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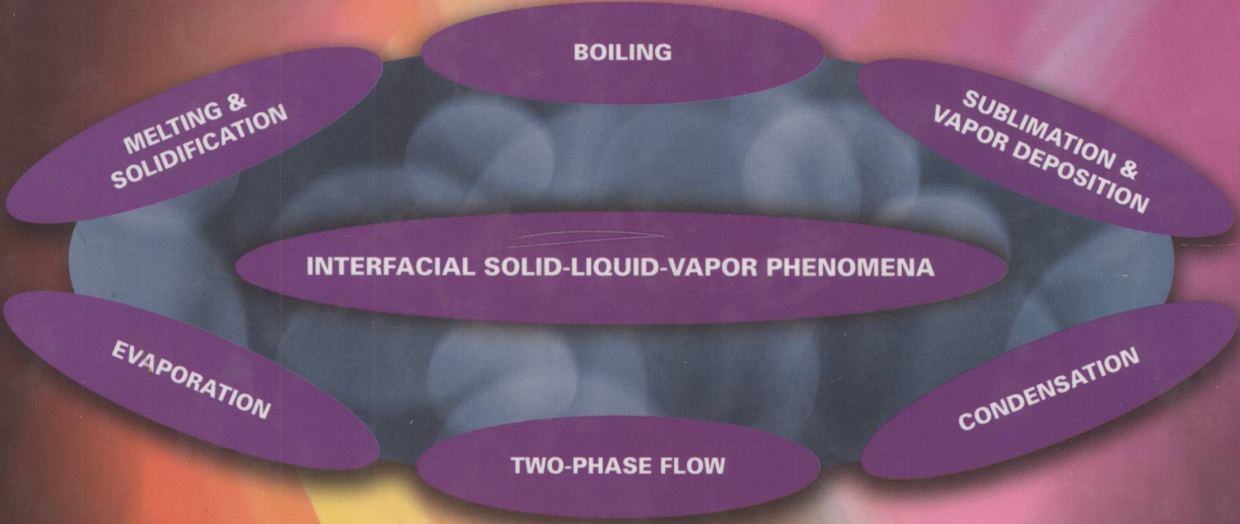
TRANSPORT PHENOMENA IN MULTIPHASE SYSTEMS



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Transport Phenomena in Multiphase Systems

To Our Families

Pouran, Tanaz, and Ali Faghri
Jennifer, Angela, and Joanna Zhang

Whose Love and Support
Make All Things Possible

Preface

Transport phenomena in multiphase systems with phase change is of great interest to scientists and engineers working in the power, nuclear, chemical processes, environmental, microelectronics, biotechnology, nano-technology, polymer science, food processing, cryogenics, space, and many other industries, from the established to emerging multidisciplinary technologies. For example, almost two-thirds of industrial heat exchangers undergo phase change; therefore, physical understanding and development of the first principal models are not only of interest in fundamental research, they also are greatly needed for a more accurate and reliable design of multiphase thermal systems.

The subject of transport phenomena in a multiphase system with phase change is important, because a unified physical/mathematical treatment is essential for engineering practitioners in the 21st century, who must cope with issues such as high heat flux and micro- or nanoscale systems for various applications.

Our motive in preparing this new textbook was to address the challenges and opportunities facing graduate education and teaching in thermal sciences within the mechanical engineering discipline and/or advanced transport phenomena in chemical engineering, which have remained basically unchanged for five decades. For example, the convection and/or conduction courses offered by most mechanical engineering departments as core courses in thermal sciences focus almost exclusively on single-phase, single-component, simple geometry such as channel flows or flat plates with the goal of an analytical solution with the continuum approach. Similarly, advanced transport phenomena in chemical engineering are based mostly on the excellent classical book by Bird *et al.*, which was originally published in 1960.

In contrast with their educational training, practicing engineers working in the thermal sciences or scientists in academia and the private sector have in recent years focused mostly on multiphase, multicomponent, non-conventional geometries, with coupled heat and mass transfer and phase change, with the goal of developing a numerical simulation using a continuum or non-continuum approach. We therefore developed this new textbook with the intention of helping instructors to bridge classroom learning and engineering practice by offering them advanced fundamental and general course materials that can replace conventional, limited, approaches for teaching advanced heat and mass transfer or transport phenomena.

The purpose of this textbook is to accurately present the basic principles for analyzing transport phenomena in multiphase systems and to demonstrate their wide variety of possible applications. Since it would take many book volumes to do justice to all aspects of multiphase systems, the scope of this book is limited to thermodynamics and momentum, heat and mass transfer fundamentals, with emphases on melting, solidification, sublimation, vapor deposition, condensation, evaporation, boiling, and two-phase flow.

Several books over the last 20 years have summarized the state of the art in liquid vapor systems. No serious attempts were made to bring all three forms of phase change, i.e., liquid vapor, solid liquid, and solid vapor, into one volume and to describe them from one perspective (in this text, pairs of arrows, , are used to portray energy and mass exchange associated with multiphase transfer between the phases listed). Furthermore, most of the existing texts were developed as monographs rather than textbooks. In writing this textbook, our goal was to provide basic engineering fundamentals related to transport phenomena in multiphase systems with phase change, including microscale and porosity effects. In most cases, the basic physical phenomena are presented with different mathematical models.

Historically, the field of transport phenomena has developed successful textbooks for momentum, heat and mass transfer in single-phase systems because these are straightforward and well developed concepts, in terms of physical and mathematical modeling. The same is not true for multiphase systems, which involve some components of the semi-empirical approach, are much more complex, and are thus less well understood. However, because of significant developments in transport phenomena in multiphase systems with phase change during the last two decades, we have much better physical, analytical, and numerical tools to model these types of problems: this is the purpose of our textbook.

Furthermore, traditionally three approaches were used to present transport phenomena: microscopic (differential), macroscopic (integral) and molecular level. Most heat transfer textbooks place the emphasis on microscopic and/or macroscopic. With the importance of microscale heat transfer or transport phenomena in applications of nanotechnology and biotechnology, as well as molecular dynamic simulations, it is important to discuss the molecular approach and the connection between the molecular and microscopic approaches. In this textbook, an attempt is made to better describe this relationship. For example, the generalized conservation equations in Chapter 3 have been developed not only microscopically and macroscopically using the continuum approach, but also using the Boltzmann equation.

There are three types of information available in the area of transport phenomena in multiphase systems that can be covered in a textbook of this nature:

1. Significant existing experimental work and correlations
2. Analytical and physical models
3. Numerical simulation modeling due to recent significant advances in digital computers and computational methodologies

We have not presented much in the way of item 1 except well established semi-empirical correlations that have been accepted in practice. The emphasis in this book is on the last two items. With respect to the final item, note that this is not a numerical method book; however, we have set up the framework so that students who wish to pursue this approach are equipped with the basic background material necessary to use existing commercial computer codes. Numerical methodologies and approaches are presented if they are specific to multiphase systems with phase change. Analytical and numerical physical models of transport phenomena in multiphase systems are the main focus in this textbook.

Chapters 1 through 4 present materials that are fundamental to the entire text. These chapters should be considered before proceeding to other chapters.

Chapter 1 begins with a review of the concept of phases of matter and a discussion of the role of phases in systems that include, simultaneously, more than one phase. This is followed by a review of transport phenomena with detailed emphasis in multicomponent systems, microscale heat transfer, dimensional analysis, and scaling. The processes of phase change between solid, liquid, and vapor are also reviewed, and the classification of multiphase systems is presented. Finally, some typical practical applications are described, which require students to understand the operational principles of these multiphase devices for further understanding and application in homeworks and examples in future chapters.

The thermodynamics of multiphase systems is presented in Chapter 2, which begins with a review of single-phase thermodynamics, including thermodynamic laws and relations, and proceeds to the concepts of equilibrium and stability. This is followed by discussion of thermodynamic surfaces and phase diagrams for single- and multicomponent systems. Also discussed are equilibrium criteria for single and multicomponent multiphase systems and the metastable equilibrium that exists in a multiphase system. Chapter 2 concludes with a discussion of thermodynamics at the interface and the effects of surface tension and disjoining pressure, including the superheat effect.

Chapter 3 presents the generalized macroscopic (integral) and microscopic (differential) governing equations for multiphase systems in local-instance formulations. The instantaneous formulation requires a differential balance for each phase, combined with appropriate jump and boundary conditions to match the solution of these differential equations at the interfaces. Also discussed in Chapter 3 are a rarefied vapor self-diffusion model and the application of the differential formulations to combustion. The generalized governing equations for multiphase systems in averaged formulations are presented in Chapter 4. The averaged formulations are obtained by averaging the governing equations within a small time interval (time average) or a small control volume (spatial average). The governing equations for the multidimensional multi-fluid and homogeneous models, as well as area-averaged governing equations for one-dimensional flows, are also discussed. Chapter 4 also covers single- and multiphase transport phenomena in porous media, including multi-fluid and mixture models. Finally, Boltzmann statistical averaging, including a detailed discussion of the Boltzmann equation and the Lattice Boltzmann method for modeling both single and multiphase systems, is presented.

Vector and tensor notations have been used in the development of generalized governing equations in Chapters 3 and 4. The neatness, generality, and compactness of vector and tensor notations are considered sufficient to overcome the criticism of those who may consider the subject too sophisticated. Examples in Chapters 3 and 4 and applications of these in non-vectorial one-, two-, or three-dimensional forms for various geometries in following chapters will provide adequate experience. In many examples, equations for simple one-dimensional processes are also developed based on actual physical mass, momentum, and energy balance, so that students appreciate the physical significance of various terms.

Chapter 5 introduces the concepts of surface tension, wetting phenomena, and contact angle, which are followed by a discussion on motion induced by capillarity. Additional detailed descriptions are presented for interfacial balances and boundary conditions for mass, momentum, energy, and species for multicomponent and multiphase interface. Also considered in Chapter 5 are heat and mass transfer through the thin film region during evaporation and condensation, including the effect of interfacial resistance and disjoining pressure. The dynamics of interfaces, including stability and wave effects, are presented. Finally, numerical simulations of interfaces and free surfaces using both continuum and non-continuum approaches are provided.

Solid-liquid phase change, including melting and solidification, is treated in Chapter 6, starting with the classification of solid-liquid phase changes and generalized boundary conditions at the interface. Different approaches to the solution of melting and solidification problems, including exact, integral approximate, and numerical solutions, are introduced. Solidification in binary solution systems, contact melting, melting and solidification in porous media, applications of solid-liquid phase change, and microscale solid-liquid phase change are also presented. Solid-vapor phase change, including sublimation and vapor deposition, is introduced in Chapter 7. The discussion begins with a brief overview of solid-vapor phase change and proceeds to detailed analyses on sublimation without and with chemical reaction, as well as physical and chemical vapor deposition.

Chapter 8 begins with a discussion of two main modes of liquid droplet embryo formation in condensation: homogeneous and heterogeneous, followed by a detailed examination of dropwise and filmwise condensation at both macro- and microscale levels. Applications of condensation in micro-gravity and condensation in porous media are also discussed. Chapter 9 presents criteria and classification of evaporation, evaporation from an adiabatic wall, evaporation from a heated wall, evaporation in porous media, evaporation in micro/miniature channels, as well as direct-contact evaporation.

Chapter 10 introduces the pool boiling curve and characterizes the various boiling regimes (free convection, nucleate, transition, and film boiling), followed by detailed discussions of each of the four pool boiling regimes, critical heat flux, minimum heat flux, and direct numerical simulation. Also discussed in Chapter 10 are the Leidenfrost phenomena as well as physical phenomena of boiling in porous media. Chapter 11 starts with definitions of various parameters for two-phase flow and flow patterns in vertical and horizontal tubes. This is followed by two-phase flow models as well as prediction of pressure drops and void fractions. Finally, the two-phase flow regimes and heat transfer characteristics for forced convective condensation and boiling at both macro- and microscale levels are presented.

The International System of Units (SI) is used throughout the book, and the conversion factors for different unit systems are provided in Appendix A. The complete thermophysical properties for all phases of various substances, along with empirical correlations of thermal properties as functions of temperature, are provided in Appendix B. Appendix C provides a brief review of vector and tensor operations.

We have used consistent symbols throughout the book. However, we have used some symbols for more than one purpose in a number of cases. We believe the context, as well as the nomenclature section, will clarify the meaning of the symbols used in these cases.

This textbook is designed for use as an advanced-level undergraduate or graduate textbook in mechanical engineering, chemical engineering, material science and engineering, nuclear engineering, biomedical engineering, or environmental engineering. It offers examples and homework problems as well as references from engineering and research applications related to multiphase systems. The only prerequisite courses necessary for the material are undergraduate thermodynamics, and heat transfer or transport phenomena. No graduate course in convection, conduction, or transport phenomena is required. In fact, convection, conduction, and/or transport phenomena are special cases of the general material presented here, if taught properly. We recognize a new trend at a number of universities to offer a single course in transport phenomena of multiphase system for all disciplines, and therefore we have tried to cover the materials that various departments might wish to have included in such a course. The materials included in this text may require more than one semester of instruction depending on the desired level of completeness. Therefore, it is recommended that the instructor choose the materials to be covered based on the background and needs of the students.

This text is not intended as a reference tool or handbook summarizing the state-of-the-art, nor does it to detail the history of multiphase systems with phase change. Part of the text was developed originally from lecture notes prepared by one of the authors (AF) who was teaching a graduate-level course at the University of Connecticut. Materials have been considerably rewritten by both authors and used as lecture notes for senior elective and/or graduate-level courses taught by the authors at the University of Connecticut, New Mexico State University, and the University of Missouri-Columbia. This textbook is suitable for students from a wide variety of backgrounds. The examples and homework problems were added to provide students a better physical understanding of theoretical concepts and uses for various applications. While the examples are designed to confer a better physical understanding, including mathematical modeling and a feeling for the order of magnitude of variables, end-of-chapter homework problems will help students appreciate fundamental concepts. There are three types of problems we have developed for this textbook: (1) simple numerical manipulation, (2) detailed physical and analytical models, and (3) open-ended problems. It is important that students gain experience in solving all three types of problems. A copyrighted solution manual and Microsoft PowerPoint presentation package are provided only to those instructors who adopt the book for the course.

The authors would like to express their deep thanks to a number of distinguished members of the heat transfer community who shared their expertise and time in reviewing this book: Thomas Avedisian, Christopher Beckermann, Arthur Bergles, F.B. Cheung, John Howell, Raymond Viskanta, and Ralph Webb. In addition, we wish to thank the following individuals who generously reviewed individual chapters or part of the book: Yutaka Asako, Theodore Bergman, Yiding Cao, Baki Cetegen, Wilson Chiu, Emily Green, Hongbin Ma, Robert McGurgan, Dmitry Khrustalev, Roop Mahajan, Gregory Jewett, Ugur Pasaogullari, Ranga Pitchumani, Jeremy Rice, Scott Thomas, and Kambiz Vafai. We are grateful to these dedicated professionals for their support, sage advice, improvements, and additions, which resulted in a superior and more comprehensive text than we envisioned. It is important to acknowledge the contributions of students over the last several years who were taught from the manuscripts out of which this book evolved. Our special thanks to Nan Cooper and Emily Jerome for their expert editing of the manuscripts.

This textbook provides an opportunity to cover fundamentals of transport phenomena in multiphase systems with all forms of phase change from one perspective. It is our hope that this textbook will influence some engineering colleges to treat transport phenomena in multiphase systems as a core requirement of the graduate curriculum in mechanical, chemical, environmental, nuclear, biomedical, and materials science disciplines. Your recommendations, comments, and criticisms are appreciated.

Amir Faghri
Yuwen Zhang

Nomenclature

A	area, m^2
A_ℓ	cross-sectional area of liquid flow passage, m^2
A_v	cross-sectional area of vapor flow passage, m^2
A'	dispersion constant, J
Bi	Biot number, hL/k (k is thermal conductivity of solid)
Bo	Bond number, $(\rho_\ell - \rho_v)gL^2/\sigma$
c	specific heat, J/(kg-K); velocity of the molecular random motion; wave velocity, m/s; speed of sound, m/s
$\mathbf{c}$	particle velocity (m/s)
c_i	molar concentration of the i^{th} species, kmol/m^3
c_p	specific heat at constant pressure, J/kg-K
c_v	specific heat at constant volume, J/kg-K
C	heat capacity, J/K; parameter in Chisholm correlation
C_D	drag coefficient
C_f	friction coefficient
Co	convective number, $(\rho_v/\rho_\ell)^{0.5}[(1-x)/x]^{0.8}$
D	diameter, m; self diffusivity, m^2/s
$\mathbf{D}$	rate of strain tensor, 1/s
D_h	hydraulic diameter, m
D_{ij}	binary diffusivity, m^2/s
$Ð_{ij}$	Maxwell-Stefan diffusivity, m^2/s
$\mathbb{D}_{ij}$	multicomponent Fick diffusivity, m^2/s
D_i^T	multicomponent thermal diffusivity, m^2/s
D/Dt	substantial derivative
e	specific internal energy, J/kg; kinetic energy of molecules, J
E	internal energy or surface free energy, J; emissive power, W/m^2
$\hat{E}$	total energy, J
f	degree of freedom; solid fraction; wave frequency, 1/s , molecular velocity distribution function.
F	force, N; Helmholtz free energy, J/kg-K
$\mathbf{F}$	force vector, N
Fo	Fourier number, $\alpha t/L^2$

Fr	Froude number, $U/\sqrt{gL}$ or $U^2/(gL)$
g	gravitational acceleration, m/s^2; specific Gibbs free energy, J/kg
G	Gibbs free energy, J; electron-lattice coupling factor, W/m^3-K
Gr	Grashof number, $g\beta\Delta TL^3/\nu^2$
h	heat transfer coefficient, W/(m^2-K); specific enthalpy, J/kg
$\bar{h}$	average heat transfer coefficient, W/m^2-K
$\tilde{h}$	average enthalpy of the multiphase mixture, J/kg
$h_{\ell v}$	latent heat of vaporization, J/kg
$h'_{\ell v}$	modified latent heat of vaporization, J/kg
h_G	heat transfer coefficient in noncondensable gas section, W/ m^2-K
h_m	convective mass transfer coefficient, m/s
$h_{m,G}$	mass transfer coefficient in noncondensable gas section, m/s
$h_{s\ell}$	latent heat of fusion, J/kg
h_x	local heat transfer coefficient, W/ m^2-K
h_{sv}	latent heat of sublimation, J/kg
H	enthalpy, J; height, m; Henry's constant
$\boldsymbol{I}$	identity tensor
j	volume flux, m/s; superficial velocity, m/s
J_0	Bessel function of the zeroth order
J_1	Bessel function of the first order
$\mathbf{J}_i$	mass flux of the i^{th} species relative to mass-averaged velocity, kg/m^2-s
$\mathbf{J}_i^*$	molar flux of the i^{th} species relative to molar-averaged velocity, kmol/m^2-s
Ja	Jakob number, $c_p\Delta T/h_{\ell v}$
k	thermal conductivity, W /(m-K)
k'	reaction rate constant
k_b	Boltzmann constant, J/K
K	interface curvature, 1/m; Permeability, m^2
K_0'	Arrhenius constant
K_{jk}	momentum exchange coefficient between phases j and k, kg/(m^3-s)
Ka	Kapitza number, $\mu_\ell^4 g/[(\rho_\ell-\rho_v)\sigma^3]$
L	(characteristic) length, m
Le	Lewis number, α/D
L_b	bubble or capillary scale, $\sqrt{\sigma/[g(\rho_\ell-\rho_g)]}$, m
m	mass, kg
$\dot{m}$	mass flow rate, kg/s
$\dot{m}''$	absolute mass flux relative to stationary coordinate system, kg/m^2-s
$\dot{\mathbf{m}}''$	mass flux vector, kg/m^2-s
$\dot{m}'''$	mass source per unit volume, kg/m^3-s
M	molecular mass, kg/kmol

Ma	Mach number, U/c; Marangoni number, $(dT/dy)(d\sigma/dT)\delta^2/(\alpha_\ell\mu_\ell)$
n	number of moles; number of horizontal tubes in an array
$\boldsymbol{n}$	unit normal vector
n_b''	number of vapor bubbles released per unit area and release cycle, $1/m^2$
n_D''	liquid droplet size distribution, $1/m^3$
n_i	number of moles for the i^{th} component in a multicomponent system
$\dot{n}_i''$	absolute molar flux of the i^{th} component relative to stationary coordinate system, kmol/s
N	number of components, number of molecules
$\mathfrak{N}$	number density of the molecules
N_A	Avogadro's number (1/mol)
N_a''	number density of nucleation sites
Nu	Nusselt number, hL/k
$\overline{\text{Nu}}$	average Nusselt number, $\overline{h}L/k$
Nu_x	local Nusselt number, $h_x x/k$
p	pressure, Pa
p_d	disjoining pressure, Pa
P	thermodynamic probability; perimeter, m; laser power, W
Pe	Peclet number, UL/α
Pr	Prandtl number, ν/α
Pr_t	turbulent Prandtl number
q	heat rate, W
q'	heat rate per unit length, W/m
q''	heat flux, W/m^2
$\mathbf{q}''$	heat flux vector, W/m^2
q''_{max}	maximum (critical) heat flux in boiling, W/m^2
q''_{min}	minimum heat flux in boiling, W/m^2
q'''	internal heat generation per unit volume, W/m^3
Q	total heat transfer, J; volume flow rate, m^3/s
r	radial coordinate, m
r_{eff}	effective pore radius, m
R	radius, m; radius of curvature, m; dimensionless radius, r/r_i; resistance, K/W
R_b	bubble radius, m
R_g	gas constant, R_u/M, kJ/kg-K
$\mathfrak{R}_j$	net reaction rate of the j^{th} chemical reaction
R_{men}	radius of curvature of the meniscus, m
R_r	characteristic micro roughness size, m
R_u	universal gas constant, 8.3144 kJ/kmol-K

R_v vapor space radius, m

R_δ interfacial thermal resistance, m^2-K/W

Ra Rayleigh number, $g\beta\Delta TL^3/(\nu\alpha)$

Re Reynolds number, UL/ν ; $4\Gamma/\mu_\ell$ (for film condensation or evaporation)

s specific entropy, J/(kg-K); interface location, m

$\dot{s}'''_{gen}$ entropy generation rate per unit volume, W/kg-K-m^3

S entropy, J/K; nondimensional interface location, s/L; source intensity in microscale melting; slip ratio, w_v/w_ℓ; solubility, kmol/Pa-m^3

Sc Schmidt number, ν/D; subcooling parameter, $c_{ps}(T_m-T_i)/h_{s\ell}$

Sh Sherwood number, h_mL/D_{12}

St Stanton number, $h/(\rho c_pU)$

Ste Stefan number, $c_p\left|T_w-T_m\right|/h_{s\ell}$

t time, s

$\mathbf{t}$ unit tangential vector

t_p laser pulse duration, s

T temperature, K

T_m melting point, K

T_{sat} saturation temperature, K

T_w wall temperature, K

T_∞ temperature of environment, K

u velocity in the x- direction, m/s

U velocity, m/s

$\bar{U}$ mean velocity, m/s

u_b laser beam scanning velocity, m/s

u_c critical Helmholtz velocity, m/s

u_f frictional velocity, $(\tau_w/\rho_\ell)^{1/2}$

$\bar{u}$ mean velocity, m/s

v specific volume, m^3/kg; velocity in the y-direction, m/s; radial velocity, m/s; or vapor velocity along the η-coordinate, m/s

V volume, m^3

$\mathbf{V}$ velocity vector, m/s

$\tilde{\mathbf{V}}$ mass-averaged velocity vector, $\left(\sum_{k=1}^{\Pi}\varepsilon_k\langle\rho_k\rangle^k\langle\mathbf{V}_k\rangle^k\right)\Big/\langle\rho\rangle$, m/s

w velocity in the z-direction or axial velocity, m/s

w_ℓ liquid phase axial velocity, m/s

w_v vapor phase axial velocity, m/s

w_w wave velocity, m/s

W work, J; width of the cavity, m; width of a capillary groove, m

We Weber number, $\rho U^2L/\sigma$

x Cartesian coordinate, m; vapor quality
x_i molar fraction of the i^{th} species
X material coordinate, m; dimensionless coordinate, x/L
$X_{k,i}$ body force per unit mass acting on the i^{th} species in the k^{th} phase, m/s^2
y Cartesian coordinate, m
Y material coordinate, m;
z Cartesian coordinate, axial coordinate, m
Z material coordinate, m; compressibility factor, pv/RT

GREEK SYMBOLS

α thermal diffusivity, m^2/s; accommodation coefficient; wave number, $2\pi/\lambda$; void fraction
α_a absorptivity
β coefficient of thermal expansion, 1/K; contact angle measured in degrees; volumetric flow fraction, $j_v/(j_\ell + j_v)$
β_m composition coefficient of volume expansion
Γ liquid mass flow rate per unit width, kg/m-s
δ liquid or vapor film thickness, m; thermal penetration depth, m; laser irradiation penetration depth, m; thickness of the deposited film, m
δ^+ nondimensional film thickness, $\delta u_f/\nu_\ell$
δ^* nondimensional film thickness, δ/L_F, or $\delta\left(\nu_\ell^2/g\right)^{-1/3}$
Δ dimensionless thermal penetration depth, $\delta/(\alpha_1\rho_1 h_{s\ell}/q_0'')$; dimensionless liquid layer thickness in contact melting, δ/W
Δt time interval for time average; time step in numerical solution, s
ΔT half of the width of phase change temperature range, K; temperature difference, K
ΔV volume element for volume average, m^3
ε porosity; volume fraction; emissivity; eddy diffusivity, m^2/s
ζ transformed coordinate, $(2\pi/\lambda)(x-ct)$
η dimensionless variable for binary solidification, $X/(2\sqrt{\tau})$; dimensionless coordinate, y/δ; coordinate normal to the solid-liquid interface, m
θ inclination angle, rad; contact or wetting angle, rad; dimensionless temperature
θ_f contact angle obtained from the smooth-surface model
θ_{men} meniscus contact angle
κ surface roughness, m
κ_T isothermal compressibility, $-(\partial V/\partial p)_T/V$
λ wavelength, m; constant in solid-liquid phase change, $S/(2\tau^{1/2})$
λ_c critical wavelength, m
λ_D most dangerous wavelength, m

μ	dynamic viscosity, kg/(m-s); chemical potential, J/mol
ν	kinematic viscosity, m^2/s
Π	number of phases
π_s	surface pressure, N/m^2
ζ	degree of advancement of the chemical reaction
ρ	density, kg/m^3
ρ_i	mass concentration of species i, kg/m^3
σ	surface tension force, N/m; collision diameter, Å
σ_{SB}	Stefan-Boltzmann constant, $5.67 \times 10^{-8}\ W/m^2\text{-}K^4$
τ	shear stress, N/m^2; thermal relaxation time, s; dimensionless time, $\alpha t / L^2$
$\boldsymbol{\tau}'$	stress tensor, N/m^2
$\boldsymbol{\tau}$	viscous stress tensor, N/m^2
τ_I, τ_δ	interfacial shear stress, N/m^2
τ_w	shear stress at wall, N/m^2
ϕ	Lennard-Jones potential, J; specific value of general property, Φ; wave amplitude; pressure drop multiplier in two-phase flow
φ	similarity variable for sublimation or evaporation; dependent variable in combustion of spherical droplet
Φ	general property
ψ	stream function, m^2/s
Ψ	availability, kJ
ω_i	mass fraction of species i
$\boldsymbol{\Omega}$	general vector quantity; surface tension parameter
∇	Laplace operator vector

Subscripts

0	reference variables; initial condition; reservoir conditions
a	adiabatic, acceleration
c	critical point, condenser
cap	capillary
CV	control volume
e	equilibrium; evaporator
f	final; fuel; thin film
g	gas (vapor); gravity
H	homogeneous
i	i^{th} component; initial; inner
I	interface
k	k^{th} phase in a multiphase system
ℓ	liquid
ℓo	single liquid phase
L	left; characteristic length
m	melting point or mushy zone
men	meniscus