PLANE AND SPHERICAL TRIGONOMETRY

BY

CLAUDE IRWIN PALMER

Late Professor of Mathematics and Dean of Students, Armour Institute of Technology; Author of a Series of Mathematics Texts

AND

CHARLES WILBER LEIGH

Professor Emeritus of Analytic Mechanics, Armour Institute of Technology, Author of Practical Mechanics

FOURTH EDITION
THIRTEENTH IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK AND LONDON
1934

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PLANE AND SPHERICAL TRIGONOMETRY

BOOKS BY

C. I. PALMER

(Published by the McGraw-Hill Book Company, Inc.)

PALMER AND BIBB'S
Practical Mathematics
- Part I—Arithmetic with Applications
- Part II—Algebra with Applications
- Part III—Geometry with Applications
- Part IV—Trigonometry and Logarithms

PALMER'S
Practical Mathematics for Home Study

PALMER'S
Practical Calculus for Home Study

PALMER AND LEIGH'S
Plane and Spherical Trigonometry with Tables

PALMER AND KRATHWOHL'S
Analytic Geometry

PALMER AND MISER'S
College Algebra

(Published by Scott, Foresman and Company)

PALMER, TAYLOR, AND FARNUM'S
Plane Geometry

PALMER, TAYLOR, AND FARNUM'S
Solid Geometry

PALMER, TAYLOR, AND FARNUM'S
Plane and Solid Geometry

PREFACE TO THE FOURTH EDITION

This edition presents a new set of problems in Plane Trigonometry. The type of problem has been preserved, but the details have been changed. The undersigned acknowledges indebtedness to the members of the Department of Mathematics at the Armour Institute of Technology for valuable suggestions and criticisms. He is especially indebted to Profs. S. F. Bibb and W. A. Spencer for their contribution of many new identities and equations and also expresses thanks to Mr. Clark Palmer, son of the late Dean Palmer, for assisting in checking answers to problems and in proofreading and for offering many constructive criticisms.

CHARLES WILBER LEIGH.

CHICAGO,
June, 1934.

PREFACE TO THE FIRST EDITION

This text has been written because the authors felt the need of a treatment of trigonometry that duly emphasized those parts necessary to a proper understanding of the courses taken in schools of technology. Yet it is hoped that teachers of mathematics in classical colleges and universities as well will find it suited to their needs. It is useless to claim any great originality in treatment or in the selection of subject matter. No attempt has been made to be novel only; but the best ideas and treatment have been used, no matter how often they have appeared in other works on trigonometry.

The following points are to be especially noted:

(1) The measurement of angles is considered at the beginning.

(2) The trigonometric functions are defined at once for any angle, then specialized for the acute angle; not first defined for acute angles, then for obtuse angles, and then for general angles. To do this, use is made of Cartesian coordinates, which are now almost universally taught in elementary algebra.

(3) The treatment of triangles comes in its natural and logical order and is not *forced* to the first pages of the book.

(4) Considerable use is made of the line representation of the trigonometric functions. This makes the proof of certain theorems easier of comprehension and lends itself to many useful applications.

(5) Trigonometric equations are introduced early and used often.

(6) Anti-trigonometric functions are used throughout the work, not placed in a short chapter at the close. They are used in the solutions of equations and triangles. Much stress is laid upon the principal values of anti-trigonometric functions as used later in the more advanced subjects of mathematics.

(7) A limited use is made of the so-called "laboratory method" to impress upon the student certain fundamental ideas.

(8) Numerous carefully graded practical problems are given and an abundance of drill exercises.

(9) There is a chapter on complex numbers, series, and hyperbolic functions.

(10) A very complete treatment is given on the use of logarithmic and trigonometric tables. This is printed in connection with the tables, and so does not break up the continuity of the trigonometry proper.

(11) The tables are carefully compiled and are based upon those of Gauss. Particular attention has been given to the determination of angles near 0 and 90°, and to the functions of such angles. The tables are printed in an unshaded type, and the arrangement on the pages has received careful study.

The authors take this opportunity to express their indebtedness to Prof. D. F. Campbell of the Armour Institute of Technology, Prof. N. C. Riggs of the Carnegie Institute of Technology, and Prof. W. B. Carver of Cornell University, who have read the work in manuscript and proof and have made many valuable suggestions and criticisms.

THE AUTHORS.

CHICAGO,
September, 1914.

CONTENTS

CHAPTER III

RELATIONS BETWEEN TRIGONOMETRIC FUNCTIONS

CHAPTER IV

RIGHT TRIANGLES

CHAPTER V

FUNCTIONS OF LARGE ANGLES

CHAPTER VI

GRAPHICAL REPRESENTATION OF TRIGONOMETRIC FUNCTIONS

CHAPTER VII

PRACTICAL APPLICATIONS AND RELATED PROBLEMS

CHAPTER VIII

FUNCTIONS INVOLVING MORE THAN ONE ANGLE

CHAPTER IX

OBLIQUE TRIANGLES

CHAPTER X

MISCELLANEOUS TRIGONOMETRIC EQUATIONS

CHAPTER XI

COMPLEX NUMBERS, DEMOIVRE'S THEOREM, SERIES

CHAPTER XII

SPHERICAL TRIGONOMETRY

The contents for the Logarithmic and Trigonometric Tables and Explanatory Chapter is printed with the tables.

PLANE AND SPHERICAL TRIGONOMETRY

CHAPTER I

INTRODUCTION

GEOMETRY

1. Introductory remarks.—The word trigonometry is derived from two Greek words, *τριγωνον* (trigonon), meaning triangle, and *μετρια* (metria), meaning measurement. While the derivation of the word would seem to confine the subject to triangles, the measurement of triangles is merely a part of the general subject which includes many other investigations involving angles.

Trigonometry is both geometric and algebraic in nature. Historically, trigonometry developed in connection with astronomy, where distances that could not be measured directly were computed by means of angles and lines that could be measured. The beginning of these methods may be traced to Babylon and Ancient Egypt.

The noted Greek astronomer Hipparchus is often called the founder of trigonometry. He did his chief work between 146 and 126 B. C. and developed trigonometry as an aid in measuring angles and lines in connection with astronomy. The subject of trigonometry was separated from astronomy and established as a distinct branch of mathematics by the great mathematician Leonhard Euler, who lived from 1707 to 1783.

To pursue the subject of trigonometry successfully, the student should know the subjects usually treated in algebra up to and including quadratic equations, and be familiar with plane geometry, especially the theorems on triangles and circles.

Frequent use is made of the protractor, compasses, and the straightedge in constructing figures.

While parts of trigonometry can be applied at once to the solution of various interesting and practical problems, much of it is studied because it is very frequently used in more advanced subjects in mathematics.

ANGLES

2. Definitions.—The definition of an angle as given in geometry admits of a clear conception of small angles only. In trigonometry, we wish to consider *positive* and *negative* angles and these of any size whatever; hence we need a more comprehensive definition of an angle.

If a line, starting from the position OX (Fig. 1), is revolved about the point O and always kept in the same plane, we say the line **generates** an angle. If it revolves from the position OX to the position OA, in the direction indicated by the arrow, the angle XOA is generated.

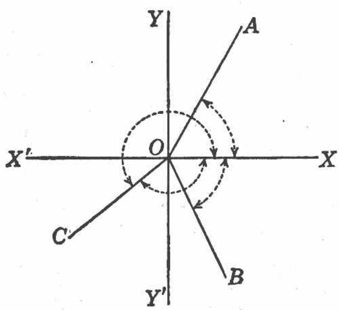

FIG. 1.

The original position OX of the generating line is called the **initial side,** and the final position OA, the **terminal side** of the angle.

If the rotation of the generating line is *counterclockwise,* as already taken, the angle is said to be **positive.** If OX revolves in a *clockwise* direction to a position, as OB, the angle generated is said to be **negative.**

In reading an angle, the letter on the initial side is read first to give the proper sense of direction. If the angle is read in the opposite sense, the negative of the angle is meant.

Thus, $$\angle AOX = -\angle XOA.$$

It is easily seen that this conception of an angle makes it possible to think of an angle as being of any size whatever. Thus, the generating line, when it has reached the position OY, having made a quarter of a revolution in a counterclockwise direction, has generated a right angle; when it has reached the position OX' it has generated two right angles. A complete revolution generates an angle containing four right angles; two revolutions, eight right angles; and so on for any amount of turning.

The right angle is divided into 90 equal parts called degrees (°), each degree is divided into 60 equal parts called minutes ('), and each minute into 60 equal parts called seconds (").

Starting from any position as initial side, it is evident that for each position of the terminal side, there are two angles less than 360°, one positive and one negative. Thus, in Fig. 1, OC is the terminal side for the positive angle XOC or for the negative angle XOC.

3. Quadrants.—It is convenient to divide the plane formed by a complete revolution of the generating line into four parts by the two perpendicular lines $X'X$ and $Y'Y$. These parts are called **first, second, third,** and **fourth quadrants,** respectively. They are placed as shown by the Roman numerals in Fig. 2.

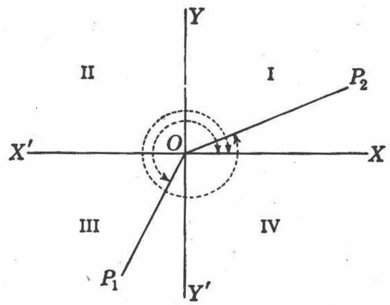

FIG. 2.

If OX is taken as the initial side of an angle, the angle is said to lie in the quadrant in which its terminal side lies. Thus, XOP_1 (Fig. 2) lies in the third quadrant, and XOP_2, formed by more than one revolution, lies in the first quadrant.

An angle lies between two quadrants if its terminal side lies on the line between two quadrants.

4. Graphical addition and subtraction of angles.—Two angles are added by placing them in the same plane with their vertices together and the initial side of the second on the terminal side of the first. The sum is the angle from the initial side of the first to the terminal side of the second.

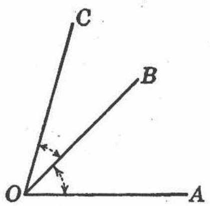

FIG. 3.

Subtraction is performed by adding the negative of the subtrahend to the minuend.

Thus, in Fig. 3,

$$\angle AOB + \angle BOC = \angle AOC.$$
$$\angle AOC - \angle BOC = \angle AOC + \angle COB = \angle AOB.$$
$$\angle BOC - \angle AOC = \angle BOC + \angle COA = \angle BOA.$$

EXERCISES

Use the protractor in laying off the angles in the following exercises:

1. Choose an initial side and lay off the following angles. Indicate each angle by a circular arrow. 75°; 145°; 243°; 729°; 456°; 976°. State the quadrant in which each angle lies.

2. Lay off the following angles and state the quadrant that each is in: −40°; −147°; −295°; −456°; −1048°.

3. Lay off the following pairs of angles, using the same initial side for each pair: 170° and −190°; −40° and 320°; 150° and −210°.

4. Give a positive angle that has the same terminal side as each of the following: 30°; 165°; −90°; −210°; −45°; 395°; −390°.

5. Show by a figure the position of the revolving line when it has generated each of the following: 3 right angles; $2\frac{1}{2}$ right angles; $1\frac{1}{3}$ right angles; $4\frac{2}{3}$ right angles.

Unite graphically, using the protractor:

6. 40° + 70°; 25° + 36°; 95° + 125°; 243° + 725°.

7. 75° − 43°; 125° − 59°; 23° − 49°; 743° − 542°; 90° − 270°.

8. 45° + 30° + 25°; 125° + 46° + 95°; 327° + 25° + 400°.

9. 45° − 56° + 85°; 325° − 256° + 400°.

10. Draw two angles lying in the first quadrant but differing by 360°. Two negative angles in the fourth quadrant and differing by 360°.

11. Draw the following angles and their complements: 30°; 210°; 345°; −45°; −300°; −150°.

5. **Angle measurement.**—Several systems for measuring angles are in use. The system is chosen that is best adapted to the purpose for which it is used.

(1) *The right angle.*—The most familiar unit of measure of an angle is the right angle. It is easy to construct, enters frequently into the practical uses of life, and is almost always used in geometry. It has no subdivisions and does not lend itself readily to computations.

(2) *The sexagesimal system.*—The **sexagesimal system** has for its fundamental unit the degree, which is defined to be the angle formed by $\frac{1}{360}$ part of a revolution of the generating line. This is the system used by engineers and others in making practical numerical computations. The subdivisions of the degree are the minute and the second, as stated in **Art. 2.** The word "sexagesimal" is derived from the Latin word *sexagesimus*, meaning one-sixtieth.

(3) *The centesimal system.*—Another system for measuring angles was proposed in France somewhat over a century ago. This is the **centesimal system.** In it the right angle is divided into 100 equal parts called **grades,** the grade into 100 equal parts called minutes, and the minute into 100 equal parts called seconds. While this system has many admirable features, its use could not become general without recomputing with a great expenditure of labor many of the existing tables.

(4) *The circular or natural system.*—In the **circular** or **natural system** for measuring angles, sometimes called **radian measure** or **π-measure**, the fundamental unit is the radian.

The radian is defined to be the *angle which, when placed with its vertex at the center of a circle, intercepts an arc equal in length*