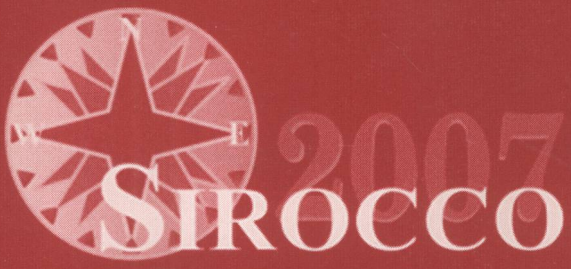
Giuseppe Prencipe
Shmuel Zaks (Eds.)

LNCS 4474

Structural Information and Communication Complexity

14th International Colloquium, SIROCCO 2007
Castiglioncello, Italy, June 2007
Proceedings



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Springer

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Giuseppe Prencipe
Università degli Studi di Pisa
Dipartimento di Informatica
Largo Bruno Pontecorvo 3, 56127 Pisa, Italy
E-mail: prencipe@di.unipi.it

Shmuel Zaks
Technion
Department of Computer Science
Haifa, Israel
E-mail: zaks@cs.technion.ac.il

Library of Congress Control Number: 2007927754

CR Subject Classification (1998): F.2, C.2, G.2, E.1

LNCS Sublibrary: SL 1 – Theoretical Computer Science and General Issues

ISSN 0302-9743
ISBN-10 3-540-72918-6 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-72918-1 Springer Berlin Heidelberg New York

Typesetting: Camera-ready by author, data conversion by Scientific Publishing Services, Chennai, India
Printed on acid-free paper SPIN: 12072859 06/3180 5 4 3 2 1 0

Lecture Notes in Computer Science 4474

Commenced Publication in 1973
Founding and Former Series Editors:
Gerhard Goos, Juris Hartmanis, and Jan van Leeuwen

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Preface

The Colloquium on Structural Information and Communication Complexity (SIROCCO) is an annual meeting focused on the relationship between algorithmic aspects of computing and communication. Over its 14 years of existence, SIROCCO has become an acknowledged forum bringing together specialists interested in the fundamental principles underlying the interplay between information, communication, and computing. SIROCCO covers topics such as distributed computing, high-speed networks, interconnection networks, mobile computing, optical computing, parallel computing, sensor networks, wireless networks, and autonomous robots. Its topics of interest include communication complexity, distributed algorithms and data structures, information dissemination, mobile agent computing, models of communication, network topologies, routing protocols, sense of direction, structural properties, and selfish computing.

SIROCCO 2007 was held in Castiglioncello (LI), Italy, June 5–8, 2007. The previous 13 SIROCCO colloquia took place in Ottawa (1994), Olympia (1995), Siena (1996), Ascona (1997), Amalfi (1998), Lacanau-Océan (1999), L'Aquila (2000), Val de Nuria (2001), Andros (2002), Umeå (2003), Smolenice Castle (2004), Mont Saint-Michel (2005), and Chester (2006).

The 66 contributions submitted to SIROCCO 2007 were subject to a thorough refereeing process, and 23 high-quality submissions were selected for publication. We would like to thank the authors of all the submitted papers. The excellent quality of the final program is also due to the dedicated and careful work of the Program Committee members. Our gratitude extends to them. We also thank the numerous sub-referees for their valuable help.

We had four invited speakers: Hans Bodlaender (Utrecht), Luisa Gargano (Salerno), S. Muthukrishnan (Google), and Alessandro Panconesi (Rome). We thank them for accepting our invitation to share their insights on new developments in their areas of interest.

We would like to express our gratitude to the conference Chair Pierre Fraigniaud (CNRS and Paris) for his enthusiasm and invaluable consultations.

Special thanks go to the local Organizing Team of the Dipartimento di Informatica, Università di Pisa, and in particular to Vincenzo Gervasi.

We acknowledge the use of the EasyChair system (for handling the submission of papers, managing the refereeing process, and generating these proceedings).

We thank the Università di Pisa, and its Dipartimento di Informatica, for their generous support. SIROCCO 2007 was co-located with FUN 2007 and with a meeting of the EU COST 293 action (GRAAL - Graphs and Algorithms in Communication Networks). The two sessions of the invited talks were held

jointly with the GRAAL meeting. We thank the Management Committee of GRAAL, and especially their past and present Chairs Xavier Munoz and Arie Koster, for supporting the idea of this joint event and for their generous support.

June 2007

Giuseppe Prencipe
Shmuel Zaks

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Fast Distributed Algorithms Via Primal-Dual (Extended Abstract)

Alessandro Panconesi

Informatica, Sapienza University
via Salaria 113, 00198 Roma, Italy
ale@di.uniroma1.it

When a trick works once, it is a trick.
If it works twice, it is a technique.
If it works three times, it is a method.

Juris Hartmanis

1 Introduction

We would like to discuss what seems to be a general methodology to develop fast distributed algorithms for optimization problems on graphs, based on the primal-dual schema. The kind of problems we have in mind are of the following type. We have a synchronous, message-passing network that is to compute a global function of its own topology. Examples of such functions are maximal independent sets, vertex and edge colorings, small dominating sets, vertex covers and so on. Crucially, nodes only know their neighbours and have very little or no global information. In what follows, the only global information allowed will be n, the number of nodes in the network (or an upper bound on it). In this setting the running time of a protocol is given by the number of communication rounds needed to compute the output. By the end of the algorithm each node or edge will have decided its final status: its own color, whether or not to be part of the dominating set etc. In many situations of interest the cost of communication is orders of magnitude larger than local computation cost, and the model provides a rough, but quite useful, quantitative framework to develop and analyze interesting algorithms.

The combinatorial objects that we are interested in computing are useful both on theoretical and practical grounds. Small dominating sets for instance are the method of choice to set up the routing infrastructure of ad-hoc networks (the so-called backbone). Edge colorings have been repeatedly used to parallelize data transfers in distributed architectures, and so on. Maximal independent sets on the other hand appear to be a basic building block of many distributed algorithms.

Note the basic challenge here: If a protocol runs for t many rounds, each node will be able to collect information only from nodes at distance t. If t is much smaller than the diameter of the network, what we are trying to do is to compute a global function of the entire network, based on local information alone.

Distributed algorithms for graph problems is a very wide and active area. In what follows we do not try to be complete or encyclopaedic. Rather, we focus on the issues

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that are immediately relevant to our topic. This means that we will not do a proper job of acknowledging the large and relevant body of brilliant literature that exists, and we offer our apologies in advance for this lack of completeness. The papers we cite and the references therein provide a good entry point for the research areas we will be discussing.

2 Distributed Algorithms Via the Primal-Dual Schema

The primal-dual schema is a very powerful methodology to develop efficient algorithms for combinatorial optimization problems. In recent years it has been applied with good success to NP-hard problems for which it yields many sophisticated approximation algorithms with performance guarantee. The main thesis of this talk is that in general a primal-dual algorithm exhibits certain locality properties that make it amenable to a fast, distributed implementation. This point of view is cogently developed in the PhD dissertation [8] which is the basis of the discussion to follow. We shall outline the method by discussing the example of vertex cover. The algorithm we describe is the first example of this general methodology [6]. The algorithm was developed in the dark days when people were interested in PRAM algorithms, but the solution is in fact fully distributed.

As it is well-known, in the vertex cover problem we are given an undirected network and we are to compute a so-called cover, i.e. a set of vertices such that, for every edge at least one of the two endpoints lies in the cover. Among all covers, we are interested in computing one of the smallest possible size. When vertices have positive integer costs we seek a cover of the smallest possible aggregate cost. This problem is NP-hard even with unit costs. What we will do is the following: (a) develop a sequential 2-approximated algorithm for it, and then (b) show how to parallelize it efficiently by means of a distributed implementation. Note that there are two separate issues here: to make the process distributed *and* fast. To do so, one must be able to exploit the inherent parallelism.

We begin by formulating the problem as an integer program (IP):

$$\begin{aligned} \min \quad & \sum_{v \in V} c(v) \cdot x_v & & \text{(IP)} \\ \text{s.t} \quad & x_v + x_u \geq 1 & \forall e = (u, v) \in E & \quad (1) \\ & x_v \in \{0, 1\} & \forall v \in V & \quad (2) \end{aligned}$$

The cover is defined to be the set of all vertices v such that the corresponding binary indicator variable $x_v = 1$. The set of constraints (1) ensure that it is indeed a cover, for at least one endpoint for every edge must be included in it.

We now let (LP) be the standard LP relaxation obtained from (IP) by replacing the constraints (2) by

$$x_v \geq 0 \qquad \forall v \in V \qquad (3)$$

In the linear-programming dual of (LP) we associate a variable α_e with constraint (1) for every $e \in E$. The linear programming dual (D) of (LP) is then

$$\max \quad \sum_{e \in E} \alpha_e \tag{D}$$

$$\text{s.t} \quad \sum_{e=(u,v) \in E} \alpha_e \leq c(v) \qquad \forall v \in V \tag{4}$$

$$\alpha_e \geq 0 \qquad \forall e \in E \tag{5}$$

We will build a cover working with the dual variables. Note that we have a constraint of type (4) for every node v, denoted as $(4)_v$. Consider the following continuous process. We let all the variables α_e grow at uniform speed. Sooner or later a constraint of type (4) will be satisfied with equality. If $(4)_v$ is the constraint we say that it becomes *tight*. When $(4)_v$ becomes tight we add v to the cover that we are computing. We do this by setting $x_v = 1$ (initially all primal variables are set to 0). Then we freeze the values α_e of the edges incident to v. The α-values of frozen edges stop growing, so that the constraint considered remains tight. The process continues with the remaining edges, until all edges are frozen.

At this point we have a set of vertices C containing all vertices v such that $x_v = 1$. We want to show that (a) it is a cover and that (b) its size is at most twice the optimum cost. To see why it is a cover, suppose not. But then there is an edge $e = uv$ which is not covered, i.e. $x_u = x_v = 0$. This means that the constraints $(4)_u$ and $(4)_v$ corresponding to u and v are not tight and α_e can continue to grow, a contradiction.

The cost of C is upper-bounded by twice the cost of the dual solution:

$$\sum_{v \in V} c(v)\, x_v = \sum_{v \in C} c(v) \leq \sum_{v \in C} \sum_{e=(u,v) \in E} \alpha_e \leq 2 \sum_{e \in E} \alpha_e.$$

You can think of this chain of inequalities in the following way. At the end of the algorithm we have a value α_e for every edge in the network. By doubling each α_e we have enough "money" to pay for the cost of vertices we put in C. This is true locally: for every $v \in C$, since v is tight, the sum of the α_e's, where e is incident to v, is equal to its cost. Since we doubled every α_e, each edge e has enough cash to pay the cost of both vertices it is incident to. Thus, twice the sum of the α_e's covers the cost of the solution we computed.

That the solution computed is 2-approximate follows by weak duality. Weak duality states that any dual feasible solution is no more than any primal solution. That is, denoting with z^* the optimal value of the primal solution we have

$$\sum_{e \in E} \alpha_e \leq z^*.$$

The primal is a relaxation of the original integer program and thus $z^* \leq \mathsf{opt}$. The claim follows.

The continuous process above can be easily turned into a discrete one that runs in polynomial-time. The problem is how to make it both distributed and fast. It is apparent