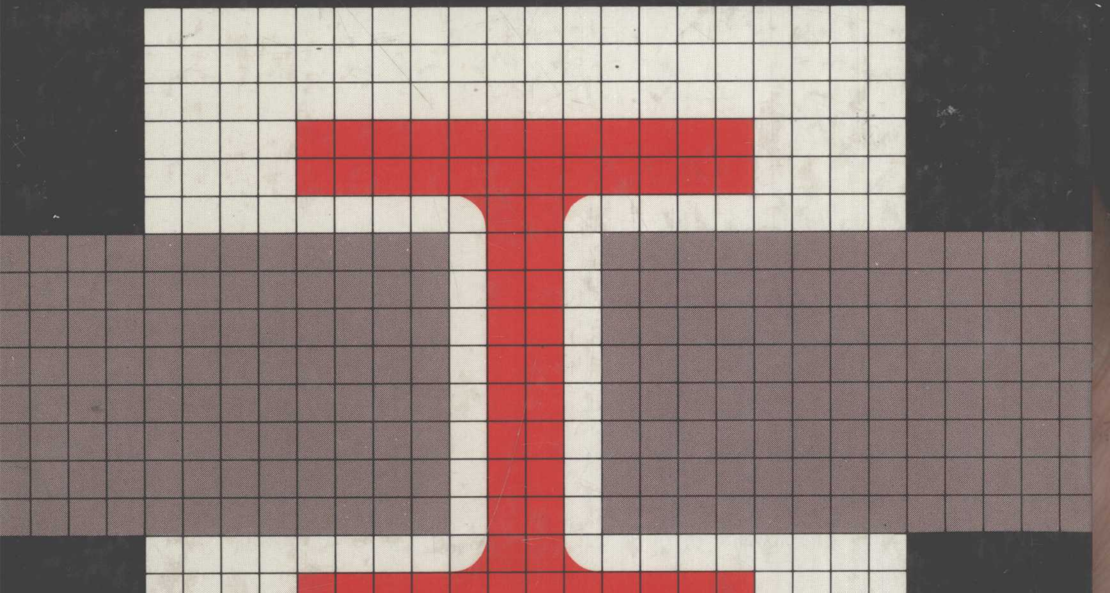
STRUCTURAL STABILITY

THEORY AND IMPLEMENTATION



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PREFACE

This book presents a simple, concise, and reasonably comprehensive introduction to the principles and theory of structural stability that are the basis for structural steel design and shows how they may be used in the solution of practical building frame design problems. It provides the necessary background for the transition for students of structural engineering from fundamental theories of structural stability of members and frames to practical design rules in AISC Specifications. It was written for upper level undergraduate or beginning graduate students in colleges and universities on the one hand, and those in engineering practice on the other.

The scope of the book is indicated by its contents. The concepts and principles of structural stability presented in Chapter 1 form the basis for the elastic and plastic theories of stability of members and frames which are discussed separately in Chapter 2 (Columns), Chapter 3 (Beam-Columns), Chapter 4 (Rigid Frames), and Chapter 5 (Beams). The energy and numerical methods of analyzing a structure for its stability limit load are described in Chapter 6.

Each of these later chapters sets out initially to state the basic principles of structural stability, followed by the derivation of the necessary basic governing differential equations based on idealized conditions. These classical solutions and their physical significance are then examined. The chapter goes on to show how these solutions are affected by the inelasticity of the material and imperfection of the structural member and system associated with a real structure, using both hand techniques and modern computer capabilities. It finally outlines some of the popularly used techniques by which this voluminous information may be utilized to provide design rules and calculation techniques suitable for design office use. In this way, the reader not only will obtain an understanding of the fundamental principles and theory of structural stability from an idealized elastic, perfect system, but also to an inelastic imperfect system that leads to the necessary links between the code rules, design office practice, and the actual structural system in the real world.

The continued rapid development in computer hardware and software in recent years has made it possible for engineers and designers to predict structural behavior quite accurately. The advancement in structural analysis techniques coupled with the increased understanding of structural behavior has made it possible for engineers to adopt the Limit States Design philosophy. A limit state is defined as a condition at which a structural system or its component ceases to perform its intended function under normal conditions (Serviceability Limit State) or failure under severe conditions (Ultimate Limit State). The recently published Load and Resistance Factor Design (LRFD) Specification by the American Institute of Steel Construction (AISC) is based on the limit states philosophy and thus represents a more rational approach to the design of steel structures.

This book is not therefore just another book that presents Timoshenko's basic elastic theory (S. P. Timoshenko and J. M. Gere, "Theory of Elastic Stability," McGraw-Hill, 1961), or Bleich's inelastic buckling theory (F. Bleich, "Buckling Strength of Metal Structures," McGraw-Hill, 1952), or Chen's numerical analysis (W. F. Chen and T. Atsuta, "Theory of Beam-Columns," two-volume, McGraw-Hill, 1976, 1977) in a new style. Instead it presents theory and principles of structural stability in its most up-to-date form. This volume includes not only the state-of-the-art methods in the analysis and design of columns as individual members and as members of a structure, but also an introduction to engineers as to how these new developments have been implemented as the stability design criteria for members and frames in AISC/LRFD Specification.

This book is based on a series of lectures that Professor Chen gave at Purdue University and Lehigh University under the general heading of "Structural Stability." The preparation of the 1985 T. R. Higgins Lectureship Award paper entitled "Columns with End Restraint and Bending in Load and Resistance Factor Design" for AISC Engineering Journal (3rd Quarter, Vol. 22, No. 3, 1985) inspired us to attempt to create a useful textbook for the undergraduate and beginning graduate students in structural engineering as well as practicing structural engineers who are less familiar with the stability design criteria of members and frames in the newly published LRFD Specification.

Professor Chen wishes to extend his thanks to AISC for the 1985 T. R. Higgins Lectureship Award, when the book began to take shape, to

Professor H. L. Michael of Purdue University for continuing support over many years, and to the graduate students, C. Cheng, L. Duan, and F. H. Wu, among others, for preparing the Answers to Some Selected Problems during their course work on Structural Stability in the spring semester of 1986 in the School of Civil Engineering at Purdue University.

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NOTATION

LOAD AND MOMENT

P = axial load

P_e = $\dfrac{\pi^2 EI}{L^2}$ = Euler buckling load

P_{cr} = elastic buckling load

P_{ek} = $\dfrac{\pi^2 EI}{(KL)^2}$

= elastic buckling load considering column end conditions

P_f = failure load by the elastic–plastic analysis

P_p = plastic collapse load or limit load by the simple plastic analysis

P_r = $P_e \dfrac{E_r}{E}$ = reduced modulus load

P_t = $P_e \dfrac{E_t}{E}$ = tangent modulus load

P_u = ultimate strength considering geometric imperfections and material plasticity

P_y = AF_y = yield load

M_a = amplified (design) moment

M_{cr} = elastic buckling moment

M_{ocr} = $\dfrac{\pi}{L}\sqrt{EI_y GJ}\sqrt{1+W^2}$, where $W^2 = \dfrac{\pi^2}{L^2}\left(\dfrac{EC_w}{GJ}\right)$

= elastic buckling moment under uniform moment

M_{eq} = $C_m M_2$ = equivalent moment

M_{ext} = moment at a section due to externally applied loads
M_{int} = internal resisting moment of the section
M_m = transition moment (in Plastic Design)
M_n = nominal flexural strength
M_p = ZF_y = plastic moment
M_{pcx} = $1.18M_{px}\left[1-\left(\frac{P}{P_y}\right)\right] \leq M_{px}$
for H-section about strong axis.
= plastic moment capacity about the strong axis considering the influence of axial load
M_{pcy} = $1.19M_{py}\left[1-\left(\frac{P}{P_y}\right)^2\right] \leq M_{py}$
for H-section about weak axis.
= plastic moment capacity about the weak axis considering the influence of axial load
M_u = ultimate moment capacity considering geometric imperfections and material plasticity
M_y = SF_y = yield moment
T_{sv} = $GJ\frac{d\gamma}{dz}$ = St. Venant (or uniform) torsion
T_w = $-EC_w\frac{d^3\gamma}{dz^3}$ = warping restraint (or non-uniform) torsion
σ = stress
σ_{ij} = stress tensor
ε = strain
ε_{ij} = strain tensor

ENERGY AND WORK

U = $\frac{1}{2}\int_v \sigma_{ij}\varepsilon_{ij}\,dv = U_a + U_b + U_{sv} + U_w = -W_{int}$
= strain energy of a linear elastic system
U_a = $\frac{1}{2}\int_0^L \frac{P^2}{EA}\,dz = \frac{1}{2}\int_0^L EA\left(\frac{du}{dz}\right)^2 dz$
= strain energy due to axial shortening
U_b = $\frac{1}{2}\int_0^L \frac{M^2}{EI}\,dz = \frac{1}{2}\int_0^L EI\left(\frac{d^2v}{dz^2}\right)^2 dz$
= strain energy due to bending
U_{sv} = $\frac{1}{2}\int_0^L \frac{T_{sv}^2}{GJ}\,dz = \frac{1}{2}\int_0^L GJ\left(\frac{d\gamma}{dz}\right)^2 dz$
= strain energy due to St. Venant torsion

U_w = $\frac{1}{2}\int_0^L EC_w\left(\frac{d^2\gamma}{dz^2}\right)^2 dz$
= strain energy due to warping restraint torsion
V = $-W_{ext}$ = potential energy
W_{int} = $-U$ = work done by the internal resisting forces
W_{ext} = $-V$ = work done by the external applied forces
Π = $U + V$ = total potential energy

GEOMETRY AND DIMENSIONS

A = cross sectional area
b_f = flange width
C_w = warping constant
= $\frac{1}{2} I_f h^2$ for I section
d = depth
h = distance between centroid of flanges
I = Ar^2 = moment of inertia
I_f = moment of inertia of one flange
J = uniform torsional (or St. Venant) constant
= $\sum_{i=1}^{n} \frac{1}{3} b_i t_i^3$ for a thin-walled open section
L = length
r = $\sqrt{\frac{I}{A}}$ = radius of gyration
S = elastic section modulus
t = thickness
u = displacement in the X-direction
v = displacement in the Y-direction
W = $\frac{\pi}{L}\sqrt{\frac{EC_w}{GJ}}$
Z = plastic section modulus
ϕ = curvature
λ_b = $\sqrt{\frac{M_p}{M_{cr}}}$ = beam slenderness parameter
λ_c = $\sqrt{\frac{P_u}{P_{ek}}} = \frac{KL}{\pi r}\sqrt{\frac{F_u}{E}}$ = column slenderness parameter
γ = angle of twist

MATERIAL PARAMETERS

E = Young's modulus
= 29,000 ksi for steel

E_{eff} = effective modulus
E_r = reduced modulus
E_t = tangent modulus
F_y, σ_y = yield stress
G = shear modulus
= $\dfrac{E}{2(1+\nu)} = 11{,}200$ ksi for steel
ν = Poisson's ratio
= 0.3 for steel

STABILITY RELATED FACTORS

A_F = amplification factor
B_1 = $P-\delta$ moment amplification factor for beam-columns in LRFD
= $\dfrac{C_m}{1-\left(\dfrac{P}{P_{ek}}\right)} \geqslant 1.0$
B_2 = $P-\Delta$ moment amplification factor for beam-columns in LRFD
= $\dfrac{1}{1-\sum\left(\dfrac{P}{P_{ek}}\right)}$ or
= $\dfrac{1}{1-\sum\left(\dfrac{P\Delta_0}{HL}\right)}$
C_b = $\dfrac{M_{cr}}{M_{ocr}}$ = equivalent moment factor for beams
= $1.75+1.05\left(\dfrac{M_1}{M_2}\right)+0.3\left(\dfrac{M_1}{M_2}\right)^2 \leqslant 2.3$ in AISC Specifications for end moment case
= $\dfrac{12}{3\dfrac{M_1}{M_{max}}+4\dfrac{M_2}{M_{max}}+3\dfrac{M_3}{M_{max}}+2}$ for other loading conditions (*see* Table 5.2b, p. 334)
C_m = equivalent moment factor for beam-columns
= $0.6-0.4\left(\dfrac{M_1}{M_2}\right) \geqslant 0.4$ in ASD for end moment case
= $0.6-0.4\left(\dfrac{M_1}{M_2}\right)$ in LRFD for end moment case

	=	$1 + \psi \dfrac{P}{P_{ek}}$ = effective length factor
κ	=	$\sqrt{\dfrac{P_e}{P_{ek}}}$ = effective length factor
r_i	=	load factors
ϕ	=	resistance factor
ϕ_b	=	resistance factor for flexure = 0.90
ϕ_c	=	resistance factor for compression = 0.85

CONTENTS

Chapter 1

GENERAL PRINCIPLES

1.1 CONCEPTS OF STABILITY

When a change in the geometry of a structure or structural component under compression will result in the loss of its ability to resist loadings, this condition is called *instability*. Because instability can lead to a catastrophic failure of a structure, it must be taken into account when one designs a structure. To help engineers to do this, among other types of failure, a new generation of designing codes have been developed based on the concept of *limit states*.

In *limit states design*, the structure or structural component is designed against all pertinent limit states that may affect the safety or performance of the structure. Basically, there are two types of limit states: The first type, *Strength limit states*, deals with the performance of structures at their maximum load-carrying capacities. Examples of strength limit states include structural failure due to either the formation of a plastic collapse mechanism or to member or frame instability. *Serviceability limit states*, on the other hand, are concerned with the performance of structures under normal service conditions. Hence, they pertain to the appearance, durability, and maintainability of a structure. Examples of serviceability limit states include deflections, drift, vibration, and corrosion.

Stability, an important constituent of the strength limit states, is dealt with explicitly in the present American Institute for Steel Construction (AISC) limit state specification.[1] Although the importance of considering stability in design is recognized by most practicing engineers, the subject still remains perplexing to some. The reason for this perplexity is that the use of *first-order structural analysis*, which is familiar to most engineers, is not permissible in a stability analysis. In a true *stability analysis*, the change in geometry of the structure must be taken into account; as a consequence, equilibrium equations must be written based on the geometry of a structure that becomes deformed under load. This is known as the *second-order analysis.* The second-order analysis is further complicated by the fact that the resulting equilibrium equations are differential equations instead of the usual algebraic equations. Consequently, a mastery of differential calculus is a must before any attempt to solve these equations.

In what follows, we will explain the nature of structural stability and ways to analyze it accurately.

The concept of stability is best illustrated by the well-known example of a ball on a curved surface (Fig. 1.1). For a ball initially in equilibrium, a slight disturbing force applied to the ball on a concave surface (Fig. 1.1a) will displace the ball by a small amount, but the ball will return to its initial equilibrium position once it is no longer being disturbed. In this case, the ball is said to be in a *stable equilibrium.* If the disturbing force is applied to a ball on a convex surface (Fig. 1.1b) and then removed, the ball will displace continuously from, and never return to, its initial equilibrium position, even if the disturbance was infinitesimal. The ball in this case is said to be in an *unstable equilibrium.* If the disturbing force is

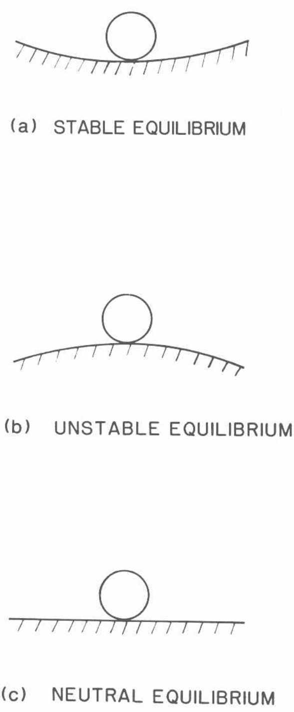
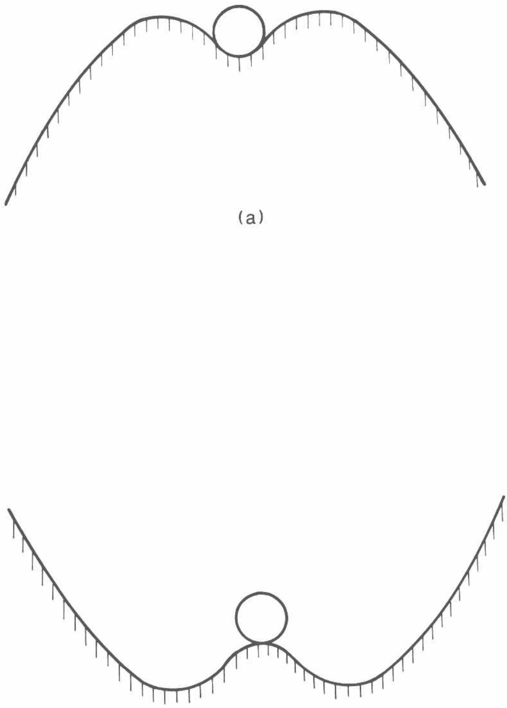


FIGURE 1.1 Stable, unstable, and neutral equilibrium



(a)

(b)

FIGURE 1.2 Effect of finite disturbance

applied to the ball on a flat surface (Fig. 1.1c), the ball will attain a new equilibrium position to which the disturbance has moved it and will stay there when the disturbance is removed. This ball is said to be in a *neutral equilibrium*.

Note that the definitions of stable and unstable equilibrium in the preceding paragraph apply only to cases in which the disturbing force is very small. These will be our *definitions of stability*. However, keep in mind that it is possible for a ball, under certain conditions (Fig. 1.2), to go from one equilibrium position to another; for example, a ball that is "stable" under a small disturbance may go to an unstable equilibrium under a large disturbance (Fig. 1.2a), or vice versa (Fig. 1.2b).

The concept of stability can also be explained by considering a system's stiffness. For an *n*-degrees-of-freedom system, the forces and displacements of the system are related by a stiffness matrix or function. If this stiffness matrix or function is *positive definite*, the system is said to be stable. The transition of the system from a state of stable to neutral