nuclear fusion

JOURNAL OF PLASMA PHYSICS AND THERMONUCLEAR FUSION

fusion nucléaire

JOURNAL DE PHYSIQUE DES PLASMAS ET FUSION THERMONUCLEAIRE

ядерный синтез

ЖУРНАЛ ПО ФИЗИКЕ ПЛАЗМЫ И ТЕРМОЯДЕРНОМУ СИНТЕЗУ

fusión nuclear

REVISTA DE FISICA DEL PLASMA Y FUSION TERMONUCLEAR

1962 SUPPLEMENT — PART 3

PLASMA PHYSICS AND CONTROLLED NUCLEAR FUSION RESEARCH

CONFERENCE PROCEEDINGS,
SALZBURG, 4—9 SEPTEMBER 1961

INTERNATIONAL ATOMIC ENERGY AGENCY - VIENNA 1963
AGENCE INTERNATIONALE DE L'ENERGIE ATOMIQUE - VIENNE 1963
МЕЖДУНАРОДНОЕ АГЕНТСТВО ПО АТОМНОЙ ЭНЕРГИИ - ВЕНА 1963
ORGANISMO INTERNACIONAL DE ENERGIA ATOMICA - VIENA 1963

NUCLEAR FUSION
FUSION NUCLEAIRE
ЯДЕРНЫЙ СИНТЕЗ
FUSION NUCLEAR

1962 SUPPLEMENT
PART 3

PROCEEDINGS OF THE CONFERENCE ON PLASMA PHYSICS AND CONTROLLED NUCLEAR FUSION RESEARCH, 4—9 SEPTEMBER 1961, SALZBURG, AUSTRIA

LES ACTES DE LA CONFERENCE SUR LA PHYSIQUE DES PLASMAS ET LA RECHERCHE CONCERNANT LA FUSION NUCLEAIRE CONTROLEE, 4—9 SEPTEMBRE 1961, SALZBOURG, AUTRICHE

ТРУДЫ КОНФЕРЕНЦИИ ПО ИССЛЕДОВАНИЯМ В ОБЛАСТИ ФИЗИКИ ПЛАЗМЫ И УПРАВЛЯЕМОГО ЯДЕРНОГО СИНТЕЗА, 4–9 СЕНТЯБРЯ 1961 г., ЗАЛЬЦБУРГ, АВСТРИЯ

LAS ACTAS DE LA CONFERENCIA SOBRE LAS INVESTIGACIONES EN MATERIA DE FISICA DEL PLASMA Y FUSION NUCLEAR CONTROLADA, 4 AL 9 DE SEPTIEMBRE DE 1961, SALZBURGO, AUSTRIA

INTERNATIONAL ATOMIC ENERGY AGENCY • AGENCE INTERNATIONALE
DE L'ENERGIE ATOMIQUE • МЕЖДУНАРОДНОЕ АГЕНТСТВО ПО АТОМНОЙ
ЭНЕРГИИ • ORGANISMO INTERNACIONAL DE ENERGIA ATOMICA

Selection and Program Committee — Comité de sélection et d'établissement du programme — Комитет по отбору докладов и разработке программы — Comité de Selección y del Programa

P. HUBERT	Chef du Service de recherches sur la fusion contrôlée, Commissariat à l'énergie atomique, France
K. V. ROBERTS	Theoretical Physics Division, Culham Laboratory, United Kingdom Atomic Energy Authority, United Kingdom
C. VAN ATTA	Associate Director, Lawrence Radiation Laboratory, University of California United States of America

Scientific Secretariat — Secrétariat scientifique — Научный секретариат — Secretaría Científica

B. BURAS	(*Co-ordinating Scientific Secretary*) Division of Research and Laboratories, International Atomic Energy Agency
P. A. DAVENPORT	(*Scientific Secretary*) Culham Laboratory, United Kingdom
C. ETIEVANT	(*Secrétaire scientifique*) Association Euratom — CEA, Service de recherches sur la fusion contrôlée, Fontenay-aux-Roses, France
W. F. GAUSTER	(*Scientific Secretary*) Oak Ridge National Laboratory, United States of America
Е. В. ПИСКАРЕВ .... (E. V. PISKAREV)	(*Ученый секретарь*) Институт атомной энергии им. И. В. Курчатова, Москва, СССР
B. TORKI	(*Assistant Scientific Secretary*) Division of Research and Laboratories, International Atomic Energy Agency
I. DRUTMAN	(*Technical Secretary*) Division of Research and Laboratories, International Atomic Energy Agency

Editors of "Nuclear Fusion" — Rédacteurs de la revue «Fusion Nucléaire» — Редакторы журнала «Ядерный синтез» — Redactores de "Fusión Nuclear"

J. G. BECKERLEY* .... R. HOBART ELLIS, JR...	Division of Scientific and Technical Information, International Atomic Energy Agency

* Present address: MIT, Nuclear Engineering, 138 Albany St., Cambridge 39, Mass., USA

Executive Secretary — Secrétaire exécutif — Исполнительный секретарь — Secretario Ejecutivo

W. LISOWSKI	Division of Scientific and Technical Information, International Atomic Energy Agency

Conference Services — Services linguistiques — Обслуживание конференции — Servicios de la Conferencia

A. BERNSTEIN	(*Chief Interpreter*) International Atomic Energy Agency
L. S. LIEBERMAN	(*Records Officer*) International Atomic Energy Agency

Public Information — Information — Общественная информация — Información pública

P. FENT	(*Press Officer*) International Atomic Energy Agency
MISS M. NAPIER	(*Special Media Officer*) International Atomic Energy Agency

FOREWORD — AVANT-PROPOS — ПРЕДИСЛОВИЕ — PREFACIO

In 1958, at the Second United Nations Conference on the Peaceful Uses of Atomic Energy, the results of research on controlled nuclear fusion obtained in a few technically advanced countries were first disclosed to the world at large. Since then, it has become more and more evident that a better understanding of fundamental phenomena is needed before the goal of energy extraction from nuclear fusion may be reached. Consequently, the intensive research undertaken in recent years has been primarily basic research in plasma physics.

The fact that such research is most complex and costly has enhanced the desirability of co-operation and exchange of information and experience between all those engaged in this field of nuclear science and technology. It has become obvious that the International Atomic Energy Agency can play an important part in promoting such co-operation on a world-wide scale.

After consulation with a number of leading scientists, the Agency convened an international conference on Plasma Physics and Controlled Nuclear Fusion Research. The extent of the interest shown by Member States did not merely confirm that such a conference was actually needed at the present time, but greatly exceeded expectations. The quality and volume of the papers submitted, the number of participants and of countries represented, all bore witness to this interest.

Today, plasma physics and controlled thermonuclear fusion research is a more-or-less academic study. All that can be said at this stage is that it should eventually lead to a practical energy source. The day may come when the energy from nuclear fusion will be needed and when the well-being of mankind may depend on the ability to draw on this almost limitless reservoir.

The publication of the conference proceedings is intended to promote international co-operation and accelerate progress in this most important field of scientific endeavor.

INTRODUCTION – ВВЕДЕНИЕ – INTRODUCCIÓN

The Conference on Plasma Physics and Controlled Nuclear Fusion Research was held in Salzburg, Austria on 4—9 September 1961. More than 500 scientists, representing 29 nations and 6 international organizations, participated in the Conference. The Proceedings are published in three parts as a 1962 supplement to this journal.

Because of the many interconnections between the various problems of plasma physics, it was decided to have no parallel plenary sessions. Accordingly, nine sessions were held during the six days of the Conference. During these sessions, 111 papers were presented. The "free" afternoons and evenings were devoted to at least fourteen informal discussions of topics of special interest to the participants. The present Proceedings do not include the records of these informal discussions (the discussions would have ceased to be "informal" if recorded), although it seems certain that new ideas generated in these discussions will lead to publication of papers elsewhere.

"Part 1" contains the texts (in original language only) of all papers delivered in Sessions I, II and IV of the Conference, the records (in English) of the discussions of these papers, as well as the texts (in English and Russian) of the two concluding speeches by Prof. Artsimovich and Dr. Rosenbluth summarizing the Conference. Translations of the abstracts of each paper (Sessions I, II, IV) are given at the end of this part of the Proceedings. In addition there is an author index.

The remainder of the Proceedings is published in "Part 2" (Sessions III, V, VI, VIII) and in "Part 3" (Sessions VII, IX). The abstracts of those papers accepted but not presented to the Conference, a list of participants, subject and author indexes for the entire Proceedings are included in the third part.

In preparing the Proceedings the Editors gratefully acknowledge the substantial help of B. Buras, P. A. Davenport, C. Etievant, W. F. Gauster, W. A. Newcomb and E. V. Piskarev.

SESSION VII — SÉANCE VII
ЗАСЕДАНИЕ VII — SESIÓN VII

8 September 1961 — 8 septembre 1961

8 сентября 1961 г. — 8 de septiembre de 1961

Chairman — Le président — Председатель — El Presidente

P. C. Thonemann (*United Kingdom*)

Scientific Secretary — Secrétaire scientifique — Ученый секретарь — Secretario Científico

P. A. Davenport (*United Kingdom*)

[Papers presented: CN-10/38, 47, 54, 57, 58, 60, 63, 68, 91, 96, 106, 150, 223, 224, 227, 238, 239, 241]

CONTENTS — SOMMAIRE — СОДЕРЖАНИЕ — INDICE

SESSION VII — SÉANCE VII — ЗАСЕДАНИЕ VII — SESIÓN VII

SESSION IX — SÉANCE IX — ЗАСЕДАНИЕ IX — SESIÓN IX

(*Contents continued, next page*)

PAPERS PRESENTED BY ABSTRACT ONLY

COMPARISON BETWEEN THEORY AND EXPERIMENT FOR THE STABILITY OF THE TOROIDAL PINCH DISCHARGE*

A. A. WARE

RESEARCH LABORATORY, ASSOCIATED ELECTRICAL INDUSTRIES

ALDERMASTON COURT, ALDERMASTON, BERKSHIRE, UNITED KINGDOM

A summary is given of the experimental observations which are relevant to the stability of the toroidal pinch discharge. This includes recent measurements on the wave velocity of fluctuations in the "Sceptre" experiments and the recent observation of steps on the magnetic field oscillograms with the torus at General Atomic. These results are first compared with the stability predictions based on the magneto-hydrodynamic (MHD) approximations (the energy principle). The experimental results indicate that of the MHD modes, only the interchange mode can be occurring with large amplitude in practice. Secondly, the stability predictions based on more accurate plasma equations are compared with experiment. These include the entropy waves discovered by Tserkovnikov and Kadomtsev and my prediction that the MHD modes move as waves. Lastly, it is shown that if instabilities keep $|\nabla p| \ll |\sigma_{\|} E_{\|} B|$, the rapid penetration of B_θ into the pinch discharge and the enhancement of B_z can be explained.

1. Introduction

The toroidal pinch discharge has been intensively studied over the past few years and many of its properties are now well known. A survey of these properties has been given elsewhere [1]; here (in Section 2 below) a summary is given of the properties relevant to stability.** Most notable of these is the gross stability of the discharge column, the discharge preserving itself in an approximately steady state for as long as the electric field is applied. Also included in the summary are the recent measurements [2, 3] of the propagation velocities of fluctuations in the discharge, and the discovery of steps on the magnetic field waveforms at the Kruskal limit and its harmonics [4].

Most treatments of the MHD stability of the pinch discharge assume that the currents are confined to a thin skin. Their results are not relevant to the toroidal pinch, since after the first few microseconds there is no skin effect. The maximum current density is, in fact, at the centre of the discharge. The more general energy principle [5] can be applied and in Section 3 its predictions are compared with experiment.

Since there are discrepancies between MHD theory and experiment, more accurate plasma equations must be considered. When higher-order terms are retained, two modifications to the stability predictions result. First, the MHD modes of instability will no longer be stationary but will propagate as waves [6]. Second, new modes of instability are possible [7, 8, 9] which have little effect on the magnetic field. KADOMTSEV [8] has given these modes the name "entropy waves." In Sections 4.1 and 4.2 these predictions are compared with the experimental results.

Among other unexplained properties of the toroidal pinch are the rapid penetration of B_θ into the plasma at the onset of the discharge, and the enhancement of B_z flux within the discharge. In Section 5 it is shown that if instabilities are assumed to keep the pressure gradient small so that $|\nabla p| \ll |\sigma_{\|} E_{\|} B|$, both these effects can be explained.

Throughout this paper all but one of the effects of the toroidal curvature are neglected, and cylindrical co-ordinates r, θ, z will be used to describe the discharge, the z-axis being taken parallel to the discharge tube. The one effect retained is the boundary condition that requires all properties of the discharge to be periodic in the z-direction with the torus circumference as the fundamental length.

2. Experimental properties of the toroidal pinch discharge

A summary is given below of the main properties of the toroidal discharge relevant to stability. The results refer in general to discharges for which the initial values of the toroidal field, namely B_{z0}, lie within the approximate range from about a quarter to unity times the maximum value of B_θ generated at the tube wall. Above this range little pinch effect occurs and with B_{z0} much below this range, if not before, the discharge is grossly unstable.

2.1 PENETRATION OF B_θ INTO THE DISCHARGE

Magnetic probe measurements have shown that B_θ penetrates into the discharge much more rapidly than predicted by electromagnetic theory [4, 10]. Kinks or steps are observed on the magnetic probe oscillograms [4], and it has been found that these occur

---

* Conference paper CN-10/47, presented by A. A. Ware. Discussion of this paper is given on page 1017. Translations of the abstract are at the end of this volume of the Conference Proceedings.

** This paper is restricted to instability frequencies ω small compared with the ion cyclotron frequency and to wavelengths comparable with or greater than the tube radius. Instabilities with higher frequencies and shorter wavelengths, such as ion waves and electron oscillations, have been the subject of considerable theoretical interest recently, but as yet there is no evidence for their existence (or absence) in the toroidal pinch discharge.

when the pitch of the magnetic lines of force, namely $\lambda_B = 2\pi r B_z / B_\theta$, satisfies $\lambda_B \approx L/n$ or $2\,L/n$ where n is an integer and L is the torus circumference. These steps appear to be associated with the penetration of B_θ, since during the time B_θ is remaining approximately constant over an outer region of the discharge, it is simultaneously increasing at smaller radii. Steps for a given value of λ_B occur later at smaller radii.

2.2 THE STEADY STATE MAGNETIC FIELD CONFIGURATION

The axial field B_z is enhanced in the discharge and its radial profile exhibits a characteristic bell shape [11, 12, 13] with a maximum at the centre of the discharge. (Since the enhancement persists for such long times with little change, it cannot be due to trapping of the field in the initial discharge constriction, and it is not explained by the paramagnetic effect of the anisotropic conductivity [11]. The corresponding profile for B_θ, when converted to axial current density, shows that j_z is a maximum at the centre of the discharge and falls off slowly with radius [11, 12, 13.] At a radius which is often about half the tube radius dB_θ/dr changes sign, and j_z is much smaller beyond this radius although j_θ is often still large.

Over the centre of the discharge the pitch of the magnetic lines of force is approximately constant [11] or varies only slowly with radius (see the published field profiles for Zeta [11], Perhapsatron S4 [14] and TA 2,000 [13], but near the walls B_z falls too rapidly with radius to maintain this condition (and sometimes goes negative), so that the pitch decreases appreciably in this region.

Among the slow toroidal pinch discharges, only in the case of Sceptre III has the plasma pressure deduced from the magnetic probes measurements been published [12]. The pressure is a maximum in the centre of the discharge and falls off with radius throughout the main core of the discharge. At approximately half the tube radius the pressure passes through a minimum and then rises towards the walls. A somewhat similar pressure profile was obtained for the faster discharge in Perhapsatron S4, except that the central maximum has a dip in it [14].

2.3 DISCHARGE STABILITY

With zero or low applied axial magnetic fields, the pinch discharge exhibits violent kink instability and large magnetic field fluctuations. Within the magnetic field range considered here this large amplitude kink instability no longer occurs [11, 15]. However, smaller magnetic field fluctuations remain whose amplitudes are usually in the range 10-20% of the unperturbed field [11, 12], and whose mean frequency is usually in the range $10^4 - 10^5$ Hz. On streak photographs light intensity variations are observed which sometimes show helical patterns and sometimes "bars" [15], these variations being correlated with the field fluctuations [16].

In the outer regions of the discharge the magnetic fluctuations often have a regular nature, and these have been identified as an $m=1$ perturbation [2]. (Discharge perturbations of the form $\exp \mathrm{i}\,(\omega t - m\theta - kz)$ are considered.) These fluctuations generally propagate in the z direction with a velocity [2] of the order $+10^6$ cm/s. (The positive direction is taken parallel to the current). Observation of the fluctuations of light emitted from these regions (spectral lines from unionised atoms and singly-ionised impurities) showed similar velocities (see Table 3). This $m=1$ instability resembles the moving $m=1$ perturbations observed throughout the whole discharge at much lower currents by ALLEN [17], BAKER *et al.* [18] and RUSBRIDGE *et al.* [19]. In the first two of these references, Doppler shift measurements are reported which showed that a wave motion was involved and not a mass motion.

In the central region of the discharge the magnetic fluctuations were small and irregular, and no wave motion could be ascertained. Light from highly ionised particles in this region, however, showed fluctuations [3] with a wave velocity of about -5×10^6 cm/s.

3. Magnetohydrodynamic stability of the discharge core

The predictions of the MHD energy principle [5] regarding the stability of the discharge core, after field penetration is complete, have been considered by WARE [1]. Since the magnetic field has a constant

TABLE I. Properties of kink and interchange types of instability

	Interchange modes ($mB_\theta + krB_z = 0$)		*Kink modes* ($mB_\theta + krB_z \neq 0$)	
Condition for minimum change in potential energy (δW)	$\nabla \cdot \boldsymbol{\xi} = \dfrac{2\,B_\theta^2\,\xi_r}{r\,(B^2 + \gamma p)}$	(1)	$\nabla \cdot \boldsymbol{\xi} = 0$	(5)
Stability condition	$\dfrac{\mathrm{d}p}{\mathrm{d}r} \geqslant -\dfrac{2\gamma p B_\theta^2}{\gamma (B^2 + \gamma p)}$	(2)	$\dfrac{\mathrm{d}p}{\mathrm{d}r} > \alpha^2 > 0$	(6)
Magnetic-field fluctuations	$\delta B_\theta \ll \xi_r \dfrac{r\mathrm{d}(B_\theta/r)}{\mathrm{d}r}$	(3)	$\delta B_\theta \approx r\,\xi_r \dfrac{\mathrm{d}(B_\theta/r)}{\mathrm{d}r}$	(7)
	$\delta B_z \ll \xi_r \dfrac{\mathrm{d}B_z}{\mathrm{d}r}$	(4)	$\delta B_z \approx \xi_r \dfrac{\mathrm{d}B_z}{\mathrm{d}r}$	(8)

(p is the pressure, γ the ratio of the specific heats (5/3) and α^2 is an undetermined positive quantity) pitch, two types of MHD instability are possible, namely interchange modes and kink modes, depending on whether the helical perturbation has the same or a different pitch to the magnetic field. The properties of these two types of instability are summarised in Table 1 below. Normal mode displacements of the form $\xi = \xi\ (r)\ \exp\ \mathrm{i}\ (\omega t - m\theta - kz)$ are considered.

The wavelength in the z direction for an interchange mode is λ_B/m, where λ_B is the pitch of the magnetic field, namely $2\pi r B_z/B_\theta$. For the kink modes the most unstable m value is 1 and the unstable wavelengths lie in a range extending above and below λ_B, the growth rate going to zero at the singular point within this range where $2\pi/k = \lambda_B$.

Since the experimental results show a substantial negative pressure gradient for the core of the discharge [12], the relation (6) of Table I predicts that the core should exhibit a gross kink instability. In the case of the interchange mode, to compare the predicted stability criterion, Table I, Eq. (2), with experiment it is necessary to know the plasma pressure. Since there is an unknown integration constant in the pressure deduced from the observed pressure gradient, it is more convenient to compare observed and predicted pressures in this case.

TABLE II. Comparison between experimental and theoretical pressures

Radius (cm)	Pressure observed by ALLEN and LILEY [12] (p_0 is the integration constant) (10^5 dynes/cm²)	Pressure from (2) using observed pressure gradient, etc. (10^5 dynes/cm²)	Percentage error with $p_0 = 1.3 \times 10^5$ dynes/cm² (%)
1	$1.04 + p_0$	2.9	+24
2	$0.84 + p_0$	2.19	+2
3	$0.56 + p_0$	2.28	+23
4	$0.30 + p_0$	1.32	+17
5	$0.10 + p_0$	0.68	−51
−1	$1.05 + p_0$	2.68	+14
−2	$0.90 + p_0$	2.33	+6
−3	$0.72 + p_0$	2.07	+2
−4	$0.52 + p_0$	1.66	−9
−5	$0.33 + p_0$	1.1	−32

In Table II, the second column shows the plasma pressures obtained experimentally by ALLEN and LILEY [12] for the discharge core. p_0, the integration constant, is the value of the pressure at $r = 6$ cm, the position of the pressure minimum. The third column shows the pressures calculated from the equality in (2) (of Table I) using the observed values for $\mathrm{d}p/\mathrm{d}r$, B_θ and B_z. The last column shows the percentage difference between the two pressures if p_0 is taken as 1.3×15^5 dynes/cm², the value for best fit. Assuming that this is the correct value for p_0, the agreement between theory and experiment is as good as can be expected, since $\mathrm{d}p/\mathrm{d}r$ will have a large experimental error.

The range of other values which could be taken for p_0 is limited from other considerations. Thus, the lowest possible value for p_0 is zero; this would make the observed negative pressure gradients twice (at $r = 2$ cm) to six times (at $r = 5$ cm) the allowable gradient for interchange stability. The upper limit for p_0 cannot be much higher than the value taken, since the integral $\int 2\pi r p\ \mathrm{d}r$ already leads to a value of N which is several times the initial gas filling [12] even if the mean ion temperature is taken as high as $10^{6\circ}$ K. (N is the number of particles per unit length of the discharge.) These considerations, therefore, suggest that the plasma pressure must be such that the negative pressure gradient is of the same order as, or somewhat greater than, the value for marginal interchange stability.

In the experimental results [12] for Sceptre III, $\mathrm{d}B_z/\mathrm{d}r$ is approximately $-\ B_z/r$ at $r = 6$ cm and hence the expected percentage fluctuation for B_z in a kink mode instability is $100\ \xi_r/r$. Since the observed fluctuations are only 15%, this means ξ_r cannot be greater than about $0.15 r \approx 1$ cm or about 6% of the tube radius. On the other hand, the interchange instability, which produces only small changes in the magnetic field, could be occurring with large amplitude. Since other workers with slow pinch discharges have observed only small fluctuations, this last conclusion must be general for such discharges.

The above results indicate that the kink instability either is not occurring in the pinch-discharge core or, at most, is occurring with only small amplitude. This reveals an apparent discrepancy between experiment and the predictions of MHD stability theory. Secondly, the observed pressure gradient suggests that the interchange instability is occurring and is limiting the negative pressure gradient to a value close to that for marginal stability with these modes.

Some possible causes for the small (or zero) amplitude of the kink mode have been discussed elsewhere [1]. These include (a) the higher-order terms omitted from the MHD equations, since the ion Larmor radius is not negligible compared with the discharge radius, and (b) the possible presence in the discharge of a large number of deuterons moving with high velocities parallel to the magnetic field so that they experience a frequency just above their cyclotron frequency. Such particles would have a stabilising effect on kink modes.

It should be noted that the above results and discussion refer only to the discharge core. In the annular region between the core and the walls an $m = 1$ perturbation has been observed [2], which could be a kink instability which has grown to a limited amplitude and then been stabilised by higher-order effects.

The steps observed on the magnetic probe oscillograms during the growth and decay of the current could be due to either changes in wavelength of an interchange instability or a kink instability. A study of the work of TAYLER [20] indicates that a discharge with uniform j_z and B_z which fills the whole tube is unstable to an $m = 1$ kink mode only for a narrow range of wavelengths $2\ \pi/k$ immediately above λ_B. Wavelength transitions would therefore be expected at values of λ_B which are just below L/n. For interchange modes to occur, the field must have approximately constant pitch and the lines of force must join on themselves. The latter condition is satisfied when $\lambda_B = mL/n$, when an m type interchange is possible. It seems likely that an interchange mode would tend to keep the magnetic pitch constant over the annular region in which it was occurring. Hence steps would occur on the magnetic field waveforms for $\lambda_B = mL/n$. The annular region in which a given interchange is occurring might move radially inwards as the current is increased. This would explain why the steps are observed later at smaller radii. This latter property suggests that the interchange modes are the cause rather than kink modes.

4. The stability predictions of more accurate plasma equations

4.1 MHD INSTABILITY WAVES

In the case of a plasma whose particle-collision frequencies are greater than the instability frequency ω (so that the particle pressures are approximately isotropic) and whose electron-collision frequency is small compared with the electron cyclotron frequency, the electric conduction equation [21] is

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} - \frac{\mathbf{j} \times \mathbf{B}}{ne} - \frac{\nabla p_e}{ne} = 0 \qquad (9)$$

(E is the electric field, V the plasma velocity, n the particle density, j the current density and p_e the electron pressure).

If $$\varrho/L \ll 1 \qquad (10)$$

and $$\omega^2 L^2 \gtrsim \mathrm{k}\, T_i/M, \qquad (11)$$

where ϱ is the ion Larmor radius, L is the characteristic length of the discharge or the instability, T_i the ion temperature and M the ion mass, the third and fourth terms in Eq. (9) are small compared with either of the first two terms [5]. Because of this, the approximate equation $\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0$ has been widely used in stability theory. When it is combined with the other plasma equations involving the MHD approximations, the predicted values for ω^2 are always real [5], and an instability perturbation ($\omega^2 < 0$) will grow continuously in time with no oscillations or wave motion involved.

However, neither of the conditions (10) and (11) are good approximations for many pinch discharge experiments. When the last two terms are retained in Eq. (9) ω^2 is no longer always real, and an instability perturbation will move as a wave [6]. A physical picture of this wave motion can be obtained by considering the "freezing-in" of magnetic lines of force in the plasma. By combining Eq. (9) with Maxwell's equation for $\nabla \times \mathbf{B}$ and the appropriate continuity equation, assuming $T_e \ll T_i$, the magnetic field is found to be frozen to the electrons [6] and not to the mean motion of the plasma. Both the unperturbed and the fluctuating components of the electron velocity will, in general, cause the instability magnetic field fluctuations to propagate, but experimentally there is evidence that the effect of the unperturbed electron motion predominates [6]. If this is the case, the wave velocity with which a helical instability will propagate in the z direction is

$$V_w \approx -\frac{1}{ne}\left(j_z - \frac{m j_\theta}{kr}\right) \qquad (12)$$

(The assumption has been made that there is no mass motion in the discharge, so that the mean electron velocity is given approximately by $\mathbf{j}/ne$.)

In the special case where the instability crests are parallel to B, as in the case of an interchange mode or a kink mode with small δk, the wave velocity reduces to [6]

$$V_w = \frac{1}{neB_\theta} \frac{\mathrm{d}p}{\mathrm{d}r} \qquad (13)$$

In Table III, the next-to-last column shows the velocity in the z-direction with which fluctuations have been observed to propagate in various toroidal discharge experiments and in the straight tube Columbus T. In order to obtain a theoretical propagation velocity from Eq. (12) or Eq. (13), it is necessary to know the wave numbers m, k, the density n and either the current densities j_θ and j_z, or the pressure gradient and B_θ. Where these are known, or where a rough estimate can be made, the values are shown in the appropriate columns in Table III and the last column shows the theoretical propagation velocity deduced from them. For the discharge core in Sceptres III and IV the instability is assumed to be an interchange mode (see Section 3 above) and the pressure gradient is taken from ALLEN and LILEY [12]. In the case of the outer region of Sceptre IV the fluctuations were very irregular and were observed to propagate sometimes in one direction and sometimes in the opposite direction. As in the case of the core fluctuations for Sceptres III and IV the values of m and k were not obtained because of the irregularity of the fluctuations.

It can be seen from Table III that the present theory gives the correct direction of propagation in every case where a theoretical value was possible, and also the right order of magnitude for the velocity.

4.2 ENTROPY WAVES

By using plasma equations which retain higher order terms in the parameter ϱ/L, several Russian workers (TSERKOVNIKOV [7], KADOMTSEV [8] and RUDAKOV and SAGDEEV [9]) have shown that, in addition to the MHD modes, new modes of instability are possible. They have taken a simple pinch discharge with $B_z = 0$ and considered only $m = 0$ perturbations. In particular Kadomtsev has used Eq. (9), together with thermal-energy equations which are more accurate than the simple adiabatic relation of the MHD approximation, and has found an instability whose growth rate has an order of magnitude which is ϱ/L times the MHD growth rates. If the negative

TABLE III

Experiment	Region of discharge	Particle density (cm^{-3})	$2\pi/k$ (cm)	m	Mean value of $j_z - mj_\theta/kr$ (amp)	$\frac{dp}{dr}$ (dynes/cm^3)	B_θ (G)	Propagation velocity of fluctuations (cm/s)	
								Experiment	Theory
Glass torus	Whole[(a)]	—	15	1	—	—	—	-10^6	—
Columbus T	Whole[(b)]	$\sim 4\times10^{14}$	15	1	$+26$	—	—	-2×10^5	-4×10^5
Sceptre III	Core[(c)]	$\sim 5\times10^{14}$	λ_B/m (assumed)	—	—	-2×10^4	10^3	-3×10^5 to -3×10^6	-2×10^6
	Outer region[(d)]	$\sim 5\times10^{14}$	67	1	-390	—	—	$+5\times10^5$ to $+4\times10^6$	$+5\times10^6$
Sceptre IV	Core[(c, e)]	$\sim 5\times10^{14}$	λ_B/m (assumed)	—	—	-2×10^4	10^3	-10^6 to -4×10^6	-2×10^6
	Outer region[(c, f)]	—	—	—	—	—	—	$\pm5\times10^5$ to $\pm4\times10^6$	—
Mark IV	Whole[(g)]	2×10^{13}	50	1	$+2.2$	—	—	-4×10^5	-7×10^5

(a) ALLEN, T. K. [17]; (b) BAKER, *et al.* [18]; (c) WILLIAMS, R. V. [22]; (d) ALLEN, N. L. [2]; (e) WILLIAMS, R. V. [3]; (f) ALLEN, N. L. [23]; (g) RUSBRIDGE, *et al.* [19].

pressure gradient exceeds that given by Eq. (2) with B put equal to B_θ, the stability condition is

$$\frac{d\ln T}{d\ln r} \geqslant 1 + \left(\frac{7}{10} + \frac{\beta}{4}\right)\frac{d\ln p}{d\ln r}, \tag{14}$$

where $\beta = 8\pi\, p/B_\theta^2$.

This instability involves different displacements (ξ_i, ξ_e) for the ions and the electrons, and these occur in such a manner that the fluctuations in p, $\mathbf{B}$ and $\mathbf{j}$ are small. The main quantities which vary in first order are n, T_e and T_i, hence these instabilities have been given the name "entropy waves". Kadomtsev has taken the unperturbed values of T_e and T_i to be equal, and in this case the fluctuations propagate with half the velocity given by Eq. (13). (Eq. (13) was derived assuming $T_e \ll T_i$.)

It can be shown [24] that in the case of a discharge with B_z, and with constant-pitch lines of force, the equations for instability modes with the same pitch as the field are very similar to those for the particular case considered by Kadomtsev with $m=0$, $B_z=0$. The only differences are that the wave number k is replaced by $[k^2+m^2/r^2]^{1/2}$ and one term contains the extra factor B_θ^2/B^2. It is believed that a stability condition similar to (14) will result with $\beta = 8\,\pi p/B^2$.

The stability condition (14) is not very enlightening; however, if the faster-growing MHD interchange modes are assumed to have reduced the pressure gradient to the value for marginal stability [Eq. (2)], the condition (14) reduces to the more transparent form

$$\frac{1}{T}\frac{dT}{dr} \geqslant \frac{2}{5}\frac{1}{p}\frac{dp}{dr} \quad \text{or} \quad \frac{1}{T}\frac{dT}{dr} \geqslant \frac{2}{3}\frac{1}{n}\frac{dn}{dr} \tag{15}$$

This is the same condition as the well known stability condition for thermal convection in a gravitational field. This and other considerations suggest that the entropy waves are a form of thermal convection.

Since the marginal stability condition (2) for the MHD interchange modes can be rewritten in the form

$$\frac{1}{p}\frac{dp}{dr} = \frac{5}{3}\frac{1}{B_z}\frac{dB_z}{dr} \tag{16}$$

the marginal stability conditions for both entropy waves and MHD interchange modes, and the constant pitch condition can be summarised as follows

$$\frac{r}{B_\theta}\frac{d(B_\theta/r)}{dr} = \frac{1}{B_z}\frac{dB_z}{dr} = \frac{3}{5}\frac{1}{p}\frac{dp}{dr} = \frac{3}{2}\frac{1}{T}\frac{dT}{dr}\left(=\frac{1}{n}\frac{dn}{dr}\right). \tag{17}$$

A preliminary study suggests that in the case when the unperturbed electron and ion temperatures are unequal, each temperature gradient must satisfy relation (15) separately.

Lastly a strange coincidence in the constants of nature must be pointed out. In a discharge in a steady state such that $E_\theta = 0$ and E_z is a constant with respect to radius, from Maxwell's equation for $\nabla\times\mathbf{B}$, and the relation $j_\parallel = \sigma_\parallel E_\parallel$ (where σ is the electrical conductivity and the subscripts refer to the components parallel to $\mathbf{B}$ in each case) it follows that constant pitch requires

$$\frac{1}{B_z}\frac{dB_z}{dr} = \frac{1}{\sigma_\parallel}\frac{d\sigma_\parallel}{dr} \tag{18}$$

and hence for completely ionised hydrogen

$$\frac{1}{B_z}\frac{\mathrm{d}B_z}{\mathrm{d}r} = \frac{3}{2}\frac{1}{T_e}\frac{\mathrm{d}T_e}{\mathrm{d}r} \tag{19}$$

This is identical with the condition for marginal stability for the entropy wave. It is a coincidence since here the factor 3/2 arises from Coulomb scattering, whereas in the stability condition it arises because of the ratio of the specific heats. It suggests that the constant-pitch configuration may arise as a result of the entropy waves. Also, provided the MHD kink instability does not occur, it suggests that a discharge satisfying equations (17) will maintain itself in a steady state indefinitely as long as the electric field is applied.

5. Field penetration and enhancement

5.1 THE CONDUCTIVITY OF A PLASMA PERPENDICULAR TO A MAGNETIC FIELD

In a plasma in equilibrium, currents flow perpendicular to the magnetic field only if there is a pressure gradient, since $\mathbf{j}\times\mathbf{B}=\nabla p$. If, therefore, instabilities in a discharge prevent the generation of an appreciable pressure gradient, $j_\perp$ will remain small. In particular if the pressure gradient is limited such that

$$\left|\frac{\mathrm{d}p}{\mathrm{d}r}\right| \ll |\sigma_\parallel E_\parallel B| \tag{20}$$

then

$$|j_\perp| \ll |j_\parallel| \tag{21}$$

and the plasma will behave approximately as if it has zero conductivity perpendicular to the magnetic field. In the case of the experimental pressure gradient considered in Section 3 the highest value of $|\mathrm{d}p/\mathrm{d}r|$ was about a quarter of $|\sigma_\parallel E_\parallel B|$.

When condition (21) is satisfied the components of Maxwell's curl **B** equation for a cylindrically symmetric discharge reduce to

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r} = 4\pi\sigma_\parallel\left(E_z\frac{B_z^2}{B^2}+E_\theta\frac{B_\theta B_z}{B^2}\right) \tag{22}$$

$$-\frac{\partial B_z}{\partial r} = 4\pi\sigma_\parallel\left(E_z\frac{B_\theta B_z}{B^2}+E_\theta\frac{B_\theta^2}{B^2}\right) \tag{23}$$

5.2 THE ENHANCEMENT OF B_z

The solution of Eqs. (23) and (24) for the conditions corresponding to the steady-state pinch discharge at peak current will be considered first. Since the components of $\partial\mathbf{B}/\partial t$ are small, E_z is approximately constant with respect to radius and $E_\theta \ll E_z$. Eqs. (22) and (23) have therefore been solved on a computer for E_z = constant, $E_\theta = 0$ and also $\sigma_\parallel$ assumed constant. Example solutions for B_θ and B_z are shown in a dimensionless form in Fig. 1. The scaling factor $B_{\theta\mathrm{w}}$ is the value of B_θ at the edge of the discharge and the pairs of curves are for various values of B_z at the tube wall ($B_{z\mathrm{w}}$).

The enhancement of B_z within the discharge for low values of $B_{z\mathrm{w}}$ at the wall is very marked. Secondly, it should be noted that for low $B_{z\mathrm{w}}$ there is a fairly abrupt change of $\mathrm{d}B_\theta/\mathrm{d}r$ within the discharge, indicating a small j_z beyond this radius. In the past, experimenters have been inclined to interpret such a change of slope as the edge of the discharge. The present theory shows that it could be due to the anisotropic conductivity in a uniform plasma.

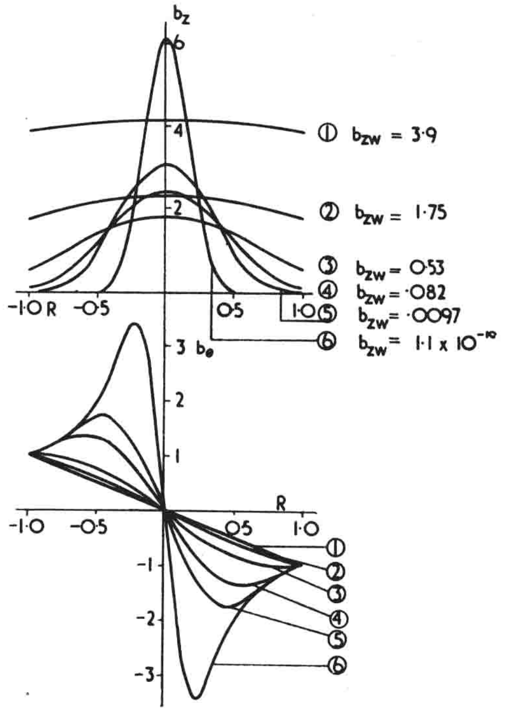

Fig. 1 Example solutions of Eqs. (22) and (23) for the values of $b_{z\mathrm{w}}$ indicated ($b_z = B_z/B_{\theta\mathrm{w}}$, $b_\theta = B_\theta/B_{\theta\mathrm{w}}$, $R = 4\pi\sigma_\parallel E_z r/B_{\theta\mathrm{w}}$ and the subscript w denotes the value at the tube wall).

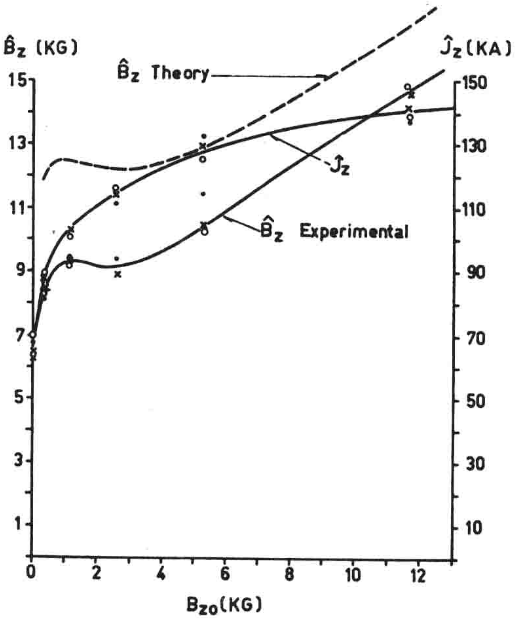

Fig. 2. Comparison between theoretical and experimental values for $\hat{B}_z$, which is the value of B_z at the centre of the discharge. The abscissa B_{z0}, is the value of B_z at the tube wall since in these experiments this field remained approximately equal to the initial field. The peak gas current, J_z, is shown for comparison.

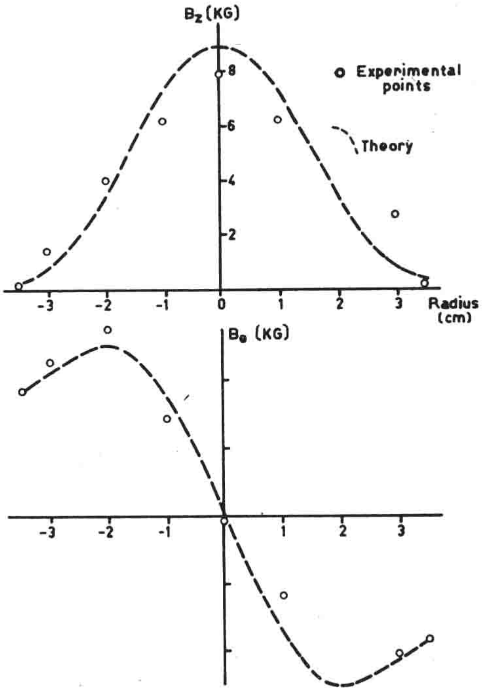

Fig. 3 Comparison between theory and experiment for the radial variation of B_z and B_θ.

In Fig. 2 the enhancement of B_z is compared with the experimental results of WARE *et al.* [4]; the observed and theoretical values of B_z at the centre of the discharge (namely B_z) are plotted against B_{z0}, the initial field. (In this experimental work the coil producing the initial B_z had a high-impedance source, and the value of B_z at the tube wall (B_{zw}) remained approximately equal to the initial value B_{z0}.) Qualitatively, the shape of the theoretical curve agrees very well with experiment; even the small bump at $B_{z0} = 1$ kG is reproduced. Quantitatively the predicted fields are higher than the observed fields, the error varying from 13% at 12 kG to 40% at 300 G.

In Fig. 3 theoretical profiles for B_z and B_θ are compared with the experimental measurements of B_z and B_θ across the tube for a low B_{zw} case. It can be seen that there is reasonably good agreement in the magnitude of both B_θ and B_z at all radii.

Since the theoretical values of B_z exceed the observed values, the theory is an adequate explanation of the enhancement of B_z. The fact that the observed enhancement of B_z is less than that predicted by theory is probably due to the fact that $\mathrm{d}p/\mathrm{d}r$ and $\mathrm{d}\sigma_\parallel/\mathrm{d}r$ are not accurately zero, as assumed in the theory, but have small negative values.

In experiments where the metal sheath is continuous in the θ-direction, the general shape of the magnetic-field profiles is again given, as has been pointed out by LEES and RUSBRIDGE [10], but the fact that B_z goes negative close to the tube wall is not explained.

5.3 THE PENETRATION OF B_θ

To predict the rate of penetration of B_θ Eqs. (22) and (23) must be solved simultaneously with Maxwell's equation for curl **E**. This has not yet been carried out, but the following qualitative discussion shows that a more rapid penetration is predicted for low B_{z0}.

First, for $B_z \gg B_\theta$ the term containing E_θ in Eq. (22) will be small and the first term will be approximately equal to $4\,\pi\,\sigma_\parallel\,E_z$. In this case, therefore, B_θ will penetrate at the same rate as into a normal conductor with isotropic conductivity $\sigma_\parallel$. At low B_z, however, the effective conductivity in the first term in Eq. (22) will be reduced by the factor $B_z^2/B^2 \approx B_z^2/B_\theta^2$ and hance as far as this term is concerned the penetration will be more rapid. Further, since B_z is increasing in the discharge, E_θ will have a negative value and will make the second term in Eq. (22) negative. Hence, at any instant of time when the values of B_θ, B_z and $\partial\,B_\theta/\partial\,r$ are given, E_z must have a larger value in order to satisfy Eq. (22). That is, the effect of the second term is to make $\partial\,B_\theta/\partial\,t$ larger. Both terms, therefore, have the effect of causing a more rapid penetration of B_θ.

6. Conclusions

1. The value of the pressure gradient, and the smallness of the magnetic fluctuations, in the core of the high current toroidal discharge, are evidence that only the interchange mode of MHD instability can be occurring with large amplitude, although MHD theory predicts that the kink modes should be unstable.

2. In the outer regions of the discharge there is an $m=1$ kink distortion which has reached a certain amplitude and ceased to grow, but moves along the tube.

3. More accurate plasma equations than the MHD approximations show that for $T_e \ll T_i$ the magnetic field is trapped to the electrons and, in general, instabilities will propagate as waves. Observations on the propagation of fluctuations in the discharge, and the $m=1$ distortion in particular, show that the correct direction of propagation and the right order of magnitude for the wave velocity are predicted.

4. The Russian workers have shown that the more accurate plasma equations lead to new modes of low-frequency instabilities (entropy waves). If the pressure gradient is close to the value for marginal stability against interchange modes [$\mathrm{d}\ln p/\mathrm{d}r = \gamma\,\mathrm{d}\ln B_z/\mathrm{d}r$] the new modes are stable when $\mathrm{d}\ln T/\mathrm{d}r \geqslant (2/5)\,\mathrm{d}\ln p/\mathrm{d}r$ [i.e. $\mathrm{d}\ln T/\mathrm{d}r \geqslant (2/3)\,\mathrm{d}\ln B_z/\mathrm{d}r$] which is identical with the well-known stability condition for thermal convection.

5. The marginal stability condition for the entropy waves is the same as the condition for a constant-pitch magnetic field in a steady-state discharge for which the electrical conductivity parallel to **B** is proportional to $T_e^{3/2}$. The entropy waves may therefore be the cause of the observed constant configuration in the discharge core.

6. If instabilities keep $|\nabla p| \ll |\sigma_\parallel E_\parallel B|$, the plasma will behave approximately as if it has zero electrical conductivity perpendicular to **B**, and the enhancement of B_z and the rapid penetration of B_θ can be explained.

Acknowledgements

The work reported in Sections 3, 4.2 and 5 was undertaken while the author was on leave of absence at the John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California, and the author gratefully acknowledges the assistance of discussions with Marshall Rosenbluth, Donald Kerst, Norman Rostoker and other colleagues at that laboratory. The author also thanks Bruce Liley and other colleagues at Associated Electrical Industries Research Laboratory, for helpful discussions, Norman Allen and Roy Williams for permission to include unpublished results in Table III, and Dr. T. E. Allibone, C.B.E., F.R.S., Director of the Laboratory, for permission to publish this paper. Part of the work at the Associated Electrical Industries Research Laboratory was done under contract to the Atomic Energy Research Establishment, Harwell.

References

[1] Ware, A. A., *Phys. Rev.* **123** (1961) 19.
[2] Allen, N. L., *Nature* **187** (1960) 279.
[3] Williams, R. V., *J. nucl. Energy, Part C* **3** (1961) 31.
[4] Ware, A. A., Forsen, H. K., Schupp, A. A., *Phys. Rev.* **125** (1962) 417.
[5] Bernstein, I. B., Frieman, E. A., Kruskal M. D., Kulsrud, R. M., *Proc. Roy. Soc.* (*London*) **A244** (1958) 17.
[6] Ware, A. A., *J. nucl. Energy, Part C*, **3** (1961) 93.
[7] Tserkovnikov, Yu. A., *Zh. exp. teor. Phys.* (*USSR*) **32** (1957) 67.
[8] Kadomtsev, B. B., *Zh. exp. teor. Phys.* (*USSR*) **37** (1959) 1096.
[9] Rudakov, L. I., Sagdeev, R. Z., *Zh. exp. teor. Phys.* (*USSR*) **37** (1959) 1337.
[10] Lees, D. J., Rusbridge, M. G., Proc. 4th Conf. Ionisation Phenomena in Gases (Uppsala 17—21 Aug. 1959) **2** North Holland Publ. Co., Amsterdam (1960) 876.
[11] Butt, E. P., *et al.*, in Proc. 2nd U.N. Intern. Conf. PUAE **32** United Nations, Geneva (1958) 42.
[12] Allen, N. L., Liley, B. S., in proceedings of Ref. 10, **2**, 937.
[13] Etievant, C., *et al.*, in proceedings of Ref. 10, **2**, 967.
[14] Connor, J. P., *et al.*, in proceedings of Ref. 11, **32**, 297.
[15] Allibone, T. E., Chick, D. R., Thomson, G. P., Ware, A. A., in proceedings of Ref. 11, **32**, 169.
[16] Ware, A. A., in proceedings of Ref. 10, **2**, 931.
[17] Allen, T. K., in Proc. 3rd Intern. Conf. Ionisation Phenomena in Gases, (Venice, 11—15 June 1957), **1**, North Holland Publishing Co., Amsterdam (1959) 40.
[18] Baker, D. A., Sawyer, G. A., Stratton, T. E., in proceedings of Ref. 11, **32**, 34; Sawyer, G. A., Scott, P. L., Stratton, T. F., *Phys. Fluids* **2** (1959) 47.
[19] Rusbridge, M. G. *et al.*, *J. nucl. Energy, Part C*, **3** (1961) 98.
[20] Tayler, R. J., *Proc. Phys. Soc.* (*London*) **70** (1957) 1049.
[21] Spitzer, L., Jr., Physics of Fully Ionised Gases (Interscience, New York, 1956) 21.
[22] Williams, R. V., to be published.
[23] Allen, N. L., to be published.
[24] Ware, A. A., to be published.

NUCLEAR FUSION: 1962 SUPPLEMENT, PART 3

THE INFLUENCE OF THE HALL EFFECT ON A SIMPLE HYDROMAGNETIC STABILITY PROBLEM*

R. J. Tayler

Culham Laboratory

Culham, Berkshire, United Kingdom

One factor that is commonly neglected in hydromagnetic stability problems is the Hall effect. It has been pointed out that, with the introduction of this term in the generalized Ohm's law, the differential equations governing the problem are no longer self-adjoint. Thus complex growth rates can be expected to occur, and ordinary instabilities may be replaced by instability waves (overstability). However, most of the arguments have been qualitative rather than quantitative.

In the present paper an exact solution is given for one very simple problem in which the Hall current is included. This is the stability of an incompressible conducting fluid containing a uniform axial magnetic field and a uniform axial current density and filling a conducting tube. For this problem it is shown that overstability does occur but that the system including the Hall effect is never overstable when the system without the Hall effect is stable. In addition, when overstability occurs, the real part of the growth rate is smaller than it is when the Hall effect is absent; thus the instability waves grow more slowly than the original instabilities. If the ion gyration frequency is much less than the characteristic hydromagnetic frequency the band of unstable wavenumbers becomes very small.

1. Introduction

Recently Ware [1] has suggested that an important factor that has been overlooked in many stability investigations is the Hall effect. He suggests that when the Hall effect is included, the pure instability predicted by the idealized hydromagnetic equations will be replaced by an instability wave (overstability). Furthermore he suggests that the approximate wave velocity will be the component of the electron drift velocity perpendicular to the instability. He points out that in many experiments such instability waves are observed and that their wave velocities are in reasonable agreement with simple theory.

The simplest argument for waves of this type is that the equation

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}/c=0, \tag{1.1}$$

is not an adequate approximation to Ohm's law. This equation predicts that the magnetic field lines are tied to the motion of the material whereas the equation

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}/c-\mathbf{j}\times\mathbf{B}/n\,e\,c=0 \tag{1.2}$$

which is the next approximation to Ohm's law in a low-pressure plasma, implies that the magnetic field is tied to the motion of the electrons. Thus, if an equilibrium is considered in which a current flows so that the electrons have an appreciable zero-order drift velocity relative to the ions, there is a probability that any perturbations in the magnetic field will be convected with the drift velocity of the electrons.

In Ware's paper no problem is solved exactly. It seems worthwhile to give a complete solution of one very simple problem for which the equations can be solved and that is the object of the present paper. The configuration considered is a very simple one which has been much studied previously [2, 3]. A conducting fluid fills a conducting cylindrical tube and contains a uniform axial magnetic field and a uniform axial current density. To obtain exact results the conducting fluid is taken to be incompressible; this should not be serious as it should be possible to study the effects of compressibility and the Hall effect independently.

The results obtained in the present paper are as follows:

1. If the system was previously stable, it is not made unstable by the introduction of the Hall effect.
2. If the system was previously unstable it may now be stable; otherwise the pure instability is replaced by an instability wave. The band of unstable wavenumbers rapidly becomes very small when the ion gyration frequency is much less than the characteristic hydromagnetic frequency (c_H/r_0).
3. As long as overstability occurs, the ratio of the phase velocity of the wave to the electron drift velocity is independent of the ion gyration frequency. An upper limit to the phase velocity is half the electron velocity but this is not usually attained. At the moment detailed numerical results have only been obtained for axisymmetric perturbations; further results are being obtained and will be reported in detail elsewhere. It must be stressed that results have only been obtained for one specially simple problem and have not been shown to have greater generality.

Although the problem as posed has been completely solved, the range of validity of the solutions is not clear. It will be seen later that the condition for the Hall-effect term to be of any importance is that the

---

* Conference paper CN-10/63, presented by R. J. Tayler. Translations of the abstract are at the end of this volume of the Conference Proceedings.