Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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J. L. Rubio de Francia J. Vinuesa (Eds.)

Orthogonal Polynomials and their Applications

Proceedings, Segovia 1986

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Orthogonal Polynomials and their Applications

Proceedings of an International Symposium held in Segovia, Spain, Sept. 22–27, 1986

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Editors

Manuel Alfaro
Departamento de Matemáticas, Universidad de Zaragoza
50009 Zaragoza, Spain

Jesús S. Dehesa
Departamento de Física Moderna, Universidad de Granada
18071 Granada, Spain

Francisco J. Marcellan
Departamento de Matemática Aplicada, E.T.S. de Ingenieros Industriales
28006 Madrid, Spain

José L. Rubio de Francia †
Departamento de Matemáticas, Universidad Autónoma de Madrid
28049 Madrid, Spain

Jaime Vinuesa
Departamento de Matématicas, Universidad de Cantabria
39080 Santander, Spain

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This volume is dedicated to the memory of Jose Luis Rubio de Francia, Professor of Mathematics of the Autonomous University of Madrid, who died on February 1988. His death is a great loss to his country, to science and education, and to his many friends. We wish to record here our profound sense of loss. We hope that the completed volume is faithful to his vision of a book that not only instructs but also inspires.

INTRODUCTION

The present volume contains the Proceedings of the Second International Symposium on Orthogonal Polynomials and Their Applications held in Segovia, Spain, September 22nd to 27th, 1986.

This Symposium continues the idea born at Bar-le-Duc (France) on the occasion of the 150th anniversary of Edmond Nicolas Laguerre's birthday.

It intends to present a comprehensive view of the main achievements, fundamentals and working problems of the orthogonal polynomials and its relationship with other mathematical fields and applied sciences.

The core of the Symposium was a series of ten invited lectures, each covering some of the more fundamental aspects. In addition, there were 60 half-hour communications, including particular aspects of orthogonal polynomials on the real line and over the complex plane, special functions, orthogonal expansions and numerical methods. A special session devoted to open problems in the field of orthogonal polynomials also took place.

This volume contains nine invited lectures, more than ten seminars and a collection of some open questions.

Many institutions and individuals helped make the Symposium possible. We particularly thank, for their financial support, the Comisión Asesora de Investigación Científica y Técnica (CAICYT), Consejo Superior de Investigaciones Científicas (CSIC), Confederación Española de Centros de Investigación Matemática y Estadística (CECIME), Universidad Politécnica de Madrid, IBM-España S.A., Diputación Provincial de Segovia, Ayuntamiento de Segovia y Caja de Ahorros y Monte de Piedad de Segovia. For the hospitality offered to the participants, we thank the Ayuntamiento and Diputación Provincial de Segovia. Finally it is a pleasure to acknowledge Mónica Garay and Russell di Napoli for their secretarial help. We are also greatly indebted to the series editors of the Lecture Notes in Mathematics for giving us the possiblity to publish these Proceedings in this series.

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LIST OF PARTICIPANTS

ALFARO, M.P.
Dpartamento de Matemáticas
Univ. de Zaragoza. Spain.

ALFARO, M.
Departamento de Matemáticas
Univ. de Zaragoza. Spain.

AL SALAM, W.A.
Department of Mathematics.
University of Alberta
Edmonton, Alberta T6G 2G1
Canada

ALVAREZ, M.
Departamento de Matemáticas
E.T.S. de Ingenieros Industriales
Univ. Politécnica de Madrid
Spain

ANTOLIN, J.
Departamento de Física Teórica
Univ. de Zaragoza. Spain.

ARIAS DE VELASCO, L.
Departamento de Matemáticas
E.T.S. de Ingenieros Industriales
Univ. de Oviedo, Gijón. Spain.

BACHENE, M.
U.E.R. Analyse, Probabilité et
Applications.
Univ. Pierre et Marie Curie.
Paris, France.

BACRY, H.
Centre de Physique Theorique.
Luminy
Marseille, France.

BARREIRO, A.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales
Univ. Politécnica de Madrid.
Spain.

BAVINK, H.
Department of Mathematics
and Computer Science.
Technical University Delft.
The Netherlands.

BELMEHDI, S.
Department de Mathematiques
Université de Savoie.
Chambery, France

BEN CHEIKH, Y.
Department de Mathematiques
Faculté de Sciences et Techniques
Monastir. Tunisia

BOUKHEMIS, A.
U.E.R. Analyse, Probabilités et
Applications.
Univ. Pierre et Marie Curie.
Paris, France.

BREZINSKI, C.
U.E.R. d'I.E.E.A.
Univ. des Sciences et Techniques
Lille I.
Villeneuve d'Ascq. France.

CACHAFEIRO, A.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales
Univ. de Santiago. Vigo. Spain.

CASASUS, L.
Dpto. de Ecuaciones Funcionales
Univ. de La Laguna. Spain.

DEHESA, J.S.
Dpto. de Física Teórica
Univ. de Granada. Spain.

DJEHAICHE, F.
U.E.R. Analyse, Probabilités et
Applications.
Univ. Pierre et Marie Curie.
Paris, France.

DOUAK, K.
U.E.R. Analyse, Probabilités et
Applications
Univ. Pierre et Marie Curie.
Paris, France.

DRAIDI, N.
Laboratoire de Mathématique
Université d'Orsay. France.

DRAUX, A.
U.E.R. d'I.E.E.A.
Univ. des Sciences et Techniques
Lille I.
Villeneuve d'Ascq. France.

EIERMANN, M.
Institut fur Praktische Matematik
Universitat Karlsruhe. FRG.

FERNANDEZ, L.
Departamento de Matemáticas
E.T.S. de Ingenieros Industriales
Univ. Politécnica de Madrid.
Spain.

GARCIA LAZARO, P.
Departamento de Matemáticas
E.T.S. de Ingenieros Industriales
Univ. Politécnica de Madrid
Spain.

GATTESCHI, L.
Dipartimento di Matematica
Universitá di Torino. Italy.

GAUTSCHI, W.
Department of Computer Science
Purdue University.
West Lafayette,
Indiana. U.S.A.

GERONIMO, J.S.
School of Mathematics.
Georgia Institute of Technoloty.
Atlanta, Georgia, U.S.A.

GILEWICZ, J.
Centre de Physique Teorique.
Luminy.
Marseille, France.

GIORDANO, C.
Dipartimento di Matematica
Universita di Torino. Italy.

GODOY, E.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales.
Univ. de Santiago. Vigo, Spain.

GONZALEZ VERA, P.
Dpto. de Ecuaciones Funcionales
Univ. de La Laguna. Spain.

GONZALEZ-CONCEPCION, C.
Dpto. de Ecuaciones Funcionales
Univ. de La Laguna. Spain.

GORI, L.
Dipartimento di Metodi e Modelli
Matematici per le Scienze
Applicate.
Universita degli Studi di Roma,
"La Sapienza". Italy.

GUADALUPE, J.J.
Departamento de Matemáticas
Univ. de Zaragoza. Spain.

GUADALUPE, R.
Departamento de Matemáticas
Facultad de Informática
Univ. Politécnica de Madrid. Spain.

GUERFI, M.
U.E.R. Analyse, Probabilités et
Applications.
Univ. Pierre et Marie Curie.
Paris, France.

HENDRIKSEN, E.
Department of Mathematics.
University of Amsterdam.
Amsterdam 1018 WB. The Netherlands.

ISMAIL, M.E.
Departament of Mathematics.
Arizona State University.
Tempe, Arizona. U.S.A.

JACOBSEN, L.
Institutt for Mathematik.
Universitet Trondheim AVH.
Dragvoll, Norway.

KOORNWINDER, T.H.
Centre for Mathematics and Computer
Science
Amsterdam 1009 AB. The Netherlands.

KRALL, A.M.
Departament of Mathematics.
Pennsylvania State University.
University Park,
Pennsylvania. U.S.A.

KURATSUBO, S.
Department of Mathematics.
Faculty of Science.
Hirosaki University.
Aomori 036, Japan.

LAFORGIA, A.
Dipartimento di Matematica
Universita di Torino, Italy.

LASCOUX, A.
U.E.R. Mathematiques
Université Paris VII, France.

LITTLEJOHN, L.L.
Department of Mathematics.
Utah State University.
Logan, Utah. U.S.A.

LOPEZ, G.
Facultad de Matemáticas y
Cibernética.
Univ. de La Habana, Cuba.

MAGNUS, A.
Institut Mathematique.
Univ. de Louvain-la-Neuve,
Belgium.

MARCELLAN, F.
Departamento de Matemáticas
E.T.S. de Ingenieros Industriales
Univ. Politécnica de Madrid,
Spain.

MARKETT, C.
Lehrstuhl A fur Mathematik.
RWTH Aachen. FRG.

MARONI, P.
U.E.R. Analyse, Probabilités et
Applications.
Univ. Pierre et Marie Curie.
Paris, France.

MASON, J.
Computational Mathematics Group.
Royal Military College of
Science.
Shrivenham, Swindon, Wilts. U.K.

MEHTA, M.L.
Laboratoire de Physique
Theorique.
C.E.N. de Saclay.
Gift-sur-Yvette, France.

MEIJER, H.G.
Department of Mathematics and
Computer Science.
Technical University Delft.
The Netherlands.

MENA, A.C.
Grupo de Matemática Aplicada
Univ. de Porto. Portugal.

MICCHELLI, C.A.
Thomas Watson I.B.M.
Research Center
Yorktown Heights, New York,
U.S.A.

MONTANER, J.
Dpto. de Matemática Aplicada
Univ. de Zaragoza. Spain.

MORAL, L.
Dpto. de Matemática Aplicada
Univ. de Zaragoza. Spain.

NEVAI, P.G.
Department of Mathematics.
Ohio State University
Columbus, Ohio, U.S.A.

NJASTAD, O.
Departament of Mathematics.
Univ. of Trondheim NTH
Trondheim, Norway.

ORSI, A.
Dipartimento di Matematica.
Facolta di Ingegneria.
Politecnico di Torino, Italy.

ORTIZ, J.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales.
Univ. de Oviedo, Gijón. Spain.

ORTIZ, E.L.
Department of Mathematics.
Imperial College of Science &
Technology.
Queen,s Gate, London SW7 2BZ. U.K.

ORSTEIN, A.
Department of Mathematics.
Institute of Technology
Technion-Israel
Haifa,Israel.

OSOWSKI, S.
Institute of Electrical Engineering
Technical University.
Warsaw. Poland.

PEREZ ACOSTA, F.
Dpto. de Ecuaciones Funcionales.
Univ. de La Laguna. Spain.

PEREZ GRASA, I.
Departamento de Matemáticas.
Facultad de Ciencias Empresariales.
Univ. de Zaragoza. Spain.

PEREZ RIERA, M.
Departamento de Matemáticas.
Univ. de Zaragoza. Spain.

PIÑAR, M.
Depto. de Matemática Aplicada.
Colegio Universitario de Jaén.
Spain.

PITTALUGA, G.
Dipartimento di Matematica.
Universita di Torino. Italy.

POPOV
Macedonian Academy of Sciences.
Skopje. Yugoslavia.

RAHMANOV, E.A.
Steklov Institute of Mathematics.
USSR Academy of Sciences.
Moscow. U.S.S.R.

REZOLA, M.L.
Departamento de Matemáticas.
Univ. de Zaragoza. Spain.

RIVLIN, T.J.
Thomas Watson IBM Research Center
Yorktown Heights, New York. U.S.A.

RODRIGUES, M.J.
Grupo de Matemática Aplicada
Univ. de Porto. Portugal.

RODRIGUEZ, I.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales.
Univ. de Oviedo, Gijón, Spain.

RODRIGUEZ SOCORRO, G.
Esc. Formación Prof. de E.G.S.
"Santa María".
Univ. Autónoma de Madrid, Spain.

RONVEAUX, A.
Departament de Physique.
Facultés Universitaires Notre
Dame de la Paix.
Namur, Belgium.

RUBIO, J.L.
División de Matemáticas.
Universidad Autónoma de
Madrid, Spain.

SABLONNIERE, P.
U.E.R. d'I.E.E.A.
Univ. des Sciences et Techniques
de Lille I.
Villeneuve d'Ascq. France.

SACRIPANTE, L.
Dipartimento di Matematica
Universita di Torino. Italy.

SAFF, E.B.
Department of Mathematics.
Univ. of South Florida.
Tampa, Florida, U.S.A.

SANSIGRE, G.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales.
Univ. Politécnica de Madrid. Spain.

SMAILI, N.
U.E.R. Analyse, Probabilités et
Applications.
Univ. Pierre et Marie Curie,
Paris, France.

SOTTAS, G.
Section de Mathematiques.
Univ. de Geneve. Switzerland.

STAHL, H.
Department of Mathematics.
Technical University Berlin.
Berlin, F.R.G.

SZAFRANIEC, E.H.
Instytut Matematyki.
Universytet Jagiellonski.
Krakow, Poland.

TASIS, C.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales.
Univ. de Oviedo, Gijón, Spain.

TEMME, N.M.
Centre for Mathematics and
Computer Science.
Amsterdam 1009 AB.
The Netherlands.

TORRANO, E.
Dpto. de Matemáticas para Químicos
Univ. Complutense
Madrid, Spain.

VALENT, G.
Laboratoire de Physique Theorique
et des Hautes Energies.
Université Paris, France.

VAN DOORN, E.A.
Dep. of Applied Mathematics.
Twente University of Tecnology.
Enschede AE 7500. The Netherlands.

VAN ISEGEHM
U.E.R. d'I.E.E.A.
Université des Sciences et
Techniques Lille I.
Villeneuve d'Ascq. France.

VAN ROSSUM, H.
Instituut voor Propedeutische
Wiskunde.
Univ. of Amsterdam. The Netherlands.

VARONA, J.
Departamento de Matemáticas.
Colegio Universitario de La Rioja
Univ. de Zaragoza. Spain.

VIGIL, L.
Departamento de Matemáticas.
Univ. de Zaragoza. Spain.

VINUESA, J.
Dpto. de Teoría de Funciones.
Univ. de Cantabria.
Santander. Spain.

WAADELAND, H.
Institutt for Mathematikk.
Universitet Trondheim AVH.
Dragvoll, Norway.

WALLIN, H.
Department of Mathematics.
University of Umea. Sweden.

WIMP, J.
Department of Mathematics.
Drexel University. Philadelphia
Pensylvania 19104. U.S.A.

ZARZO, A.
Departamento de Matemáticas.
E.T.S. de Ingenieros Industriales.
Univ. Politécnica de Madrid. Spain.

ZAYED, A.I.
Departament of Mathematics.
California Polytechnic State
University.
San Luis Obispo. California, U.S.A.

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ERROR ESTIMATE IN PADE APPROXIMATION

Claude BREZINSKI

Laboratoire d'Analyse Numérique et d'Optimisation
U.F.R. I.E.E.A.
Université des Sciences et Techniques de Lille Flandres-Artois
59655 - Villeneuve d'Ascq - Cedex
FRANCE

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ABSTRACT.

The aim of this paper is to extend Kronrod's procedure for estimating the error in Gaussian quadrature formulas to Padé approximation. Some results on Stieltjes polynomials are also given. Examples show the effectiveness of the method. The procedure is then applied to the ε-algorithm, which is a convergence acceleration method related to Padé approximation. General principles for estimating the error in series approximations and sequence transformations are also brought to light.

INTRODUCTION.

In 1965, A.S. Kronrod published a small book [10] in which he proposed an heuristic method for estimating the error in Gaussian quadratures. It simply consists in comparing the results obtained by two different quadrature formulas. If, for example, one makes use of two Gaussian quadratures with respectively n and n+1 nodes then one has to evaluate the integrand at 2n+1 points. This procedure is not the best since a Gaussian quadrature with n+1 nodes exactly integrates polynomials up to the degree 2n+1 while we are using 2n+1 points for the same accuracy. We are wasting the integrand evaluations. Thus, Kronrod's idea was to use a Gaussian quadrature formula first and then to add it new nodes in an optimal way that is in order to obtain a new formula exact for polynomials of the highest possible degree. The difference between the two results gives an estimate of the error.

Since Padé approximants can be viewed as formal Gaussian quadratures [2], the aim of this paper is to extend Kronrod's idea to Padé approximation.

Let us recall some results. Let f be a formal power series

$$f(t) = \sum_{i=0}^{\infty} c_i t^i \quad , \quad c_i \in \mathbb{C}$$

Let c be the linear functional acting on the space of complex polynomials, defined by

$$c(x^i) = c_i \quad i = 0,1,\ldots$$

We formaly have

$$f(t) = c((1-xt)^{-1}).$$

Let v be an arbitrary polynomial of degree k. We define w as

$$w(t) = c\left(\frac{v(x)-v(t)}{x-t}\right)$$

where c acts on the variable x, t being a parameter. w is a polynomial of degree k-1. w is the polynomial associated to v. We set

$$\tilde{v}(t) = t^k v(t^{-1}) \quad \text{and} \quad \tilde{w}(t) = t^{k-1} w(t^{-1}).$$

The rational fraction $\tilde{w}(t)/\tilde{v}(t)$ is called a Padé-type approximant of f. It is denoted by $(k-1/k)_f(t)$. v is called its generating polynomial. We have

$$f(t) - (k-1/k)_f(t) = \frac{t^k}{\tilde{v}(t)} \, c\left(\frac{v(x)}{1-xt}\right) = O(t^k).$$

Padé-type approximants can be related to polynomial interpolation. Let P be the Hermite interpolation polynomial of $(1-xt)^{-1}$ (as a function of x, t being a parameter) at the zeros of v. We have

$$(k-1/k)_f(t) = c(P(x)).$$

Thus (k-1/k) can be viewed as a formal interpolatory quadrature formula for $(1-xt)^{-1}$ with the zeros of v as nodes. The above error expression is just the classical property of an interpolatory quadrature formula with k nodes to be exact for polynomials of degree at most k-1. Moreover, from that expression, we have

$$f(t)-(k-1/k)_f(t) = \frac{t^k}{\tilde{v}(t)} \{c(v(x)) + c(xv(x))t + \ldots + c(x^{k-1} v(x))t^{k-1} + t^k \, c\left(\frac{x^k v(x)}{1-xt}\right)\}$$

Since v can be arbitrarily chosen, let us take it such that

$$c(x^i v(x)) = 0 \quad i = 0,\ldots,k-1.$$

This is always possible if the Hankel determinant

$$H_k^{(0)} = \begin{vmatrix} c_0 & c_1 & \cdots & c_{k-1} \\ c_1 & c_2 & \cdots & c_k \\ \vdots & & & \vdots \\ c_{k-1} & c_k & \cdots & c_{2k-2} \end{vmatrix}$$