-holonomic systems can be obtained without using Lagrange multipliers in the analysis. In the end an interesting observation for a class of mechanical systems assembled solely with cylindrical joints is pointed out which serves as a justification, in part, for the use of duals in dynamic analysis.

The rest of the paper is divided into five main sections. The following section outlines the method for analysis of displacement, kinematics and constraints of a multibody chain. The kinetics and the equations of motion for the system is formulated in the second section. In the third section, a general algorithm to obtain the equations of motion for a multibody system is presented. The fourth section contains two solved examples followed by a brief discussion at the end of the paper.

1. Description and Kinematics of the system

The contents of this section elaborate on five main topics. The first introduces the notations used in the analysis and the significance of bound unit vectors as opposed to free ones. The second mentions the possible dual representations for a general displacement of a body in a coordinate frame. The third topic relates the dual velocity of two points on a rigid body and the effect of joint motion on a coincident point on two adjacent bodies. The fourth heading introduces the ideas of generalized coordinates, generalized speeds and degrees of freedom for a system subjected to holonomic and non-holonomic constraints. In the end the dual holonomic and non-holonomic partials corresponding to the degrees of freedom are defined. These topics, together, provide complete information of the position and velocity of each particle of the multibody chain as functions of the generalized coordinates and speeds defining the system.

In the analysis presented in this paper, a rigid body is a reference frame. In a reference frame, coordinate frames are fixed passing through a point formed by a basis of mutually orthogonal unit vectors. These unit vectors which are bound to lines and pass through a particular point on the body are bound unit vectors. The bound unit vectors are different from free unit vectors because of additional constraint to pass through a specified point. In this framework, a set of bound unit vectors which are parallel are distinct from each other as opposed to parallel free unit vectors, which are identical. These bound unit vectors are denoted by bold faced small letters with hats. In this paper, the term "unit vector" refers to a bound unit vector, unless otherwise mentioned. To resolve any ambiguity of notation, a unit vector $\mathbf{x}$ through a point A on body B has the notation $\hat{\mathbf{x}}_A^B$ (fig. 1). The components of the dual vector $\hat{\mathbf{a}}$ in the coordinate frame at A are components of the free vector $\mathbf{a} + \epsilon(\mathbf{r} \times \mathbf{a})$, with property $\epsilon^2 = 0$.

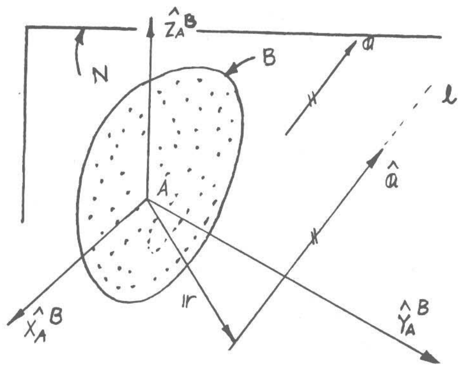

Fig.1 A Unit Vector in a Coordinate Frame at A

It requires six scalar parameters to characterize a spatial displacement of a rigid body. Some alternative ways of representing a displacement in 3-dimensions are listed in reference [1]. They include (4×4) transformation matrices with real numbers or more compactly (3×3) matrices with dual elements. A dual displacement of a body about a unit vector by a dual angle $\hat{\theta} = \theta + \epsilon l$ is defined as the rotation θ and translation l of the body along the unit vector (fig. 2). In reference [6], for general orientation of a rigid body, transformation matrices for 24 sequences of 3-rotations are listed. These formula obtained for spherical kinematics could be dualized to give the dual transformation matrices for spatial motion given by the same sequence of three dual displacements. The dual transformation matrix derived in reference [15] is equivalent to a body two sequence 3-1-3 in the notation of reference [6]. Due to special configuration of joints in a mechanical chain, the number of parameters required to represent successive bodies are less than six, out of which variables may be even fewer. For example, in the description of a manipulator chain by Denavit and Hartenberg's notation, only two dual displacements are needed to position and orient adjacent bodies, and out of the four parameters needed for this description, usually one is a variable.

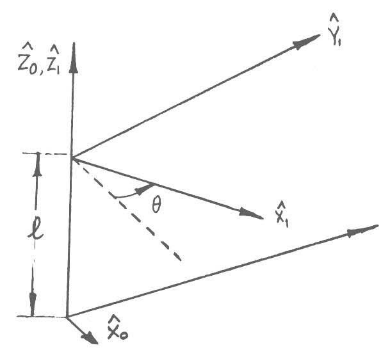

Fig. 2. Dual Displacement of a Rigid Body

The velocity of a point of the system is known if the angular velocities of the bodies and the linear velocity of at least one point on each body are known. The *dual velocity* of a body B, for a point P, in a reference frame A has the notation ${}^A\hat{\mathbf{V}}^P$, defined as $({}^A\omega^B + \epsilon {}^A\mathbf{V}^P)$, where ${}^A\omega^B$ is the angular velocity of the body B in A and ${}^A\mathbf{V}^P$ is the linear velocity of P in A. In eqs.(1) and (2), it will be shown that once the dual velocity of a body for a point P, in terms of the unit vectors attatched at P is known, expressing these unit vectors in terms of unit vectors at Q gives the dual velocity of the body for Q. At each instant, the motion of a rigid body, in a given reference frame can be expressed as sliding and turning about an instantaneous screw axis. The dual velocity of the body for the set of points R on the instantaneous screw axis is given by eq.(1), where $\hat{\mathbf{s}}$ is the unit vector along the instantaneous screw axis, fig.(3).

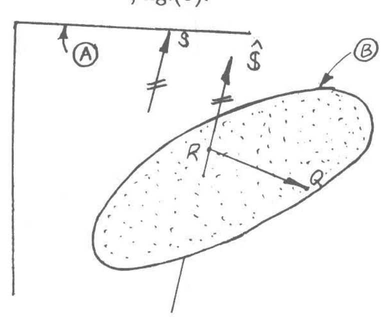

Fig. 3. Instantaneous screw motion of B along $\hat{\mathbf{s}}$

$$ {}^A\hat{\mathbf{V}}^R = ({}^A\omega^B + \epsilon {}^A\mathbf{V}^R)\hat{\mathbf{s}} \qquad (1) $$

The representation of $\hat{\mathbf{s}}$ in terms of unit vectors at Q is $[\mathbf{s} + \epsilon(QR \times \mathbf{s})]$, where (QR) is a vector from Q to a point on R and $(\times)$ denotes the cross product. On substitution, eq.(1) leads to eq. (2) and one can recognize the dual part as the linear velocity of point Q

of B in reference frame A.

$$^A\hat{\mathbf{V}}^Q = {}^A\omega^B\mathbf{s} + \epsilon[{}^A\mathbf{V}^R\mathbf{s} + ({}^A\omega^B\mathbf{s} \times RQ)] \quad (2)$$

The dual velocity for a coincident point P on two adjacent bodies i and $i-1$ for a slide-turn motion about the joint, $\dot{\hat{\theta}}_i = \dot{\theta}_i + \epsilon \dot{l}_i$ is given by eq.(3), where $\hat{\mathbf{z}}_i$ is the unit vector along the joint axis. The use of eqs.(2) and (3) enable one to complete velocity analysis for the entire multibody chain.

$$\hat{\mathbf{V}}^{P_i} = \hat{\mathbf{V}}^{P_{i-1}} + \dot{\hat{\theta}}_i\hat{\mathbf{z}}_i \quad (3)$$

Let the configuration of a multibody chain be specified by k independent dual displacements about unit vectors fixed on rigid bodies of the chain to result in $2k$ independent variables of motion. In special cases of pure rotation or translation between adjacent bodies, the variables describing the system are reduced to say, n variables. These independent variables are the generalized coordinates denoted by symbols (q_k, $k = 1,..,n$), and may be θ_i or l_i. Consistent with Kane's notation, n generalized speeds u_k, (k=1,..,n) are defined which satisfy eq.(4), where Y_{rs} and Z_r are functions of q_i and time t.

$$u_r = \sum_{s=1}^{n} Y_{rs}\dot{q}_s + Z_r, \quad (r = 1,..n) \quad (4)$$

For a holonomic system, where the generalized speeds are independent, there will be n dynamical equations of motion, corresponding to each of the n generalized speeds. If the constraints on the system are non-holonomic, such as encountered in rolling of bodies, out of n generalized speeds, only p are independent and there are $(n-p)$ non-holonomic constraint equations relating u_i, eq.(5).

$$u_r = \sum_{s=1}^{p} A_{rs}u_s + B_r, \quad (r = p+1, ..., n) \quad (5)$$

The degrees of freedom for the system are the number of independent generalized speeds. The set of p independent generalized speeds lead to p independent dynamic equations. The motion of the system is governed by $2n$ first order differential equations, p dynamic equations of motion, $(n-p)$ constraint equations (eq. 5) and n equations relating generalized coordinates to the speeds (eq. 4) with special case of $(p = n)$ for holonomic systems.

The dual velocity of a body B for a point P in a reference frame A can be expressed by eq.(6) for a holonomic system and eq.(7) for a non-holonomic system, where $\hat{\mathbf{v}}_i$ and $\tilde{\hat{\mathbf{v}}}_i$ are defined as the dual holonomic and non-holonomic partial velocity, analogous to Kane's formulation. The dual non holonomic partial velocity for a body is obtained using eqs.(5) and (6) by solving for the dependent generalized speeds in terms of independent ones.

$$^A\hat{\mathbf{V}}^P = \sum_{i=1}^{n} \hat{\mathbf{v}}_i u_i + \hat{\mathbf{v}}_t \quad (6)$$

$$^A\tilde{\hat{\mathbf{V}}}^P = \sum_{i=1}^{p} \tilde{\hat{\mathbf{v}}}_i u_i + \tilde{\hat{\mathbf{v}}}_t \quad (7)$$

Before concluding this section, it will be appropriate to point out one of the features of a class of holonomic systems with cylindrical joints. If the generalized speeds chosen in the analysis are the angular and linear rates about the joint axis, the dual partial velocity for a body of the chain for the generalized speeds corresponding to $\dot{\theta}_i$ and $\dot{l}_i$ bear a special relationship, the latter is ϵ times the former. Later in the analysis, it will be shown that for such systems it is sufficient to do the dynamic analysis for either of the two generalized speeds and the the result for the other can be extracted out of the same expression.

2. Kinetics and Equations of motion for the system

This section is organized to cover two broad ideas. The first is to introduce the concept of dual momentum and from it derive the dual inertia force. The second is to obtain the equations of motion and show its equivalence to Kane's formulation.

The dual momentum of a rigid body B for the mass center B^* in A is defined by eq.(8), where $^A\mathbf{P}^B$ is the linear momentum and $^A\mathbf{H}^{B^*}$ is the angular momentum of the body B for B^* in A.

$$^A\hat{\mathbf{H}}^B = {}^A\mathbf{P}^B + \epsilon{}^A\mathbf{H}^{B^*} \quad (8)$$

A dual force $\hat{\mathbf{F}}$ on a rigid body at a point P is defined as $(\mathbf{F}+\epsilon\mathbf{M})$, with the equivalent force $\mathbf{F}$ and the moment of couple $\mathbf{M}$ at P. Differentiating eq.(8) in A and rearranging the terms leads to the expression for the dual inertia force vector $\hat{\mathbf{F}}^*$ of eq.(9). The expression in eq.(9) can be shown to be equal to $(\mathbf{F}^* + \epsilon\mathbf{M}^*)$, where $\mathbf{F}^*$ and $\mathbf{M}^*$ are the real and dual parts of the expression , $[ma^* + \epsilon(\mathbf{I}\cdot\alpha + \omega \times \mathbf{I}\cdot\omega)]$. $\mathbf{I}$ is the inertia dyadic of the body at the mass center, ω and α are the angular velocity and acceleration of B in A and the symbol $(\cdot)$ denotes the dyadic product.

$$\hat{\mathbf{F}}^* = \frac{^A d}{dt}\hat{\mathbf{H}} + \hat{\mathbf{V}} \times \hat{\mathbf{H}} \quad (9)$$

In the solved examples, eq.(9) is evaluated in terms of the components in coordinate frames attatched to the bodies at the mass centers.

Using d'Alembert's principle, the system can be viewed as being in equilibrium under the effect of dual external and inertial forces, computed in eq.(9). If P is a point on the body and $\mathbf{F}_P$ is the dual force acting at P, eq.(10) should hold for all generalized speeds i. These are the n dynamical equations of motion for the system. In eq.(10), the summation index goes over all P and these include the mass center of each body. The forces of constraints are assumed ideal and non-contributing and thus need not be included in the analysis. In analysis of non-holonomic systems, the dual holonomic partials are replaced by non-holonomic partials of eq.(7) leading to p dynamical equations of motion for the system. The equivalence of this algorithm with Kane's method can be seen by expanding terms in eq.(10). The dual part of the expression is Kane's dynamical equations of motion.

$$dual(\sum_P \hat{\mathbf{v}}_i \cdot \hat{\mathbf{F}}_P) = 0 \quad (10)$$

Extending discussion for the class of holonomic systems with cylindrical joints, where the dual holonomic partials corresponding to the two degrees of freedom at a joint are related by factor ϵ, an interesting property is observed from eq.(10). The dual part of the expression of eq.(10) is the equation of motion corresponding to the rotational degree of freedom at the joint while the proper part corresponds to the equation of motion for the slide degree of freedom about the same joint. If a system is built solely of cylindrical joints, the procedure outlined in this section needs to be carried out only for $n/2$ degrees of freedom to give the equations of motion for all n degrees of freedom.

3. Algorithm to compute the equations of motion

This section outlines the steps of the algorithm, based on the ideas already developed in the last two sections. The algorithm can be divided into five main steps which have associated details, best explained with examples. The first step of the algorithm is to identify the generalized coordinates and attatch unit vectors to the bodies to define the system. The second step is to carry out the velocity analysis for the entire system, using eqns.(2) and (3) and calculate in particular, the dual velocity of the body at points where external dual forces act and for the center of mass of each body. If the system has non-holonomic constraints, the appropriate constraint equations are derived. The third step of the algorithm is to identify a set of generalized speeds, according to eq.(4). If the constraints are non-holonomic, the constraint equations already worked out are expressed in the form of eq.(5). The dual velocity of the bodies for all relevant points are expressed in the form of eqns.(6) or (7) and the partial velocities are identified. The fourth step is to compute the dual inertia forces for the bodies, according to eq.(9) and the final step is to compute the equations of motion for the system using eq.(10).

4. Solved Examples

The method proposed in this paper is solved for two examples in this section. These examples have been taken from the references, mentioned earlier, and are illustrative of the method in the analysis of holonomic and non- holonomic systems. The notations for the problems are kept the same as those of the references.

Example 1 : The Stanford Manipulator. This example is analysed for the motion of the first three links of Stanford manipulator, which form a (R-R-P) mechanism [11], fig. 4. The symbols and assumptions in the analysis are the same as those of reference [9]. The rigid bodies are 1, 2, and 3 with center of masses at A, C and E. The three generalized coordinates are θ_1, θ_2 and s_3'. The frame O can be aligned with B by a body 3 dual displacement ($\hat{\theta}_1 = \theta_1 + \epsilon d_1$), frame B with D by body 1-3 dual displacement ($\hat{\alpha}_1 = 90° + \epsilon 0$), ($\hat{\theta}_2 = \theta_2 + \epsilon d_2$) and D with E by body 1-3 displacements ($\hat{\alpha}_2 = 90° + \epsilon 0$), ($\hat{\theta}_3 = 0 + \epsilon s_3'$). The frames at center of masses A and C are parallel to B and D respectively, and displaced along the $\mathbf{z}$ axis by $-g_1$ and $-g_2$.

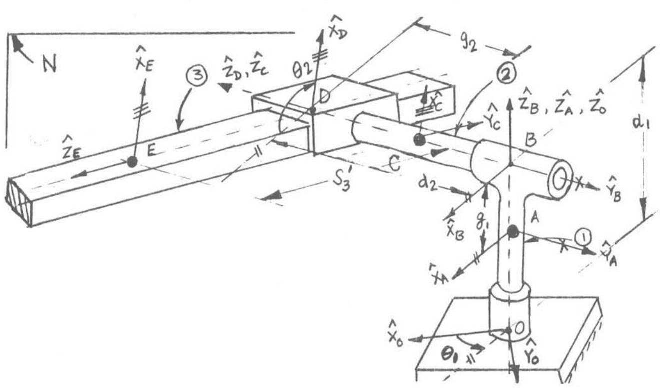

Fig. 4. The Stanford Manipulator

The generalized speeds for the system are taken as $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{s}_3'$. All dual velocities are computed in the inertial reference frame N, so the first subscript in the notation for the dual velocity is omitted. The abbreviations for *sin* and *cos* are S and C. The expressions for the dual velocity for the bodies at the points of interest are given in eqs.(13) to (19). The trigonometric identities (11) and (12) are used to simplify the terms of the transformation matrices.

$$Cos(\theta + \epsilon d) = Cos\theta - \epsilon d Sin\theta \tag{11}$$
$$Sin(\theta + \epsilon d) = Sin\theta + \epsilon d Cos\theta \tag{12}$$

$$\hat{\mathbf{V}}^{B_1} = \dot{\theta}_1 \hat{\mathbf{z}}_B \tag{13}$$
$$\hat{\mathbf{V}}^{A} = \dot{\theta}_1 \hat{\mathbf{z}}_A \tag{14}$$
$$\hat{\mathbf{V}}^{B_2} = \dot{\theta}_1 \hat{\mathbf{z}}_B + \dot{\theta}_2 \hat{\mathbf{y}}_B \tag{15}$$
$$\hat{\mathbf{V}}^{D_2} = \dot{\theta}_1 (S\hat{\theta}_2 \hat{\mathbf{x}}_D + C\hat{\theta}_2 \hat{\mathbf{y}}_D) + \dot{\theta}_2 \hat{\mathbf{z}}_D \tag{16}$$
$$\hat{\mathbf{V}}^{C} = \dot{\theta}_1 [(S\hat{\theta}_2 - \epsilon g_2 C\hat{\theta}_2)\hat{\mathbf{x}}_C + (C\hat{\theta}_2 + \epsilon g_2 S\hat{\theta}_2)\hat{\mathbf{y}}_C] + \dot{\theta}_2 \hat{\mathbf{z}}_C \tag{17}$$
$$\hat{\mathbf{V}}^{D_3} = \dot{\theta}_1 (S\hat{\theta}_2 \hat{\mathbf{x}}_D + C\hat{\theta}_2 \hat{\mathbf{y}}_D) + \dot{\theta}_2 \hat{\mathbf{z}}_D + \dot{s}_3' \epsilon \hat{\mathbf{z}}_E \tag{18}$$
$$\hat{\mathbf{V}}^{E} = \dot{\theta}_1 [S\hat{\theta}_2 (\hat{\mathbf{x}}_E - \epsilon s_3' \hat{\mathbf{y}}_E) - C\hat{\theta}_2 \hat{\mathbf{z}}_E] + \dot{\theta}_2 (\epsilon s_3' \hat{\mathbf{x}}_E + \hat{\mathbf{y}}_E) + \epsilon \dot{s}_3' \hat{\mathbf{z}}_E \tag{19}$$

speed	$\hat{\mathbf{V}}_{B_1}$	$\hat{\mathbf{V}}_{B_2}$	$\hat{\mathbf{V}}_A$	$\hat{\mathbf{V}}_C$	$\hat{\mathbf{V}}_{D_2}$	$\hat{\mathbf{V}}_{D_3}$	$\hat{\mathbf{V}}_E$
u_1	$\hat{\mathbf{z}}_B$	$\hat{\mathbf{z}}_B$	$\hat{\mathbf{z}}_A$	$(S\hat{\theta}_2 - \epsilon g_2 C\hat{\theta}_2)\hat{\mathbf{x}}_C$ $+(C\hat{\theta}_2 + \epsilon g_2 S\hat{\theta}_2)\hat{\mathbf{y}}_C$	$S\hat{\theta}_2\hat{\mathbf{x}}_D$ $+C\theta_2\hat{\mathbf{y}}_D$	$S\hat{\theta}_2\hat{\mathbf{x}}_D$ $+C\theta_2\hat{\mathbf{y}}_D$	$S\hat{\theta}_2(\hat{\mathbf{x}}_E - \epsilon s_3\prime\hat{\mathbf{y}}_E)$ $-C\hat{\theta}_2\hat{\mathbf{z}}_E$
u_2	0	$\hat{\mathbf{y}}_B$	0	$\hat{\mathbf{z}}_C$	$\hat{\mathbf{z}}_D$	$\hat{\mathbf{z}}_D$	$\epsilon s_3\prime\hat{\mathbf{x}}_E + \hat{\mathbf{y}}_E$
u_3	0	0	0	0	0	$\epsilon\hat{\mathbf{z}}_E$	$\epsilon\hat{\mathbf{z}}_E$

Table 1 : The Dual Holonomic Partials for Example 1.
($u_1 = \dot{\theta}_1$, $u_2 = \dot{\theta}_2$, $u_3 = \dot{s}_3'$)

The dual holonomic partials for these defined generalized speeds are collated in Table (1). The three bodies have masses m_1, m_2 and m_3 and their central principal axes are along the basis vectors of the coordinate frames defined at the center of mass. The central principal gyrations for the body (i, $i = 1,..,3$) are k_{i1}, k_{i2} and k_{i3} along the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ axes, respectively at the mass centers, with a simplifying constraint, $k_{i1} = k_{i2}$. The components of the dual velocity for the three bodies in the bases fixed at the mass centers are those in eq.(20)-(22).

$$\hat{\mathbf{V}}^1 = (0, 0, \dot{\theta}_1)^T \tag{20}$$
$$\hat{\mathbf{V}}^2 = [\{S\theta_2 + \epsilon(d_2 - g_2)C\theta_2\}\dot{\theta}_1, \{C\theta_2 - \epsilon(d_2 - g_2)S\theta_2\}\dot{\theta}_1, \dot{\theta}_2]^T \tag{21}$$
$$\hat{\mathbf{V}}^3 = [S\theta_2 + \epsilon d_2 C\theta_2)\dot{\theta}_1 + \epsilon s_3' \dot{\theta}_2, \dot{\theta}_2 - \epsilon s_3' S\theta_2 \dot{\theta}_1, (-C\theta_2 + \epsilon d_2 S\theta_2)\dot{\theta}_1 + \epsilon \dot{s}_3']^T \tag{22}$$

The dual momentum of the bodies in the same bases are computed in eq.(23)-(25). The expressions are obtained using eq.(8) with trigonometric simplifications.

$$\frac{\hat{\mathbf{H}}^1}{m_1} = (0, 0, \epsilon k_{13}^2 \dot{\theta}_1)^T \tag{23}$$
$$\frac{\hat{\mathbf{H}}^2}{m_2} = [\{(d_2 - g_2)C\theta_2 + \epsilon k_{21}^2 S\theta_2\}\dot{\theta}_1, \{-(d_2 - g_2)S\theta_2 + \epsilon k_{22}^2 C\theta_2\}\dot{\theta}_1, \epsilon k_{23}^2 \dot{\theta}_2]^T \tag{24}$$
$$\frac{\hat{\mathbf{H}}^3}{m_3} = [(d_2 C\theta_2 + \epsilon k_{31}^2 S\theta_2)\dot{\theta}_1 + s_3' \dot{\theta}_2, -s_3' S\theta_2 \dot{\theta}_1 + \epsilon k_{32}^2 \dot{\theta}_2, (d_2 S\theta_2 - \epsilon k_{33}^2 C\theta_2)\dot{\theta}_1 + \dot{s}_3']^T \tag{25}$$

The dual inertia force for the three bodies are computed using eq.(9) and their components, in the same bases are those in eqs.(26)-(28).

$$\frac{\hat{\mathbf{F}}_1^*}{m_1} = (0, 0, \epsilon k_{13}^2 \ddot{\theta}_1)^T \tag{26}$$

$$\frac{\hat{\mathbf{F}}_2^*}{m_2} = \begin{pmatrix} (d_2 - g_2)C\theta_2\ddot{\theta}_1 + \epsilon(k_{21}^2 S\theta_2\ddot{\theta}_1 + k_{23}^2 C\theta_2\dot{\theta}_1\dot{\theta}_2) \\ -(d_2 - g_2)S\theta_2\ddot{\theta}_1 + \epsilon(k_{22}^2 C\theta_2\ddot{\theta}_1 - k_{23}^2 S\theta_2\dot{\theta}_1\dot{\theta}_2) \\ -(d_2 - g_2)\dot{\theta}_1^2 + \epsilon k_{23}^2 \ddot{\theta}_2 \end{pmatrix} \tag{27}$$

$$\frac{\hat{\mathbf{F}}_3^*}{m_3} = \begin{pmatrix} [d_2 C\theta_2\ddot{\theta}_1 + 2\dot{\theta}_2\dot{s}_3' + s_3'\ddot{\theta}_2 - s_3' S\theta_2 C\theta_2\dot{\theta}_1^2 \\ +\epsilon\{k_{31}^2 S\theta_2\ddot{\theta}_1 + C\theta_2\dot{\theta}_1\dot{\theta}_2(2k_{31}^2 - k_{33}^2)\}] \\ [-s_3'(S\theta_2\ddot{\theta}_1 + 2C\theta_2\dot{\theta}_1\theta_2) - 2\dot{s}_3' S\theta_2\dot{\theta}_1 - d_2\dot{\theta}_1^2 \\ +\epsilon\{k_{32}^2\ddot{\theta}_2 + C\theta_2 S\theta_2\dot{\theta}_1^2(k_{33}^2 - k_{31}^2)\}] \\ [\ddot{s}_3' + d_2 S\theta_2\ddot{\theta}_1 - s_3'(\dot{\theta}_2^2 + S^2\theta_2\dot{\theta}_1^2) \\ -\epsilon k_{33}^2(C\theta_2\ddot{\theta}_1 - S\theta_2\dot{\theta}_1\dot{\theta}_2)] \end{pmatrix} \tag{28}$$

The last step of this algorithm is to use eq.(10). The external forces acting on the system are the actuator torques/thrust provided by the revolute /prismatic joints at O, B, D, and the negative of the dual inertia vectors calculated in eq.(26)-(28). These actuator torques/thrust are working forces as they contribute to relative motion between the bodies along their lines of action, however all other constraint forces/moments between the bodies are non-working and can be neglected from the analysis. The equations of motion for the system, corresponding to the three generalized speeds u_1, u_2 and u_3, are the eq.(29)-(31), which are obtained carrying the computation of eq.(10), where T_1, T_2, F_3 are the actuator torques/thrust at the joints and A_1 is a variable defined in eq.(32).

$$F_3 = m_3(d_2 S\theta_2\ddot{\theta}_1 + \ddot{s}_3' - s_3' S^2\theta_2\dot{\theta}_1^2 - s_3'\dot{\theta}_2^2) \tag{29}$$

$$\begin{aligned} T_2 =& (m_2 k_{23}^2 + m_3 s_3'^2 + m_3 k_{31}^2)\ddot{\theta}_2 + m_3 s_3' d_2 C\theta_2\ddot{\theta}_1 \\ &+ 2m_3 s_3'\dot{\theta}_2\dot{s}_3' + m_3(k_{33}^2 - k_{31}^2 - s_3'^2)S\theta_2 C\theta_2\dot{\theta}_1^2 \end{aligned} \tag{30}$$

$$\begin{aligned} T_1 =& A_1\ddot{\theta}_1 + m_3 d_2 s_3' C\theta_2\ddot{\theta}_2 + m_3 d_2 S\theta_2\ddot{s}_3' - m_3 d_2 s_3' S\theta_2\dot{\theta}_2^2 \\ &+ 2m_3 d_2 C\theta_2\dot{\theta}_2\dot{s}_3' - 2m_3(k_{33}^2 - k_{31}^2 - s_3'^2)S\theta_2 C\theta_2\dot{\theta}_1\dot{\theta}_2 \\ &+ 2m_3 s_3' S^2\theta_2\dot{\theta}_1\dot{s}_3' \end{aligned} \tag{31}$$

$$\begin{aligned} A_1 =& m_1 k_{13}^2 + (m_2 + m_3)d_2^2 + m_2(k_{21}^2 + g_2^2) + m_3 k_{33}^2 C\theta_2^2 \\ &+ m_3 s_3'^2 S\theta_2^2 - 2m_2 g_2 d_2 + m_3 k_{31}^2 S\theta_2^2 \end{aligned} \tag{32}$$

These equations match the results obtained from EMDEG [2], a symbolic manipulation code, which generates equations of motion for a single open chain manipulator. However, the equations of motion for the same problem in reference [9] have a few different terms which may be typographical mistakes.

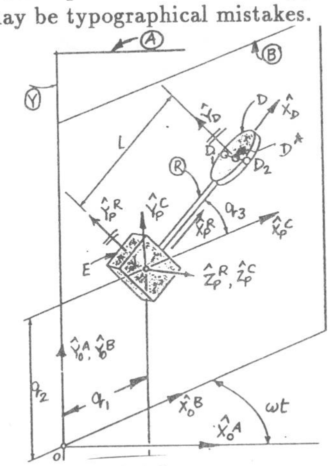

Fig. 5. Motion of R between two parallel plates

Example 2 : Motion of a sharp edged disk sandwiched between parallel glass plates, fig. (5). This example has been taken from reference [7] and describes the motion of a body R, which is formed by a block E and a disk D attached rigidly at the two ends of R. R can slide and rotate inside B, formed by two parallel sheets of glass. The axis of the disk is normal to the rod connecting E and D and is parallel to the plane in which R moves. The edges of the disk are sharp edged and allow the motion of D only along the line joining E and D. The body B has a simple rotational motion in A about the axis Y. In the analysis, an auxilliary body C is introduced, which has the same motion of translation as R but has no rotational motion in B.

The rigid body R, for dynamic analysis, is modelled as two particles of mass m_1 and m_2, lumped at the geometric centers P and D^*. The coordinate frames introduced are shown in the figure. The frame O is aligned with the frame at P attached to body R by body 2-1-3 displacements of $(\omega t + \epsilon q_2)$, ϵq_1 and q_3. The frame at P is aligned with D by body 1 dispacement ϵl. The generalized coordinates defining the system are $(q_i,\ i = 1, .., 3)$. The constraints imposed due to the assumption of sharp edged disk are eqs.(33) and (34) and are non-integrable constraint equations.

$$dual({}^B\hat{\mathbf{V}}^{D^*}) \cdot \hat{\mathbf{y}}_D = 0 \tag{33}$$

$$dual({}^B\hat{\mathbf{V}}^{D^*}) \cdot \hat{\mathbf{z}}_D = 0 \tag{34}$$

The velocity analysis for the system results in eqns.(35)-(40). The eqs.(35)- (39) refer to the velocity in the inertially fixed reference frame A, while eq.(40) is in a reference frame attatched to B.

$$\hat{\mathbf{V}}^{O_B} = \omega\hat{\mathbf{y}}_O^B \tag{35}$$

$$\hat{\mathbf{V}}^{O_C} = (\omega + \epsilon\dot{q}_2)\hat{\mathbf{y}}_O^B + \epsilon\dot{q}_1\hat{\mathbf{x}}_O^B \tag{36}$$

$$\hat{\mathbf{V}}^{P_C} = (\omega + \epsilon\dot{q}_2)\hat{\mathbf{y}}_P^C + \epsilon\dot{q}_1\hat{\mathbf{x}}_P^C - \epsilon\omega q_1\hat{\mathbf{z}}_P^C \tag{37}$$

$$\begin{aligned} \hat{\mathbf{V}}^{P_R} =& \omega(Sq_3\hat{\mathbf{x}}_P^R + Cq_3\hat{\mathbf{y}}_P^R) + \dot{q}_3\hat{\mathbf{z}}_P^R + \epsilon\{(\dot{q}_2 Sq_3 + \dot{q}_1 Cq_3)\hat{\mathbf{x}}_P^R \\ &+ (\dot{q}_2 Cq_3 - \dot{q}_1 Sq_3)\hat{\mathbf{y}}_P^R - \epsilon\omega q_1\hat{\mathbf{z}}_P^R\} \end{aligned} \tag{38}$$

$$\begin{aligned} \hat{\mathbf{V}}^{D^*} =& \omega(Sq_3\hat{\mathbf{x}}_D + Cq_3\hat{\mathbf{y}}_D) + \dot{q}_3\hat{\mathbf{z}}_D + \epsilon\{(\dot{q}_2 Sq_3 + \dot{q}_1 Cq_3)\hat{\mathbf{x}}_D \\ &+ \epsilon(\dot{q}_2 Cq_3 - \dot{q}_1 Sq_3 + \dot{q}_3 l)\hat{\mathbf{y}}_D - \epsilon\omega(q_1 + lCq_3)\hat{\mathbf{z}}_D\} \end{aligned} \tag{39}$$

$$\begin{aligned} {}^B\hat{\mathbf{V}}^{D^*} =& \dot{q}_3\hat{\mathbf{z}}_D + \epsilon\{(\dot{q}_2 Sq_3 + \dot{q}_1 Cq_3)\hat{\mathbf{x}}_D \\ &+ (\dot{q}_2 Cq_3 - \dot{q}_1 Sq_3 + \dot{q}_3 l)\hat{\mathbf{y}}_D\} \end{aligned} \tag{40}$$

generalized speeds	$\tilde{\hat{\mathbf{V}}}_{P_R}$	$\tilde{\hat{\mathbf{V}}}_{D^*}$
u_1	$\epsilon\hat{\mathbf{x}}_P^R$	$\epsilon\hat{\mathbf{x}}_D$
u_2	$-\frac{\hat{\mathbf{z}}_P^R}{l} + \epsilon\hat{\mathbf{y}}_P^R$	$-\frac{\hat{\mathbf{z}}_D}{l}$

Table 2. The dual Non-holonomic partials for Example 2

The non-holonomic constraints (34) is identically satisfied, while eq.(33) results in eq.(41).

$$\dot{q}_2 Cq_3 - \dot{q}_1 Sq_3 + \dot{q}_3 l = 0 \tag{41}$$

The three generalized speeds $(u_i, i = 1, .., 3)$ are defined according to eqs.(42)-(44). The constraint eq.(40) expressed in terms of u_i is eq.(45).

$$u_1 = \dot{q}_1 Cq_3 + \dot{q}_2 Sq_3 \tag{42}$$

$$u_2 = -\dot{q}_1 S q_3 + \dot{q}_2 C q_3 \tag{43}$$
$$u_3 = \dot{q}_1 \tag{44}$$
$$u_3 = -\frac{u_2}{l} \tag{45}$$

The non-holonomic partials for the body R corresponding to the points P and D^* are collated in Table (2). These expressions were obtained by substituting expressions for u_i from eqs.(42)-(44) in eqs.(38) and (39) and making use of the constraint condition (45). The dual velocity of body R for points P and D^* are expressed in their bases and are given in eq.(46)-(47). The expressions for the dual momentum are in eq.(48)-(49) and the non-holonomic dual inertia forces in (50)-(51), expressed in their bases.

$$\hat{\mathbf{V}}_P = (\omega S q_3 + \epsilon u_1, \omega C q_3 + \epsilon u_2, -\frac{u_2}{l} - \epsilon \omega q_1)^T \tag{46}$$
$$\hat{\mathbf{V}}_{D^*} = \{(\omega S q_3 + \epsilon u_1, \omega C q_3, -\frac{u_2}{l} - \epsilon \omega (q_1 + l C q_3)\}^T \tag{47}$$
$$\frac{\hat{\mathbf{H}}_P}{m_1} = (u_1, u_2, -\omega q_1)^T \tag{48}$$
$$\frac{\hat{\mathbf{H}}_{D^*}}{m_2} = \{u_1, 0, -\omega(q_1 + l C q_3)\}^T \tag{49}$$
$$\frac{\hat{\mathbf{F}}^*_P}{m_1} = \{\dot{u}_1 - \omega^2 C q_3 q_1 + \frac{u_2^2}{l}, \dot{u}_2 + \omega^2 S q_3 q_1 - \frac{u_1 u_2}{l}, \omega(-\dot{q}_1 + S q_3 u_2 - C q_3 u_1\}^T \tag{50}$$
$$\frac{\hat{\mathbf{F}}^*_{D^*}}{m_2} = \{\dot{u}_1 - \omega^2 C q_3 (q_1 + l C q_3), \omega^2 S q_3 (q_1 + l C q_3) - \frac{u_1 u_2}{l}, \omega(-\dot{q}_1 + l S q_3 \dot{q}_3 - C q_3 u_1)\}^T \tag{51}$$

The gravity is modelled to act vertically downwards through the masses. The dynamical equations of motion obtained are the eq.(52)-(53), for the generalized speeds u_1 and u_2, respectively. These were obtained by evaluating eq.(10). These equations of motion are identical to those obtained in the reference.

$$\dot{u}_1 = -g S q_3 + \omega^2 C q_3 q_1 + \frac{1}{(m_1 + m_2)}(m_2 l \omega^2 C q_3^2 - m_1 \frac{u_2^2}{l}) \tag{52}$$
$$\dot{u}_2 = -g C q_3 + \omega^2 S q_3 q_1 + \frac{u_1 u_2}{l} \tag{53}$$

5. Discussion and Conclusion

This paper provides a general framework to study motion of multibody systems based on d'Alembert's principle using dual vectors. The outlined algorithm was demonstrated to be the dual version of Kane's formulation. The formulation was worked out for examples from two distinct classes of systems, systems subjected to holonomic and non-holonomic constraints. This makes the method comprehensive enough to tackle any mechanical system. The choice of a non-holonomic system for the worked out example is to reconfirm that one can generate the equations of motion for non-holonomic systems explicitly without introducing Lagrange multipliers. This dual method retains the features associated with Kane's formulation, such as ease of symbolic generation of dynamic equations on a computer. The use of dual vectors, dual transformation matrices, dual momenta and dual forces provide a concise representation for geometry, kinematics and kinetics of a multibody systems. The class of holonomic system built solely with cylindrical joints have added advantage when solved with duals over the other methods as one needs to carry out the analysis for exactly half the number of degrees of freedom to derive equations of motion for all degrees of freedom. However, it should be pointed out that very few mechanical systems, in practice, have cylindrical pairs at the joints but nevertheless the use of duals provides an alternative viewpoint to investigate dynamics of motion of systems.

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DYNAMIC MODELING OF SERIAL AND PARALLEL MECHANISMS/ROBOTIC SYSTEMS: PART I – METHODOLOGY

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ABSTRACT

The precise control of mechanical linkages requires knowledge (in varying degrees) of the system's dynamic model. Part I of this two-part paper presents a unified analytic approach to the dynamic modeling and analysis of the general case of rigid-link multi-degree-of-freedom mechanical devices, including the specific and elementary case of the serial manipulator. The approach is based on, but not restricted to, the transference of system dependence from one set of generalized coordinates to another (e.g., from the relative joint parameters to Cartesian referenced hand coordinates of the serial manipulator) and is shown in Part II to allow the analysis of any single-loop mechanism, multi-loop parallel-input linkages, redundant manipulators, and tightly coordinated multiple manipulator systems.

The technique involves the initial modeling of the system (or its components) in terms of simple open kinematic chain relationships. Then, using the concept of loop closure and knowledge of the kinematic constraints interrelating the various sets of possible generalized (or Lagrangian) input coordinates, the final system model is obtained in terms of the desired reference coordinate set. The derivation of the initial (open chain) dynamic model is based almost entirely on the principle of virtual work and the generalized principle of d'Alembert. The resultant model is expressed in terms of kinematic and dynamic influence coefficients and is particularly well suited for the "transfer of generalized coordinates", especially the representation of the non-linear, velocity related, acceleration and effective load terms in a quadratic format. The utility of the developed modeling approach is demonstrated in Part II of this work by its application to the previously mentioned linkage based application systems.

INTRODUCTION

A formalized procedure is presented which establishes a base technology capable of modeling, and subsequently analyzing, a wide variety of different class mechanisms/linkage systems from a single generic approach. The essence of the algorithm is neither new nor unique in that it involves the initial modeling of each system component (e.g., link, spring, dashpot, etc.) in terms of a full set of base, or at least Lagrangian coordinates[1], with the final model being obtained from the simultaneous solution of the constitutive relationships of the various components along with the constraint equations[2] relating the Lagrangian coordinates to the desired generalized coordinates.

The contribution here, and the primary difference between this method and other approaches (e.g., Newton's 2^{nd} Law [Meriam, 1966], Lagrange Multiplier [Paul, 1975] and [Luh and Zheng, 1985], Loop Closure [Uicker, et.al., 1964] and [Denavit, et.al., 1965], Zero Eigenvalue [Kamman and Huston, 1984], and D'Alembert's Principle [Murray and Lovell, 1987] and [Nakamura and Ghodoussi, 1988], is the manner in which the kinematic constraint equations are obtained and incorporated into the dynamics. This work employs a

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[1]The terminology utilized herein is adopted from Paul [1979, Chap. 8]. Base referenced cartesian coordinates are base coordinates (e.g., the coordinates associated with the Newton-Euler equations of motion for a single link). A superabundant set of coordinates (i.e., not necessarily independent) are referred to as Lagrangian (e.g., all the relative joint angles/displacements in a closed kinematic chain). The term generalized coordinates will refer to any minimum (i.e., independent and equal in number to the degrees of freedom of the system) set of Lagrangian coordinates (e.g., the relative joint angles of an open kinematic chain).

[2]Here only first- and second-order kinematic constraints are considered. For position analysis the reader is referred to Duffy [1980].

systematic approach[3] by which the linkage is dissected, modeled, and then reassembled effectively reducing the effort involved in solving the combined set[4] of constitutive (kinetic) and constraint (kinematic) equations. Additionally, the algorithm stems from a philosophi-cally geometric point of view yielding a physically tractable[5], 'partially'[6] closed form, result as opposed to the more common algebraic bent yielding trigonometric (or simply numerical) results of a less interpretable nature. This point of view (and the results to follow) is taken because it is the opinion of the authors that the ability to physically interpret the inherent and fundamental role of geometry in the solution of constrained system dynamics is extremely useful, if not essential, for true "intelligent" controller design. This follows the physically explicit reciprocal and orthogonal screw results of Ball's Theory of Screws (e.g., [Ball, 1900] and [Sugimoto and Duffy, 1982]) and helps one avoid some of the pitfalls of linear algebra [Lipkin and Duffy, 1986] (e.g., the addition of dissimilar quantities).

The presentation given here and in Part II closely follows the sequence of operations generally involved in the modeling algorithm. This sequence consists of four principal components:

1. The development of the dynamic model of an open kinematic chain in terms of relative joint angles/displacements. The method of kinematic influence coefficients is employed here [Thomas and Tesar, 1982].
2. Transfer of system dependence from one set of generalized coordinates to another (e.g., from joint space to Cartesian space). This is accomplished through the application of a set of equations developed herein allowing for an isomorphic transformation of the kinematic and dynamic models. These models are expressed in a particular "canonical" form with the nonlinear terms given quadratically.
3. Formulation of the kinematic constraints relating the total system to the desired generalized input coordinates. Integral to this is the assembly of the actual linkage structure by the closure of the open kinematic chain(s) in accordance with the physical constraints.
4. Distribution of the system's dynamic dependence to the desired generalized input coordinates. This involves the isomorphic transformation equations of step two and often results in a reduction of the system description from a set of Lagrangian coordinates (if this is not inherent in step three).

While the above wording and the text to follow imply that the final model is to be referenced to a minimal set of input coordinates, it is important to note that this is not necessary. In fact, transference between different sets of Lagrangian coordinates, or from a set of generalized coordinates to a Lagrangian set, is directly allowed (provided the influence coefficients (G,H) relating the various sets are available) as illustrated in the multi-manipulator section of Part II of this work.

The first two algorithm constituents are developed here; the modeling procedure is illustrated in full by its application, in differing degrees of detail, to a number of interesting linkages in Part II. Following the development of the two primary constituents, comments are given with regard to current drawbacks and desirable future developments.

DYNAMIC MODELING OF OPEN KINEMATIC CHAINS

Numerous researchers have proposed and investigated various methods for the analysis of open kinematic chains. For study of these methods and their relative merits, the reader is referred to Renaud [1984], Lee, et al. [1983], Silver [1982], Thomas and Tesar [1982] and Hollerbach [1980], since this is not an explicit objective of this work.

The model presented here utilizes the generalized principle of d'Alembert [Freeman and Tesar, 1986], [Lee et.al., 1983] in conjunction with what will be termed the method of kinematic influence coefficients and results in closed-form vector expressions. This approach stems principally from Benedict and Tesar [1978 a,b] and Thomas and Tesar [1982] and is based on the separation of all the kinematic (geometric) phenomena in the form of a collection of purely position dependent functions (kinematic influence coefficients) operated on by external independent functions of time (input time states). This separation approach has been found to be ideally suited to obtain the model for all classes of linkages (complex 1-DOF machines [Pollock, 1975], multi-input-output planar systems [Freeman and Tesar, 1982], hybrid parallel-serial devices [Freeman and Tesar, 1983] and [Sklar and Tesar, 1986], multi-loop spatial mechanisms and linkage systems, including multiple manipulators, multi-fingered end-effectors and walking machines [Freeman, 1985] and [Freeman and Tesar, 1986] etc.), as well as for a wide range of analyses (real-time computation [Wander and Tesar, 1986], quasi-static deformations [Fresonke et.al., 1987], flexible manipulators [Behi, 1985], design optimization [Thomas et.al., 1985], linearized state space model generation [Freeman, 1985] and [Whitehead and Kamen, 1985], etc.). Additionally, the method is being extended to the generation of system sensitivity models for parameter estimation and identification along the lines of [Neuman and Murray, 1984] and [Vukobratovic and Kircanski, 1984], as well as for time varying stability analysis. It should be noted that this method (KIC) is similar in concept to that of Stepanenko and Vukobratovic [1976], Paul [Chap. 10, 1979] and Kane and Levinson [1983], with partial velocities, velocity coefficients and first-order kinematic influence coefficients being synonymous.

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[3]The treatment given here is not as formal as the topology of networks (e.g., the Independent Loop Mobility Criteria of Paul [1979, Chap. 8]) or the Kirchhoff's Circulation Law approach of Davies [1983] but is sufficient to demonstrate the applicability of the modeling approach presented herein to extremely complicated multiple loop chains.

[4]The algorithm allows for the complete solution of the system kinematics, with the kinematics then addressable by any of the multitude of Kineto-Static Sandor and Erdman [1984] or other approaches. If one is principally concerned with the dynamic model, the combined approach is suggested in regard to computational effort.

[5]The models component expressions are given in terms of intrinsic vector relationships.

[6]The algorithm generally requires the inversion of certain Jacobian matrices. The symbolic treatment of this is under investigation but satisfactory results have not yet been obtained.

Notation

The notational scheme utilized in this paper underscores the transfer of system dependence concept central to the modeling algorithm developed herein. This is accomplished through a graphically descriptive form whereby parameters currently being considered dependent are obviously distinct from those currently being considered independent. The distinction basically results by employing subscripts when referring to independent parameters and allowing superscripts for dependent parameters. The reader is referred to Appendix A for a more detailed description of this scheme as developed in [Freeman, 1985] and excerpted from [Freeman and Tesar, 1986].

Generalized Kinematics

Here, only the general methodology and result format of the higher-order kinematics is given. The problem of position analysis is not addressable (except in an iterative sense) by the proposed unified procedure. For closed form position analysis, the reader is again referred to Duffy [1980].

Now, adopting the standard Jacobian form $[G^u_\phi]$ of representation for the velocity of a vector of P^ϕ dependent (output) parameters $\underline{u}$ in terms of a set of M generalized independent (input) coordinates $\underline{\phi}$, one has

$$\dot{\underline{u}} = [G^u_\phi]\dot{\underline{\phi}} \qquad (1)$$

Here

$$[G^u_\phi] = \left[\frac{\partial \underline{u}}{\partial \phi_1} \,\vdots\, \frac{\partial \underline{u}}{\partial \phi_2} \,\vdots\, \cdots \,\vdots\, \frac{\partial \underline{u}}{\partial \phi_M}\right]$$
$$\equiv [\underline{g}^u_1 \,\vdots\, \underline{g}^u_2 \,\vdots\, \cdots \,\vdots\, \underline{g}^u_M] \qquad (2)$$

is the Jacobian (see Table A1, examples 1 and 2) relating the coordinates ($\underline{u}$) to ($\underline{\phi}$) and is of the dimensional shape

$\rho([G^u_\phi])$ = (dimension of the dependent, superscripted parameter set ($\underline{u}$)) by (dimension of the independent, subscripted parameter set ($\underline{\phi}$))

$$= P \times M \qquad (3)$$

with the n^{th} column ($\underline{g}^u_n$) being of shape

$$\rho(\underline{g}^u_n) = P \times 1 \qquad (4)$$

Having stated the first-order kinematics in a fairly common form, the second-order kinematics are presented in a less common form. Here, a particular matrix formulation is chosen in which the non-linear, velocity related components are expressed in terms of a three-dimensional coefficient array, $[H^u_{\phi\phi}]$, (consisting of position dependent second-order partial derivatives) operated on quadratically in a "plane by plane" sense. This type of quadratic representation will be shown to be extremely useful when dealing with the transference of system dependence from one set of generalized coordinates to another due to the maintenance of the generalized coordinate velocity vector. Generally, the acceleration vector ($\ddot{\underline{u}}$) of a set of (P) dependent parameters ($\underline{u}$) is expressed, in terms of the (M) generalized coordinates ($\underline{\phi}$) as

$$\ddot{\underline{u}} = [G^u_\phi]\ddot{\underline{\phi}} + \dot{\underline{\phi}}^T[H^u_{\phi\phi}]\dot{\underline{\phi}} \qquad (5)$$

where

$$[\dot{G}^u_\phi]\dot{\underline{\phi}} = \dot{\underline{\phi}}^T[H^u_{\phi\phi}]\dot{\underline{\phi}} = \begin{pmatrix} \dot{\underline{\phi}}^T[{}^uH^1_{\phi\phi}]\dot{\underline{\phi}} \\ \dot{\underline{\phi}}^T[{}^uH^2_{\phi\phi}]\dot{\underline{\phi}} \\ \vdots \\ \dot{\underline{\phi}}^T[{}^uH^P_{\phi\phi}]\dot{\underline{\phi}} \end{pmatrix} = \text{Px1 vector} \qquad (6)$$

and

$$[{}^uH^i_{\phi\phi}] = \begin{bmatrix} \frac{\partial^2 u^i}{\partial\phi_1\partial\phi_1} & \frac{\partial^2 u^i}{\partial\phi_1\partial\phi_2} & \cdots & \frac{\partial^2 u^i}{\partial\phi_1\partial\phi_M} \\ \frac{\partial^2 u^i}{\partial\phi_2\partial\phi_1} & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ \frac{\partial^2 u^i}{\partial\phi_M\partial\phi_1} & \cdots & & \frac{\partial^2 u^i}{\partial\phi_M\partial\phi_M} \end{bmatrix} = \text{M x M Matrix} \qquad (7)$$

with the shape of the second-order influence coefficient array being

$$\rho([H^u_{\phi\phi}]) = (\text{dim. of } \underline{u})\text{x}(\text{dim. of } \phi)\text{x}(\text{dim. of } \phi)$$
$$= P \times M \times M \qquad (8)$$

With the general kinematic format established, the reader is referred to Appendix B for detailed treatment of open chain definition and kinematics. This is again largely an excerpt from [Freeman and Tesar, 1986].

Dynamics of Open Chains

The dynamic controlling equations will now be developed using the generalized principle of d'Alembert (i.e., the virtual work of the d'Alembert loads) to transfer the system dependence from the specified 6M Cartesian based local link coordinates to the M generalized parameters $\underline{\phi}$. The inertia force $\underline{f}^{jk}$ due to the centroidal acceleration ${}^j\underline{a}^c$ of the mass M_{jk} of link jk can be expressed by using Newton's equations as

$$\underline{f}^{jk} = M_{jk}\, {}^j\underline{a}^c \qquad (9)$$

The inertia moment $\underline{m}^{jk}$ can also be expressed in a modified Euler format (the benefit of this will become apparent momentarily) as

$$\underline{m}^{jk} = [\Pi^{jk}]\underline{\alpha}^{jk} + (\underline{\omega}^{jk})^T[P^{jk}]\underline{\omega}^{jk} \qquad (10)$$

where

$$[\Pi^{jk}] = [\underline{I}^{jx} \,\vdots\, \underline{I}^{jy} \,\vdots\, \underline{I}^{jz}] \; ; \text{ 3 x 3 globally referenced inertia matrix} \qquad (11)$$

$$[P^{jk}] = \left\{ \begin{array}{l} [\,\underline{0} \,\vdots\, \underline{I}^{jz} \,\vdots\, -\underline{I}^{jy}] \\ [-\underline{I}^{jz} \,\vdots\, \underline{0} \,\vdots\, \underline{I}^{jx}\,] \\ [\,\underline{I}^{jy} \,\vdots\, -\underline{I}^{jx} \,\vdots\, \underline{0}\,] \end{array} \right\} \; ; \text{ 3 x 3 x 3 globally referenced inertia array} \qquad (12)$$

From Eqs. (9) and (10) and the principle of virtual work, the generalized inertial loads ($\underline{T}^I_\phi$) of an M-link chain as referenced to the M relative joint parameters ($\underline{\phi}$) are given by

$$T^I_\phi = \sum_{j=1}^{M} ([{}^jG^c_\phi]^T f^{jk} + [G^{jk}_\phi]^T m^{jk}) \qquad (13)$$

with $[{}^jG^c_\phi]$ and $[G^{jk}_\phi]$ the associated centroidal and rotational Jacobians of link jk, respectively. In order to maintain the particular form of Eqs. (5) and (10) (which will aid in the desire to keep intrinsic quantities recognizable), the Jacobian transposes in Eq. (13) must be brought inside the quadratic forms of Eqs. (9) and (10) (recall Eqs. (B12) and (B15)). This is accomplished by using the generalized scalar (or tensor) product (•) developed in Appendix C. The reader may wish to skip ahead to the kinematic transfer derivation of Eqs. (26)-(40) where a more detailed treatment of this procedure is given. Now, with the generalized dot product (•) and the influence coefficient representation of the link kinematics, the driving inertia torques become

$$T^I_\phi = [I^*_{\phi\phi}]\ddot{\phi} + \dot{\phi}^T[P^*_{\phi\phi\phi}]\dot{\phi} \qquad (14)$$

where the MxM joint-referenced effective inertia matrix is

$$[I^*_{\phi\phi}] = \sum_{j=1}^{M} \{M_{jk}[{}^jG^c_\phi]^T[{}^jG^c_\phi] + [G^{jk}_\phi]^T[\Pi^{jk}][G^{jk}_\phi]\} \qquad (15)$$

and the MxMxM inertial power array (operated on in the same "plane by plane" manner as in Eqs. (5) and (6) for parameter accelerations ($\ddot{u}$)) is

$$[P^*_{\phi\phi\phi}] = \sum_{j=1}^{M} \{M_{jk}([{}^jG^c_\phi]^T \cdot [{}^jH^c_{\phi\phi}])$$
$$+ (([G^{jk}_\phi]^T[\Pi^{jk}]) \cdot [H^{jk}_{\phi\phi}]) + [G^{jk}_\phi]^T([G^{jk}_\phi]^T \cdot [P^{jk}])[G^{jk}_\phi]\} \qquad (16)$$

Including the effects of a set of M (one for each of the M links) externally applied load vectors $({}^jT^u)$, the required generalized driving torque vector (T_ϕ) is given by

$$T_\phi = [I^*_{\phi\phi}]\ddot{\phi} + \dot{\phi}^T[P^*_{\phi\phi\phi}]\dot{\phi} - \sum_{j=1}^{M} [{}^jG^u_\phi]^T({}^jT^u) \qquad (17)$$

For completeness, an effective damping matrix $[C^*_{\phi\phi}]$ for linear viscous damping can be expressed as

$$[C^*_{\phi\phi}] = \sum_{r=1}^{R} c_r[{}^rG^u_\phi]^T[{}^rG^u_\phi] \qquad (18)$$

yielding a generalized dissipative load at (ϕ) of

$$(T^*_\phi)^D = [C^*_{\phi\phi}]\dot{\phi} \qquad (19)$$

where, from the virtual work of the local friction force $({}^rT^u)^D$,

$$(T^*_\phi)^D = \sum_{r=1}^{R} [{}^rG^u_\phi]^T({}^rT^u)^D \qquad (20)$$

with

$$({}^rT^u)^D = c_r u^r = c_r[{}^rG^u_\phi]\dot{\phi} \qquad (21)$$

Here, u^r is the motion parameter associated with damper r having a friction coefficient of c_r. Additionally, a 'localized' effective spring rate $[K^*_{\phi\phi}]$ can be defined as

$$[K^*_{\phi\phi}] \equiv \left[\frac{d(T^*_\phi)^k}{d\phi}\right] \qquad (22)$$

With the effective generalized load $(T^*_\phi)^k$ due to all system springs given by

$$(T^*_\phi)^k = \sum_{q=1}^{Q} [{}^qG^u_\phi]^T({}^qT^u)^k \qquad (23)$$

and with the q^{th} local spring load $({}^qT^u)^k$ given by

$$({}^qT^u)^k = K_q U^q \qquad (24)$$

Eq. (22) results in

$$[K^*_{\phi\phi}] = \sum_{q=1}^{Q} \{K_q[{}^qG^u_\phi]^T[{}^qG^u_\phi] + (({}^qT^u)^k)^T \cdot [{}^qH^u_{\phi\phi}]^T\} \qquad (25)$$

Here u^q is the deformation parameter associated with spring q having a spring rate of k_q. The addition of Eq. (19) and the subtraction of Eq. (23) from the right hand side of Eq. (17) gives a more complete controlling equation and completes the formalized open-chain dynamics.

TRANSFER OF SYSTEM DEPENDENCE

In general, one is concerned with the ability to perform a transfer of dependence from one set of generalized coordinates to another only if the dynamics cannot (or at least not conveniently) be obtained directly in terms of the desired generalized coordinates. That is to say, recalling Eqs. (14) - (25), a transfer is necessary only if the required kinematic influence coefficients are not directly derivable (i.e., without inversion) from known constraint (explicit or implicit) relationships. Unfortunately, this is usually the case and the ability to transfer an analytic description of the system model from any given set of generalized coordinates to any other set is therefore essential to the proposed modeling procedure. For example, consider the serial manipulator where up to now only the kinematic (and kinetic) model in terms of the relative joint coordinates has been specifically established (Appendix B). Suppose that, as is often the case (e.g., [Whitney, 1969], [Luh, et.al., 1980], [Hewit and Burdess, 1981], [Khatib, 1983], [Hogan, 1984], [Zheng and Luh, 1986], [Hayati, 1986], etc.), one wishes to determine the relative joint kinematics/kinetics required to obtain a specified end-effector trajectory/dynamics. In other words, one wants to solve the inverse kinematics problem (again excluding position except in an iterative sense) thereby determining the relative joint velocities ($\dot{\phi}$) and accelerations ($\ddot{\phi}$) in terms of the end-effector velocities ($\dot{e}$) and accelerations ($\ddot{e}$) (or loads T_e). Except in the simplest of cases (e.g., wrist-partionable manipulators [Hollerbach and Sahar, 1983]), this is at best extremely difficult to accomplish directly in terms of the end-effector coordinates (e) (due primarily to the non-vector nature of spatial orientation).

Kinematics

While an expression for (ϕ) in terms of (e) (i.e., $\phi = f(e)$) is not generally available, the kinematic