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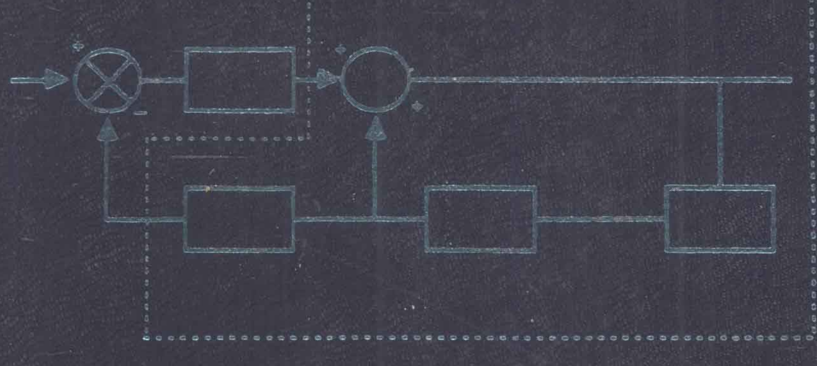
KALMAN FILTERING

Theory and Practice



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Angus P. Andrews

PRENTICE HALL INFORMATION AND SYSTEM SCIENCES SERIES

Thomas Kailath, Series Editor

KALMAN FILTERING
Theory and Practice

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Preface

We have attempted to make this a book for people who will use Kalman filters. Our objective in writing it was to give our readers a working familiarity with both the **theoretical** and **practical** aspects of the subject. For that purpose we have included some "real-world" problems in practice as illustrative examples. We also cover the more practical aspects of implementation: how to represent the problem in a mathematical model, analyze the performance of the estimator as a function of model parameters, implement the mechanization equations in numerically stable algorithms, assess its computational requirements, test the validity of results, and monitor the filter performance in operation. These are important attributes of the subject that are often overlooked in theoretical treatments but are necessary for application of the theory to real-world problems.

The enclosed 3 1/2-inch diskette contains software packages to demonstrate the workings of the filter algorithms. These are written in FORTRAN, the *lingua franca* of engineering programming, and formatted as ASCII document files in DOS format. The inclusion of the software is practically a matter of necessity, because Kalman filtering would not be very useful without computers to implement it. It is a better learning experience for the student to discover how the Kalman filter works by observing it in action.

The implementation of Kalman filtering on computers also illuminates some of the practical considerations of finite word length arithmetic, and the need for alternative algorithms to preserve the accuracy of the results. If the student wishes to apply what she or he learns, it is essential that she or he experience its workings and failings—and learn to recognize the difference.

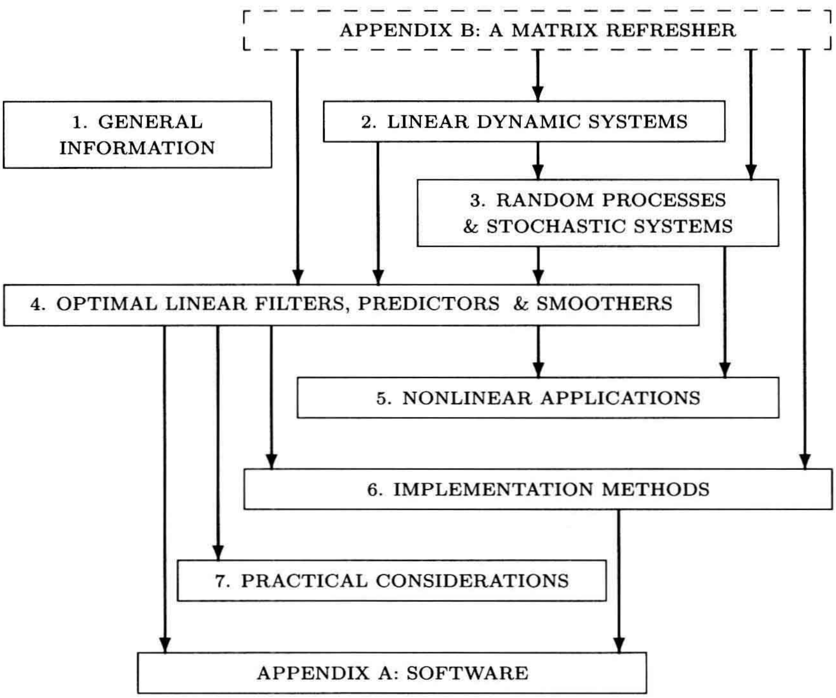
The book is organized for use as a text for an introductory course in stochastic processes at the senior level and as a first-year-graduate-level course in Kalman filtering theory and application. It could also be used for self-instruction or for purposes of review by practicing engineers and scientists who are not intimately familiar with the subject. The organization of the material is illustrated by the following chapter-level dependency graph, which shows how the subject of each chapter depends upon material in other chapters. The arrows in the figure indicate the recommended order of study. Boxes above another box and connected by arrows indicate that the material represented by the upper boxes is background material for the subject in the lower box.

Chapter 1 is an informal introduction to the general subject matter by way of its history of development and application. Chapters 2 and 3 and Appendix B cover the essential background material on linear systems, probability, stochastic processes, and modeling. These chapters could be covered in a senior-level course in electrical, computer, and systems engineering.

Chapter 4 covers linear optimal filters, predictors and smoothers with detailed examples of applications. Chapter 5 is devoted to nonlinear estimation by "extended" Kalman filters. Applications of these techniques to the identification of unknown parameters of systems are given as examples. Chapter 6 covers the more modern implementation techniques, with algorithms provided for computer implementation.

Chapter 7 deals with more practical matters of implementation and use beyond the numerical methods of Chapter 6. These matters include memory and throughput requirements (and methods to reduce them), divergence problems (and effective remedies), and practical approaches to suboptimal filtering and measurement selection.

Chapters 4 to 7 cover the essential material for a first-year graduate class in Kalman filtering theory and application, or as a basic course in digital estimation theory and application. A solutions manual for the exercises in the back of the chapters is available.



Acknowledgments

We have endeavored to give credit to the people who first introduced many of the concepts, methods, and topics covered by this book. This is for two purposes:

1. To acknowledge the debt that we all owe to these people for showing us how to do our own work a little better. In attempting to do this, we are bound to omit many important contributors—due to our own ignorance. If we have failed to give credit where it is due, we do apologize for the oversight and we would appreciate learning of our errors.
2. To give you, the reader, some useful background information. Kalman filtering is a relatively young subject, and most of its principal contributors are still involved in the field of study. If you should have the opportunity to meet them in your own technical activities, it would enrich your own experience if you could recognize them and their contributions.

The authors express their appreciation to the following individuals, especially, for their direct contributions to the preparation of this book:

Joseph Smith and **Dwayne Heckman** of Rockwell International and California State University at Fullerton, for their cooperation and collaboration since 1979 in the development of the course material from which this book has grown.

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Most of all, to our families for their support and understanding through the years of preparation. For their dedication, we dedicate this book to **Sonja Grewal**, **Sara Grewal**, **Gwen Grewal**, **Jeri Andrews**, **Eric and Nancy Andrews**, **Christopher Haley and Margaret Andrews**, and **Erin Andrews**.

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Contents