

Statistical Signal Processing of **Complex-Valued Data**

The Theory of Improper and Noncircular Signals

Peter J. Schreier and **Louis L. Scharf**

CAMBRIDGE

Statistical Signal Processing of Complex-Valued Data

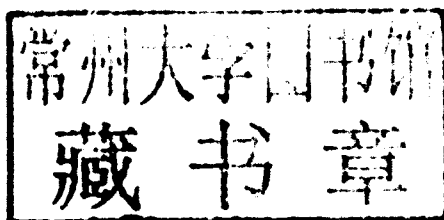
The Theory of Improper and
Noncircular Signals

PETER J. SCHREIER

University of Newcastle, New South Wales, Australia

LOUIS L. SCHARF

Colorado State University, Colorado, USA



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521897723

© Cambridge University Press 2010

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2010

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

ISBN 978-0-521-89772-3 Hardback

Statistical Signal Processing of Complex-Valued Data

Complex-valued random signals are embedded into the very fabric of science and engineering, yet the usual assumptions made about their statistical behavior are often a poor representation of the underlying physics. This book deals with improper and noncircular complex signals, which do not conform to classical assumptions, and it demonstrates how correct treatment of these signals can have significant payoffs.

The book begins with detailed coverage of the fundamental theory and presents a variety of tools and algorithms for dealing with improper and noncircular signals. It provides a comprehensive account of the main applications, covering detection, estimation, and signal analysis of stationary, nonstationary, and cyclostationary processes.

Providing a systematic development from the origin of complex signals to their probabilistic description makes the theory accessible to newcomers. This book is ideal for graduate students and researchers working with complex data in a range of research areas from communications to oceanography.

PETER J. SCHREIER is an Associate Professor in the School of Electrical Engineering and Computer Science, The University of Newcastle, Australia. He received his Ph.D. in electrical engineering from the University of Colorado at Boulder in 2003. He currently serves on the Editorial Board of the *IEEE Transactions on Signal Processing*, and on the IEEE Technical Committee *Machine Learning for Signal Processing*.

LOUIS L. SCHARF is Professor of Electrical and Computer Engineering and Statistics at Colorado State University. He received his Ph.D. from the University of Washington at Seattle. He has since received numerous awards for his research contributions to statistical signal processing, including an IEEE Distinguished Lectureship, an IEEE Third Millennium Medal, and the Technical Achievement and Society Awards from the IEEE Signal Processing Society. He is a Life Fellow of the IEEE.

Preface

Complex-valued random signals are embedded into the very fabric of science and engineering, being essential to communications, radar, sonar, geophysics, oceanography, optics, electromagnetics, acoustics, and other applied sciences. A great many problems in detection, estimation, and signal analysis may be phrased in terms of two channels' worth of real signals. It is common practice in science and engineering to place these signals into the real and imaginary parts of a complex signal. Complex representations bring economies and insights that are difficult to achieve with real representations.

In the past, it has often been assumed – usually implicitly – that complex random signals are *proper* and *circular*. A *proper* complex random variable is uncorrelated with its complex conjugate, and a *circular* complex random variable has a probability distribution that is invariant under rotation in the complex plane. These assumptions are convenient because they simplify computations and, in many aspects, make complex random signals look and behave like real random signals. Yet, while these assumptions can often be justified, there are also many cases in which proper and circular random signals are very poor models of the underlying physics. This fact has been known and appreciated by oceanographers since the early 1970s, but it has only recently been accepted across disciplines by acousticians, optical scientists, and communication theorists.

This book develops the tools and algorithms that are necessary to deal with *improper* complex random variables, which are correlated with their complex conjugate, and with *noncircular* complex random variables, whose probability distribution varies under rotation in the complex plane. Accounting for the improper and noncircular nature of complex signals can have big payoffs. In digital communications, it can lead to a significantly improved tradeoff between spectral efficiency and power consumption. In array processing, it can enable us to estimate with increased accuracy the direction of arrival of one or more signals impinging on a sensor array. In independent component analysis, it may be possible to blindly separate Gaussian sources – something that is impossible if these sources are *proper*.

In the electrical engineering literature, the story of improper and noncircular complex signals began with Brown and Crane, Gardner, van den Bos, Picinbono, and their co-workers. They have laid the foundations for the theory we aim to review and extend in this research monograph, and to them we dedicate this book. The story is continuing, with work by a number of our colleagues who are publishing new findings as we write this preface. We have tried to stay up to date with their work by referencing it as carefully as we have been able. We ask their forbearance for results not included.

Outline of this book

The book can be divided into three parts. Part I (Chapters 1 and 2) gives an overview and introduction to complex random vectors and processes. In Chapter 1, we describe the origins and uses of complex signals. The chapter answers the following question: why do engineers and applied scientists represent real measurable effects by complex signals? Chapter 2 lays the foundation for the remainder of the book by introducing important concepts and definitions for complex random vectors and processes, such as widely linear transformations, complementary correlations, the multivariate improper Gaussian distribution, and complementary power spectra of wide-sense stationary processes. Chapter 2 should be read before proceeding to any of the later chapters.

Part II (Chapters 3–7) deals with complex random vectors and their application to correlation analysis, estimation, performance bounding, and detection. In Chapter 3, we discuss in detail the second-order description of a complex random vector. In particular, we are interested in those second-order properties that are invariant under either widely unitary or widely linear transformation. This leads us to a test for impropriety and applications in independent component analysis (ICA). Chapter 4 treats the assessment of multivariate association between two complex random vectors. We provide a unifying treatment of three popular correlation-analysis techniques: canonical correlation analysis, multivariate linear regression, and partial least squares. We also present several generalized likelihood-ratio tests for the correlation structure of complex Gaussian data, such as sphericity, independence within one data set, and independence between two data sets.

Chapter 5 is on estimation. Here we are interested in linear and widely linear least-squares problems, wherein parameter estimators are constrained to be linear or widely linear in the measurement and the performance criterion is mean-squared error or squared error under a constraint. Chapter 6 deals with performance bounds for parameter estimation. We consider quadratic performance bounds of the Weiss–Weinstein class, the most notable representatives of which are the Cramér–Rao and Fisher–Bayes bound. Chapter 7 addresses detection, where the problem is to determine which of two or more competing models best describes experimental measurements. In order to demonstrate the role of widely linear and widely quadratic forms in the theory of hypothesis testing, we concentrate on hypothesis testing within Gaussian measurement models.

Part III (Chapters 8–10) deals with complex random processes, both continuous- and discrete-time. Throughout this part, we focus on second-order spectral properties, and optimum linear (or widely linear) minimum mean-squared error filtering. Chapter 8 discusses wide-sense stationary (WSS) processes, with a focus on the role of the complementary power spectral density in rotary-component and polarization analysis. WSS processes admit a spectral representation in terms of the Fourier basis, which allows a frequency interpretation. The transform-domain description of a WSS signal is a spectral process with *orthogonal* increments. For nonstationary signals, we have to sacrifice either the Fourier basis and thus its frequency interpretation, or the orthogonality of the transform-domain representation. In Chapter 9, we will discuss both possibilities,

which leads either to the Karhunen–Loève expansion or the Cramér–Loève spectral representation. The latter is the basis for bilinear time–frequency representations. Then, in Chapter 10 we treat cyclostationary processes. They are an important class of nonstationary processes that have periodically varying correlation properties. They can model periodic phenomena occurring in science and technology, including communications, meteorology, oceanography, climatology, astronomy, and economics.

Three appendices provide background material. Appendix 1 presents rudiments of matrix analysis. Appendix 2 introduces Wirtinger calculus, which enables us to compute generalized derivatives of a *real* function with respect to *complex* parameters. Finally, Appendix 3 discusses majorization, which is used at several places in this book. Majorization introduces a preordering of vectors, and it will allow us to optimize certain scalar real-valued functions with respect to real vector-valued parameters.

This book is mainly targeted at researchers and graduate students who rely on the theory of signals and systems to conduct their work in signal processing, communications, radar, sonar, optics, electromagnetics, acoustics, oceanography, geophysics, and geography. Although it is not primarily intended as a textbook, chapters of the book may be used to support a special-topics course at a second-year graduate level. We would expect readers to be familiar with basic probability theory, linear systems, and linear algebra, at a level covered in a typical first-year graduate course.

Acknowledgments

We would like to thank Dr. Patrik Wahlberg for giving us detailed feedback on many chapters of this book. We further thank Dr. Phil Meyler of Cambridge University Press for his support throughout the writing of this book. Peter Schreier acknowledges financial support from the Australian Research Council (ARC) under its Discovery Project scheme, and thanks Colorado State University, Ft. Collins, USA, for its hospitality during a five-month study leave in the winter and spring of 2008 in the northern hemisphere. Louis Scharf acknowledges years of research support by the Office of Naval Research and the National Science Foundation (NSF), and thanks the University of Newcastle, Australia, for its hospitality during a one-month study leave in the autumn of 2009 in the southern hemisphere.

Peter J. Schreier
Newcastle, New South Wales, Australia

Louis L. Scharf
Ft. Collins, Colorado, USA

Notation

Conventions

$\langle x, y \rangle$	inner product
$\ x\ $	norm (usually Euclidean)
\hat{x}	estimate of x
\tilde{x}	complementary quantity to x
$x \perp y$	x is orthogonal to y

Vectors and matrices

\mathbf{x}	column-vector with components x_i
$\mathbf{x} \prec \mathbf{y}$	\mathbf{x} is majorized by \mathbf{y}
$\mathbf{x} \prec_w \mathbf{y}$	\mathbf{x} is weakly majorized by \mathbf{y}
\mathbf{X}	matrix with components $(\mathbf{X})_{ij} = X_{ij}$
$\mathbf{X} > \mathbf{Y}$	$\mathbf{X} - \mathbf{Y}$ is positive definite
$\mathbf{X} \geq \mathbf{Y}$	$\mathbf{X} - \mathbf{Y}$ is positive semidefinite (nonnegative definite)
\mathbf{X}^*	complex conjugate
\mathbf{X}^T	transpose
$\mathbf{X}^H = (\mathbf{X}^T)^*$	Hermitian (conjugate) transpose
\mathbf{X}^\dagger	Moore–Penrose pseudo-inverse
$\langle \mathbf{X} \rangle$	subspace spanned by columns of \mathbf{X}
$\underline{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}^* \end{bmatrix}$	augmented vector
$\underline{\mathbf{X}} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 \\ \mathbf{X}_2^* & \mathbf{X}_1^* \end{bmatrix}$	augmented matrix

Functions

$x(t)$	continuous-time signal
$x[k]$	discrete-time signal
$\hat{x}(t)$	Hilbert transform of $x(t)$; estimate of $x(t)$
$X(f)$	scalar-valued Fourier transform of $x(t)$
$\mathbf{X}(f)$	vector-valued Fourier transform of $\mathbf{x}(t)$
$\mathbb{X}(f)$	matrix-valued Fourier transform of $\mathbf{X}(t)$

$X(z)$	scalar-valued z -transform of $x[k]$
$\mathbf{X}(z)$	vector-valued z -transform of $\mathbf{x}[k]$
$\mathbb{X}(z)$	matrix-valued z -transform of $\mathbf{X}[k]$
$x(t) * y(t) = (x * y)(t)$	convolution of $x(t)$ and $y(t)$

Commonly used symbols and operators

$\arg(x) = \angle x$	argument (phase) of complex x
\mathbb{C}	field of complex numbers
\mathbb{C}^{2n}	set of augmented vectors $\underline{\mathbf{x}} = [\mathbf{x}^T, \mathbf{x}^H]^T$, $\mathbf{x} \in \mathbb{C}^n$
$\delta(x)$	Dirac δ -function (distribution)
$\det(\mathbf{X})$	matrix determinant
diag (\mathbf{X})	vector of diagonal values $X_{11}, X_{22}, \dots, X_{nn}$ of \mathbf{X}
Diag (x_1, \dots, x_n)	diagonal or block-diagonal matrix with diagonal elements x_1, \dots, x_n
\mathbf{e}	error vector
$E(x)$	expectation of x
ev (\mathbf{X})	vector of eigenvalues of \mathbf{X} , ordered decreasingly
I	identity matrix
$\text{Im } x$	imaginary part of x
K	matrix of canonical/half-canonical correlations k_i
Λ	matrix of eigenvalues λ_i
\mathbf{m}_x	sample mean vector of \mathbf{x}
$\boldsymbol{\mu}_x$	mean vector of \mathbf{x}
$p_x(x)$	probability density function (pdf) of x (often used without subscript)
\mathbf{P}_U	orthogonal projection onto subspace $\langle \mathbf{U} \rangle$
$P_{xx}(f)$	power spectral density (PSD) of $x(t)$
$\tilde{P}_{xx}(f)$	complementary power spectral density (C-PSD) of $x(t)$
$\mathbb{P}_{xx}(f)$	augmented PSD matrix of $x(t)$
Q	error covariance matrix
ρ	correlation coefficient; degree of impropriety; coherence
\mathbb{R}	field of real numbers
\mathbf{R}_{xy}	cross-covariance matrix of \mathbf{x} and \mathbf{y}
$\tilde{\mathbf{R}}_{xy}$	complementary cross-covariance matrix of \mathbf{x} and \mathbf{y}
$\underline{\mathbf{R}}_{xy}$	augmented cross-covariance matrix of \mathbf{x} and \mathbf{y}
\mathbb{R}_{xy}	covariance matrix of composite vector $[\mathbf{x}^T, \mathbf{y}^T]^T$
$r_{xy}(t, \tau)$	cross-covariance function of $x(t)$ and $y(t)$
$\tilde{r}_{xy}(t, \tau)$	complementary cross-covariance function of $x(t)$ and $y(t)$
$\text{Re } x$	real part of x
$\text{sgn}(x)$	sign of x
sv (\mathbf{X})	vector of singular values of \mathbf{X} , ordered decreasingly

\mathbf{S}_{xx}	sample covariance matrix of \mathbf{x}
$\tilde{\mathbf{S}}_{xx}$	sample complementary covariance matrix of \mathbf{x}
$\underline{\mathbf{S}}_{xx}$	augmented sample covariance matrix of \mathbf{x}
$S_{xx}(v, f)$	(Loève) spectral correlation of $x(t)$
$\tilde{S}_{xx}(v, f)$	(Loève) complementary spectral correlation of $x(t)$
$\mathbf{T} = \begin{bmatrix} \mathbf{I} & \mathbf{jI} \\ \mathbf{I} & -\mathbf{jI} \end{bmatrix}$	real-to-complex transformation
$\text{tr}(\mathbf{X})$	matrix trace
\mathbf{W}	Wiener (linear or widely linear minimum mean-squared error) filter matrix
$\mathcal{W}^{m \times n}$	set of $2m \times 2n$ augmented matrices
$\mathbf{x} = \mathbf{u} + \mathbf{jv}$	complex message/source
ξ	internal (latent) description of \mathbf{x}
$x(t) = u(t) + \mathbf{j}v(t)$	complex continuous-time message/source signal
$\xi(f)$	spectral process corresponding to $x(t)$
$\mathbf{y} = \mathbf{a} + \mathbf{jb}$	complex measurement/observation
$y(t)$	complex continuous-time measurement/observation signal
$v(f)$	spectral process corresponding to $y(t)$
$\boldsymbol{\omega}$	internal (latent) description of \mathbf{y}
Ω	sample space
$\mathbf{0}$	zero vector or matrix

Contents

<i>Preface</i>	page xiii
<i>Notation</i>	xvii

Part I Introduction 1

1 The origins and uses of complex signals 3

1.1	Cartesian, polar, and complex representations of two-dimensional signals	4
1.2	Simple harmonic oscillator and phasors	5
1.3	Lissajous figures, ellipses, and electromagnetic polarization	6
1.4	Complex modulation, the Hilbert transform, and complex analytic signals	8
1.4.1	Complex modulation using the complex envelope	9
1.4.2	The Hilbert transform, phase splitter, and analytic signal	11
1.4.3	Complex demodulation	13
1.4.4	Bedrosian's theorem: the Hilbert transform of a product	14
1.4.5	Instantaneous amplitude, frequency, and phase	14
1.4.6	Hilbert transform and SSB modulation	15
1.4.7	Passband filtering at baseband	15
1.5	Complex signals for the efficient use of the FFT	17
1.5.1	Complex DFT	18
1.5.2	Twofer: two real DFTs from one complex DFT	18
1.5.3	Twofer: one real $2N$ -DFT from one complex N -DFT	19
1.6	The bivariate Gaussian distribution and its complex representation	19
1.6.1	Bivariate Gaussian distribution	20
1.6.2	Complex representation of the bivariate Gaussian distribution	21
1.6.3	Polar coordinates and marginal pdfs	23
1.7	Second-order analysis of the polarization ellipse	23
1.8	Mathematical framework	25
1.9	A brief survey of applications	27

2 Introduction to complex random vectors and processes 30

2.1	Connection between real and complex descriptions	31
2.1.1	Widely linear transformations	31
2.1.2	Inner products and quadratic forms	33

2.2	Second-order statistical properties	34
2.2.1	Extending definitions from the real to the complex domain	35
2.2.2	Characterization of augmented covariance matrices	36
2.2.3	Power and entropy	37
2.3	Probability distributions and densities	38
2.3.1	Complex Gaussian distribution	39
2.3.2	Conditional complex Gaussian distribution	41
2.3.3	Scalar complex Gaussian distribution	42
2.3.4	Complex elliptical distribution	44
2.4	Sufficient statistics and ML estimators for covariances: complex Wishart distribution	47
2.5	Characteristic function and higher-order statistical description	49
2.5.1	Characteristic functions of Gaussian and elliptical distributions	50
2.5.2	Higher-order moments	50
2.5.3	Cumulant-generating function	52
2.5.4	Circularity	53
2.6	Complex random processes	54
2.6.1	Wide-sense stationary processes	55
2.6.2	Widely linear shift-invariant filtering	57
	Notes	57

Part II Complex random vectors 59

3	Second-order description of complex random vectors	61
3.1	Eigenvalue decomposition	62
3.1.1	Principal components	63
3.1.2	Rank reduction and transform coding	64
3.2	Circularity coefficients	65
3.2.1	Entropy	67
3.2.2	Strong uncorrelating transform (SUT)	67
3.2.3	Characterization of complementary covariance matrices	69
3.3	Degree of impropriety	70
3.3.1	Upper and lower bounds	72
3.3.2	Eigenvalue spread of the augmented covariance matrix	76
3.3.3	Maximally improper vectors	76
3.4	Testing for impropriety	77
3.5	Independent component analysis	81
	Notes	84

4	Correlation analysis	85
4.1	Foundations for measuring multivariate association between two complex random vectors	86
4.1.1	Rotational, reflectional, and total correlations for complex scalars	87

4.1.2	Principle of multivariate correlation analysis	91
4.1.3	Rotational, reflectional, and total correlations for complex vectors	94
4.1.4	Transformations into latent variables	95
4.2	Invariance properties	97
4.2.1	Canonical correlations	97
4.2.2	Multivariate linear regression (half-canonical correlations)	100
4.2.3	Partial least squares	101
4.3	Correlation coefficients for complex vectors	102
4.3.1	Canonical correlations	103
4.3.2	Multivariate linear regression (half-canonical correlations)	106
4.3.3	Partial least squares	108
4.4	Correlation spread	108
4.5	Testing for correlation structure	110
4.5.1	Sphericity	112
4.5.2	Independence within one data set	112
4.5.3	Independence between two data sets	113
	Notes	114
5	Estimation	116
5.1	Hilbert-space geometry of second-order random variables	117
5.2	Minimum mean-squared error estimation	119
5.3	Linear MMSE estimation	121
5.3.1	The signal-plus-noise channel model	122
5.3.2	The measurement-plus-error channel model	123
5.3.3	Filtering models	125
5.3.4	Nonzero means	127
5.3.5	Concentration ellipsoids	127
5.3.6	Special cases	128
5.4	Widely linear MMSE estimation	129
5.4.1	Special cases	130
5.4.2	Performance comparison between LMMSE and WLMMSSE estimation	131
5.5	Reduced-rank widely linear estimation	132
5.5.1	Minimize mean-squared error (min-trace problem)	133
5.5.2	Maximize mutual information (min-det problem)	135
5.6	Linear and widely linear minimum-variance distortionless response estimators	137
5.6.1	Rank-one LMVDR receiver	138
5.6.2	Generalized sidelobe canceler	139
5.6.3	Multi-rank LMVDR receiver	141
5.6.4	Subspace identification for beamforming and spectrum analysis	142
5.6.5	Extension to WLMVDR receiver	143
5.7	Widely linear-quadratic estimation	144

5.7.1	Connection between real and complex quadratic forms	145
5.7.2	WLQMMSE estimation	146
	Notes	149
6	Performance bounds for parameter estimation	151
6.1	Frequentists and Bayesians	152
6.1.1	Bias, error covariance, and mean-squared error	154
6.1.2	Connection between frequentist and Bayesian approaches	155
6.1.3	Extension to augmented errors	157
6.2	Quadratic frequentist bounds	157
6.2.1	The virtual two-channel experiment and the quadratic frequentist bound	157
6.2.2	Projection-operator and integral-operator representations of quadratic frequentist bounds	159
6.2.3	Extension of the quadratic frequentist bound to improper errors and scores	161
6.3	Fisher score and the Cramér–Rao bound	162
6.3.1	Nuisance parameters	164
6.3.2	The Cramér–Rao bound in the proper multivariate Gaussian model	164
6.3.3	The separable linear statistical model and the geometry of the Cramér–Rao bound	165
6.3.4	Extension of Fisher score and the Cramér–Rao bound to improper errors and scores	167
6.3.5	The Cramér–Rao bound in the improper multivariate Gaussian model	168
6.3.6	Fisher score and Cramér–Rao bounds for functions of parameters	169
6.4	Quadratic Bayesian bounds	170
6.5	Fisher–Bayes score and Fisher–Bayes bound	171
6.5.1	Fisher–Bayes score and information	172
6.5.2	Fisher–Bayes bound	173
6.6	Connections and orderings among bounds	174
	Notes	175
7	Detection	177
7.1	Binary hypothesis testing	178
7.1.1	The Neyman–Pearson lemma	179
7.1.2	Bayes detectors	180
7.1.3	Adaptive Neyman–Pearson and empirical Bayes detectors	180
7.2	Sufficiency and invariance	180
7.3	Receiver operating characteristic	181
7.4	Simple hypothesis testing in the improper Gaussian model	183

7.4.1	Uncommon means and common covariance	183
7.4.2	Common mean and uncommon covariances	185
7.4.3	Comparison between linear and widely linear detection	186
7.5	Composite hypothesis testing and the Karlin–Rubin theorem	188
7.6	Invariance in hypothesis testing	189
7.6.1	Matched subspace detector	190
7.6.2	CFAR matched subspace detector	193
	Notes	194

Part III Complex random processes 195

8 Wide-sense stationary processes 197

8.1	Spectral representation and power spectral density	197
8.2	Filtering	200
8.2.1	Analytic and complex baseband signals	201
8.2.2	Noncausal Wiener filter	202
8.3	Causal Wiener filter	203
8.3.1	Spectral factorization	203
8.3.2	Causal synthesis, analysis, and Wiener filters	205
8.4	Rotary-component and polarization analysis	205
8.4.1	Rotary components	206
8.4.2	Rotary components of random signals	208
8.4.3	Polarization and coherence	211
8.4.4	Stokes and Jones vectors	213
8.4.5	Joint analysis of two signals	215
8.5	Higher-order spectra	216
8.5.1	Moment spectra and principal domains	217
8.5.2	Analytic signals	218
	Notes	221

9 Nonstationary processes 223

9.1	Karhunen–Loève expansion	224
9.1.1	Estimation	227
9.1.2	Detection	230
9.2	Cramér–Loève spectral representation	230
9.2.1	Four-corners diagram	231
9.2.2	Energy and power spectral densities	233
9.2.3	Analytic signals	235
9.2.4	Discrete-time signals	236
9.3	Rihaczek time–frequency representation	237
9.3.1	Interpretation	238
9.3.2	Kernel estimators	240
9.4	Rotary-component and polarization analysis	242

9.4.1	Ellipse properties	244
9.4.2	Analytic signals	245
9.5	Higher-order statistics	247
	Notes	248
10	Cyclostationary processes	250
10.1	Characterization and spectral properties	251
10.1.1	Cyclic power spectral density	251
10.1.2	Cyclic spectral coherence	253
10.1.3	Estimating the cyclic power-spectral density	254
10.2	Linearly modulated digital communication signals	255
10.2.1	Symbol-rate-related cyclostationarity	255
10.2.2	Carrier-frequency-related cyclostationarity	258
10.2.3	Cyclostationarity as frequency diversity	259
10.3	Cyclic Wiener filter	260
10.4	Causal filter-bank implementation of the cyclic Wiener filter	262
10.4.1	Connection between scalar CS and vector WSS processes	262
10.4.2	Sliding-window filter bank	264
10.4.3	Equivalence to FRESH filtering	265
10.4.4	Causal approximation	267
	Notes	268
	Appendix 1 Rudiments of matrix analysis	270
A1.1	Matrix factorizations	270
A1.1.1	Partitioned matrices	270
A1.1.2	Eigenvalue decomposition	270
A1.1.3	Singular value decomposition	271
A1.2	Positive definite matrices	272
A1.2.1	Matrix square root and Cholesky decomposition	272
A1.2.2	Updating the Cholesky factors of a Grammian matrix	272
A1.2.3	Partial ordering	273
A1.2.4	Inequalities	274
A1.3	Matrix inverses	274
A1.3.1	Partitioned matrices	274
A1.3.2	Moore–Penrose pseudo-inverse	275
A1.3.3	Projections	276
	Appendix 2 Complex differential calculus (Wirtinger calculus)	277
A2.1	Complex gradients	278
A2.1.1	Holomorphic functions	279
A2.1.2	Complex gradients and Jacobians	280
A2.1.3	Properties of Wirtinger derivatives	281

A2.2 Special cases	282
A2.3 Complex Hessians	283
A2.3.1 Properties	285
A2.3.2 Extension to complex-valued functions	285
Appendix 3 Introduction to majorization	287
A3.1 Basic definitions	288
A3.1.1 Majorization	288
A3.1.2 Schur-convex functions	289
A3.2 Tests for Schur-convexity	290
A3.2.1 Specialized tests	291
A3.2.2 Functions defined on \mathcal{D}	292
A3.3 Eigenvalues and singular values	293
A3.3.1 Diagonal elements and eigenvalues	293
A3.3.2 Diagonal elements and singular values	294
A3.3.3 Partitioned matrices	295
<i>References</i>	296
<i>Index</i>	305