Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1139

Differential Geometric Methods in Mathematical Physics

Proceedings, Clausthal 1983

Edited by H. D. Doebner and J. D. Hennig

Springer-Verlag
Berlin Heidelberg New York Tokyo

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1139

Differential Geometric Methods in Mathematical Physics

Proceedings of an International Conference
Held at the Technical University of Clausthal, FRG,
August 30–September 2, 1983

Edited by H. D. Doebner and J. D. Hennig

Springer-Verlag
Berlin Heidelberg New York Tokyo

Editors

Heinz-Dietrich Doebner
Jörg-Dieter Hennig
Institut für Theoretische Physik A,
Technische Universität Clausthal
3392 Clausthal-Zellerfeld, Federal Republic of Germany

Mathematics Subject Classification (1980): 49H, 53B, 53C, 58C, 58F, 70E, 81C, 81G, 83D, 73C, 55N, 20F.

ISBN 3-540-15666-6 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-15666-6 Springer-Verlag New York Heidelberg Berlin Tokyo

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2146/3140-543210

Lecture Notes in Mathematics

For information about Vols. 1–925, please contact your bookseller or Springer-Verlag.

Vol. 926: Geometric Techniques in Gauge Theories. Proceedings, 1981. Edited by R. Martini and E.M.de Jager. IX, 219 pages. 1982.

Vol. 927: Y. Z. Flicker, The Trace Formula and Base Change for GL (3). XII, 204 pages. 1982.

Vol. 928: Probability Measures on Groups. Proceedings 1981. Edited by H. Heyer. X, 477 pages. 1982.

Vol. 929: Ecole d'Eté de Probabilités de Saint-Flour X – 1980. Proceedings, 1980. Edited by P.L. Hennequin. X, 313 pages. 1982.

Vol. 930: P. Berthelot, L. Breen, et W. Messing, Theorie de Dieudonné Cristalline II. XI, 261 pages. 1982.

Vol. 931: D.M. Arnold, Finite Rank Torsion Free Abelian Groups and Rings. VII, 191 pages. 1982.

Vol. 932: Analytic Theory of Continued Fractions. Proceedings, 1981. Edited by W.B. Jones, W.J. Thron, and H. Waadeland. VI, 240 pages. 1982.

Vol. 933: Lie Algebras and Related Topics. Proceedings, 1981. Edited by D. Winter. VI, 236 pages. 1982.

Vol. 934: M. Sakai, Quadrature Domains. IV, 133 pages. 1982.

Vol. 935: R. Sot, Simple Morphisms in Algebraic Geometry. IV, 146 pages. 1982.

Vol. 936: S.M. Khaleelulla, Counterexamples in Topological Vector Spaces. XXI, 179 pages. 1982.

Vol. 937: E. Combet, Intégrales Exponentielles. VIII, 114 pages. 1982.

Vol. 938: Number Theory. Proceedings, 1981. Edited by K. Alladi. IX, 177 pages. 1982.

Vol. 939: Martingale Theory in Harmonic Analysis and Banach Spaces. Proceedings, 1981. Edited by J.-A. Chao and W.A. Woyczyński. VIII, 225 pages. 1982.

Vol. 940: S. Shelah, Proper Forcing. XXIX, 496 pages. 1982.

Vol. 941: A. Legrand, Homotopie des Espaces de Sections. VII, 132 pages. 1982.

Vol. 942: Theory and Applications of Singular Perturbations. Proceedings, 1981. Edited by W. Eckhaus and E.M. de Jager. V, 363 pages. 1982.

Vol. 943: V. Ancona, G. Tomassini, Modifications Analytiques. IV, 120 pages. 1982.

Vol. 944: Representations of Algebras. Workshop Proceedings, 1980. Edited by M. Auslander and E. Lluis. V, 258 pages. 1982.

Vol. 945: Measure Theory. Oberwolfach 1981, Proceedings. Edited by D. Kölzow and D. Maharam-Stone. XV, 431 pages. 1982.

Vol. 946: N. Spaltenstein, Classes Unipotentes et Sous-groupes de Borel. IX, 259 pages. 1982.

Vol. 947: Algebraic Threefolds. Proceedings, 1981. Edited by A. Conte. VII, 315 pages. 1982.

Vol. 948: Functional Analysis. Proceedings, 1981. Edited by D. Butković, H. Kraljević, and S. Kurepa. X, 239 pages. 1982.

Vol. 949: Harmonic Maps. Proceedings, 1980. Edited by R.J. Knill, M. Kalka and H.C.J. Sealey. V, 158 pages. 1982.

Vol. 950: Complex Analysis. Proceedings, 1980. Edited by J. Eells. IV, 428 pages. 1982.

Vol. 951: Advances in Non-Commutative Ring Theory. Proceedings, 1981. Edited by P.J. Fleury. V, 142 pages. 1982.

Vol. 952: Combinatorial Mathematics IX. Proceedings, 1981. Edited by E. Billington, S. Oates-Williams, and A.P. Street. XI, 443 pages. 1982.

Vol. 953: Iterative Solution of Nonlinear Systems of Equations. Proceedings, 1982. Edited by R. Ansorge, Th. Meis, and W. Törnig. VII, 202 pages. 1982.

Vol. 954: S.G. Pandit, S.G. Deo, Differential Systems Involving Impulses. VII, 102 pages. 1982.

Vol. 955: G. Gierz, Bundles of Topological Vector Spaces and Their Duality. IV, 296 pages. 1982.

Vol. 956: Group Actions and Vector Fields. Proceedings, 1981. Edited by J.B. Carrell. V, 144 pages. 1982.

Vol. 957: Differential Equations. Proceedings, 1981. Edited by D.G. de Figueiredo. VIII, 301 pages. 1982.

Vol. 958: F.R. Beyl, J. Tappe, Group Extensions, Representations, and the Schur Multiplicator. IV, 278 pages. 1982.

Vol. 959: Géométrie Algébrique Réelle et Formes Quadratiques. Proceedings, 1981. Edité par J.-L. Colliot-Thélène, M. Coste, L. Mahé, et M.-F. Roy. X, 458 pages. 1982.

Vol. 960: Multigrid Methods. Proceedings, 1981. Edited by W. Hackbusch and U. Trottenberg. VII, 652 pages. 1982.

Vol. 961: Algebraic Geometry. Proceedings, 1981. Edited by J.M. Aroca, R. Buchweitz, M. Giusti, and M. Merle. X, 500 pages. 1982.

Vol. 962: Category Theory. Proceedings, 1981. Edited by K.H. Kamps D. Pumplün, and W. Tholen. XV, 322 pages. 1982.

Vol. 963: R. Nottrot, Optimal Processes on Manifolds. VI, 124 pages. 1982.

Vol. 964: Ordinary and Partial Differential Equations. Proceedings, 1982. Edited by W.N. Everitt and B.D. Sleeman. XVIII, 726 pages. 1982.

Vol. 965: Topics in Numerical Analysis. Proceedings, 1981. Edited by P.R. Turner. IX, 202 pages. 1982.

Vol. 966: Algebraic K-Theory. Proceedings, 1980. Part I. Edited by R.K. Dennis. VIII, 407 pages. 1982.

Vol. 967: Algebraic K-Theory. Proceedings, 1980. Part II. VIII, 409 pages. 1982.

Vol. 968: Numerical Integration of Differential Equations and Large Linear Systems. Proceedings, 1980. Edited by J. Hinze. VI, 412 pages. 1982.

Vol. 969: Combinatorial Theory. Proceedings, 1982. Edited by D. Jungnickel and K. Vedder. V, 326 pages. 1982.

Vol. 970: Twistor Geometry and Non-Linear Systems. Proceedings, 1980. Edited by H.-D. Doebner and T.D. Palev. V, 216 pages. 1982.

Vol. 971: Kleinian Groups and Related Topics. Proceedings, 1981. Edited by D.M. Gallo and R.M. Porter. V, 117 pages. 1983.

Vol. 972: Nonlinear Filtering and Stochastic Control. Proceedings, 1981. Edited by S.K. Mitter and A. Moro. VIII, 297 pages. 1983.

Vol. 973: Matrix Pencils. Proceedings, 1982. Edited by B. Kågström and A. Ruhe. XI, 293 pages. 1983.

Vol. 974: A. Draux, Polynômes Orthogonaux Formels – Applications. VI, 625 pages. 1983.

Vol. 975: Radical Banach Algebras and Automatic Continuity. Proceedings, 1981. Edited by J.M. Bachar, W.G. Bade, P.C. Curtis Jr., H.G. Dales and M.P. Thomas. VIII, 470 pages. 1983.

Vol. 976: X. Fernique, P.W. Millar, D.W. Stroock, M. Weber, Ecole d'Eté de Probabilités de Saint-Flour XI – 1981. Edited by P.L. Hennequin. XI, 465 pages. 1983.

Vol. 977: T. Parthasarathy, On Global Univalence Theorems. VIII, 106 pages. 1983.

Vol. 978: J. Ławrynowicz, J. Krzyż, Quasiconformal Mappings in the Plane. VI, 177 pages. 1983.

Vol. 979: Mathematical Theories of Optimization. Proceedings, 1981. Edited by J.P. Cecconi and T. Zolezzi. V, 268 pages. 1983.

continued on page 345

Preface

The XII. International Conference of the series "Differential Geometric Methods in Mathematical/Theoretical Physics" (DGM-series) took place at the Institute for Theoretical Physics A, Technical University of Clausthal, Germany F.R., August 30 - September 2, 1983. It was organized by H.D. Doebner, S.I. Andersson (Clausthal) and G. Denardo (Trieste).
The DGM-series was initiated by K. Bleuler and H.D. Doebner 1971 in Bonn, continued in Bonn (1973,1975), Aix-en-Provence (1974,1979), Warsaw (1976), Clausthal (1978,1980), Salamanca (1979), Trieste (1981) and Jerusalem (1982).
The idea of the series is to promote the application of geometrical, analytical and algebraical methods and their interplay, especially differential-geometrical and -topological ones for the modelling of complex physical systems, and to exploit the often hidden geometry and symmetry of such systems. Over the years the conferences gathered quite a large number of prominent researchers in this branch of mathematics/mathematical physics and the field of the DGM series grew considerably. They stimulated also an increasing interest in developing new mathematical techniques in connection with the geometrical structure of physical systems.
The topics of the XII. DGM conference are roughly described by the following key words, which are also the titles of the chapters

- Momentum Mappings and Invariants
- Aspects of Quantizations
- Structure of Gauge Theories
- Non-Linear Systems, Integrability and Foliations
- Geometrical Modelling of Special Systems.

The articles in this volume cover only part of the material presented at the conference (48 lectures). The editors agree with the general editorial requirement of homogeneity in a lecture notes volume, which applies also to these proceedings. Hence it was not possible to include papers with a very strong bias towards physics or papers having definitely the form of a pure research announcement, a pure review or a pedagogical exposition. Some of the manuscripts were not received in time, some of the material is or will be published elsewhere.

Concerning the discussion of the geometrical and topological background of systems which were not in the focus of previous DGM conferences we refer specifically to the lectures of G. Casati, G. and G.A. Lassner, M. Rasetti, M. Epstein et al. and R. Kerner. There are strong indications that such investigations will contribute to the future development of the geometrical approach.

Acknowledgements

We wish to express our gratitude to the following organizations and persons for generous financial support and for other assistance rendering the conference and these proceedings possible

- Der Niedersächsische Minister für Wissenschaft und Kunst
- Alexander von Humboldt-Stiftung
- Deutscher Akademischer Austauschdienst, DAAD
- Deutsche Stiftung für Internationale Entwicklung, DSE
- Technische Universität Clausthal, especially the Rektor Prof.Dr. St. Schottlaender, and its Office for Continuing Education and Foreign Studies, chairman Prof.Dr. H. Quade.

We want also to thank the Springer-Verlag for their kind assistance in matters of publication.
Last but not least we thank Mrs. M. Ilgauds, Institute for Theoretical Physics, Clausthal, for the preparation of this volume and the members and students of the Institute whose help made the organization smooth and efficient.

Clausthal, January 1985
The Editors

TABLE OF CONTENTS

IV. Structure of Gauge Theories

V. Non-Linear Systems, Integrability and Foliations

VI. Geometrical Modelling of Special Systems

I. The Work of Steven M. PANEITZ

Steven M. Paneitz presented his conference contribution on "Sharp Asymptotics of Solutions to the Yang-Mills Equations and the Conformal Connection" in the afternoon session on September 1, 1983. Immediately after his excellent and very well received lecture he went together with other participants for a bath in a small lake near the conference building. While swimming he suddenly got into difficulties, within seconds he sank. Divers found his body in the lake during the night. The next day the participants of the conference paid respect to our departed colleague and friend. It was agreed to dedicate this volume to his memory.

Steven Paneitz was an outstanding and most talented mathematician and mathematical physicist. His death is a heavy loss not only for his family, his friends and collaborators but also for the community of scientists. He visited the Institute for Theoretical Physics in Clausthal several times and participated actively in the last conferences of the series on Differential Geometric Methods.
We will remember him always with honour as a strong young mathematician and as a friend.

H.D. Doebner, J.D. Hennig

PANEITZ was a man of exceptional flexibility and unusual breadth. Although he chose to work mainly in a rather coherent direction in the general field of Functional Analysis and Applications, his publications have a considerable mathematical range. They are all connected with the central theoretical physical problem of developing the mathematical consequences of causality, symmetry, and stability. The very brief description given here will be organized around these themes, and their relations to mathematical field and particle theory.

Causality: He resolved and developed questions about causality in groups and homogeneous spaces that had interested a group at M.I.T., originally in connection with theoretical physical issues. Typically, Paneitz both plumbed the depths of the original issues connected with 4-dimensional space-times, with major surprises in non-uniqueness of causal structures in the local causal group, SU(2,2), and the dependence of the stability (or positive energy) cone for wave equations on their spin; and exhaustively treated the natural generalization to arbitrary semisimple Lie groups.

Stability: He adapted the stability theory of the Krein school to the case of nonlinear invariant wave equations, again both at an abstract level and with penetrating application to interesting particular cases. This work served also to resolve a major outstanding problem in linear quantization theory, as e.g. the case of wave equations on a given curved, non-static, Lorentzian manifold,- the determination of a canonical vacuum, or equivalently, creation and annihilation operators, in addition to the quantized field operators that had earlier been established.

Symmetry: Intensive systematic work on the harmonic analysis of homogeneous vector bundles over space-times applies both to field and particle theory. Not merely a matter of group theory, the spatio-temporal labelling of vectors in the representation spaces is crucial for the formation of local interactions and other physical purposes. PANEITZ' work here verged on the monumental; he eschewed any kind of brilliant display, and his work may in part appear deceptively straightforward, but it ultimately has had, and may well continue to have, remarkable implications not otherwise attainable. Among these, for example, are the finiteness of the integrated action for general solutions of the (Lorentzian!) Yang-Mills equations on Minkowski space; and the self-adjointness of the leading term in the perturbative expansion of the S-matrix for conformally invariant quantized fields in the interaction representation.

Overall, PANEITZ was the most impressive and productive young mathematician I have known. In our joint work he usually contributed more than I did, and especially to precision and completeness. In any event, his accidental death at the age of 28 was a terrible and really significant loss to the mathematical community.

I.E. Segal

PUBLICATIONS OF STEPHEN M. PANEITZ

1. Unitarization of symplectics and stability for causal differential equations in Hilbert space. J. Funct. Anal. 41 (1981), 315-326.

2. Invariant convex cones and causality in semisimple Lie algebras and groups. J. Func. Anal. 43 (1981), 313-359.

3. Quantization of wave equations and hermitian structures in partial differential varieties. Proc. Natl. Acad. Sci. USA 77 (1980), 6943-6947. (With I.E. Segal.)

4. Essential unitarization of symplectics and applications to field quantization. J. Func. Anal. 48 (1982), 310-359.

5. Covariant chronogeometry and extreme distances: Elementary particles. Proc. Natl. Acad. Sci. 78 (1981). (With I.E. Segal, H.P. Jakobsen, B. Ørsted, and B. Speh.)

6. Analysis in space-time bundles. I. General considerations and the scalar bundle. J. Func. Anal. 47 (1982), 78-142. (With I.E. Segal.)

7. Analysis in space-time bundles, II. The spinor and form bundles. J. Func. Anal. 49 (1982), 335-414.

8. Self-adjointness of the Fourier expansion of quantized interaction field Lagrangians. Proc. Natl. Acad. Sci. USA 80 (1983), 4595-4598. (With I.E. Segal.)

9. The Yang-Mills equations on the universal cosmos. J. Func. Anal. 53 (1983), 112-150. (With Y. Choquet-Bruhat and I.E. Segal.)

10. Determination of a polarization by nonlinear scattering, and examples of the resulting quantization. Lec. Notes in Math. No. 1037, Ed. S.I. Andersson and H.D. Doebner (Proceedings, Clausthal, 1981), Springer-Verlag, Berlin, 1983.

11. Determination of invariant convex cones in simple Lie algebras. Arkiv f. mat. 21 (1983), 217-228.

12. All linear representations of the Poincaré group up to dimension 8. Ann. Inst. H. Poincaré (Phys. Theor.) 40 (1984), 35-57.

13. Parametrization of causal actions of universal covering groups and global hyperbolicity. J. Func. Anal., in press.

14. Analysis in space-time bundles. III. Higher spin bundles. J. Func. Anal. 54 (1983), 18-112.

15. Global solutions of the hyperbolic Yang-Mills equations and their sharp asymptotics. Proceedings of the Amer. Math. Soc. Summer Institute on Nonlinear Functional Analysis and Applications (Berkeley, 1983), in press.

16. Indecomposable finite dimensional representations of the Poincaré group and associated fields. These proceedings (Clausthal, 1983).

17. Indecomposable representations of the Poincaré group and associated fields. Proc. XII. International Coll. Group Theoretical Methods in Physics, Trieste, 1983 (Posth. presentation), Lecture Notes in Physics, Vol. 201 (1984), 84-87.

INDECOMPOSABLE FINITE DIMENSIONAL REPRESENTATIONS OF THE POINCARE GROUP AND ASSOCIATED FIELDS

Stephen M. Paneitz
Department of Mathematics
Massachusetts Institute of Technology
Cambridge, MA 02139 USA

Introduction

The idea that 'true' space-time may deviate in the large from Minkowski space without sacrificing group-covariance has led to an empirically accurate, parameter-free, and theoretically satisfying description of cosmological phenomena such as the redshift [1,2]. However, the idea also has implications noted long ago [3] for fundamental particle physics (where group-covariance is essential) that are thus far less quantitatively clear-cut and are currently being explored [4,5]. One facet of this program that is relevant here concerns an apparently more rigid and equally natural model [6] for the fundamental fermions, which differs from the standard spinor fields yet mathematically deforms into them as an invariant distance unit R, interpretable as the 'radius of the universe', tends to ∞.

Fundamental fields are presumed to transform under the 15-dimensional causal group $\tilde{G}$ of space-time $\tilde{M} \simeq R^1 \times SU(2)$. The group representation is assumed to be an induced representation, meaning that the transformation rules for the fields are completely determined by the transformation of field values (assumed to lie in a finite-dimensional space) at a point $p \in \tilde{M}$ under the isotropy group $G_p \subset \tilde{G}$ at that point. Now the isotropy group of $0 \times -I$ coincides with the Poincare group extended by scale transformations (an 11-dimensional group denoted $\underline{P}$) acting on Minkowski space M_0, such that M_0 is regarded as embedded in $\tilde{M}$ by e.g. conformal compactification and covering transformations (cf.[4, Part I]).

Thus $\tilde{G}$-covariant fields are determined by inducing representations R of $\underline{P} \simeq (R^1 \times SL(2,C)) \tilde{\times} H(2)$ ($\tilde{x}$ = semi-direct product, H(2) = 2×2 hermitian matrices). Conventionally R is assumed trivial on the H(2)-subgroup, representing the translations when the inducing point is the 'point at infinity' $0\times -I$. Yet from a more conservative position that also notes the large distance scale rendering the action of accessible translations relatively unobservable, this assumption may be questioned. The next section shows that the mathematical possibilities for representations of $\underline{P}$ restricting to given representations of SL(2,C) (determined by spins (s_+,s_-), s_+,s_- half-integral) is highly restricted [7]. For example, there is a unique representation of $SL(2,C) \tilde{\times} H(2)$ (up to contragredience) restricting to the direct sum of half-spin representations $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$; fields induced from this incompletely reducible representation on C^4 with a suitable conformal dimension or weight have been dubbed spannor fields (a 'wrenched' spinor, as via a spanner [6]). A means of determining the 'special' conformal weights (defined below) is sketched in the third section.

Determination of Indecomposable Representations of $\underline{P}_0$

According to the first result (cf.[7,8]), any finite-dimensional representation of the Poincare group $\underline{P}_0$ may be put in block-upper-triangular form.

Theorem 1. Let $\underline{g}$ be any real finite-dimensional Lie algebra such that $[\underline{g},\underline{n}] = \underline{n}$, where $\underline{n}$ is the maximal solvable ideal of $\underline{g}$. Let $\underline{h}$ be any semisimple subalgebra of $\underline{g}$ complementary to $\underline{n}$. Then, given any finite-dimensional representation ρ of $\underline{g}$ in a complex vector space V, there exists a direct sum decomposition $V = V_1 + \cdots + V_n$ such that the V_j are invariant and irreducible under $\rho(\underline{h})$ and such that

$$\rho(\underline{n})\, V_j \subseteq \sum_{k<j} V_k \qquad \text{for } j = 1, \ldots, n.$$

Thus a finite-dimensional representation ρ of $\underline{P}_0 \simeq SL(2,C) \tilde{\times} H(2)$ is determined by a finite sequence of spins $\{(s_j^+, s_j^-)\}_{j=1}^n$ and maps ρ_{ij} (for $1 \leq i < j \leq n$) of $H(2)$ into the space of linear transformations from the jth subspace into the ith subspace (defining the infinitesimal representation). Extending the results of [7] somewhat, it may be shown that, for any ρ, such a ρ_{ij} depends only on the spins (s_i^+, s_i^-) and (s_j^+, s_j^-), up to a scalar factor. Moreover we have

Theorem 2. ρ_{ij} equals zero unless

$$|s_i^+ - s_j^+| = \tfrac{1}{2} \quad \text{and} \quad |s_i^- - s_j^-| = \tfrac{1}{2} . \qquad (1)$$

Naturally these constraints on the ρ_{ij} are equivalent to the commutation relations between the generators of the SL(2,C) and H(2) subgroups of $\underline{P}_0$. The remaining constraints on ρ due to commutativity of $\{d\rho(0 \tilde{+} F): F \in H(2)\}$ have been computed using explicit matrix elements for standardized ρ_{ij}'s whose corresponding spins satisfy (1); however, these are less succinctly presented. Recall that a representation is indecomposable if it is not a nontrivial direct sum. By ad hoc and additional argument we have

Theorem 3. Any indecomposable representation of the Poincare group that is nontrivial on the translations and of dimension ≤ 9 is, up to duality, complex-conjugation, or anti-duality, linearly equivalent to a unique such representation of dimensions 4,5,6, or 7, or either of two 8-dimensional representations. The sequences of spins characterizing their restrictions to the homogeneous Lorentz group are, respectively, $(\frac{1}{2},0) + (0,\frac{1}{2})$, $(0,0) + (\frac{1}{2},\frac{1}{2})$, $(0,0) + (\frac{1}{2},\frac{1}{2}) + (0,0)$, $(0,1) + (\frac{1}{2},\frac{1}{2})$, $(0,0) + (1,0) + (\frac{1}{2},\frac{1}{2})$, and $(\frac{1}{2},0) + (1,\frac{1}{2})$.

Furthermore, there are only finitely many inequivalent indecomposable representations of $\underline{P}_0$ of dimension smaller than 24, and a continuum of inequivalent 24-dimensional representations. A complete determination of all such representations of $\underline{P}_0$ (or at least of the simplest, prototypical group $SL(2,R) \tilde{\times} R^2$) is not derivable by standard group-theoretical methods at present, but appears within reach.

Decomposition of Corresponding Induced Fields

Before induction to $\tilde{G}$, the representations of $\underline{P}_0$ must be extended to $\underline{P}$. As noted in [8], one convenient way to do this, given the decomposition $V_1 + \cdots + V_n$ under $SL(2,C)$ as in Theorem 1, is by assignment of a constant w_j (conformal weight) to each V_j such that $w_i = w_j + 1$ whenever $\rho_{ij} \neq 0$. Denote the resulting representation of $\underline{P}$ also by ρ.

Now define the system of conformal weights w_j (determined by $w=w_n$ if ρ is indecomposable) to be special if the representation of $\tilde{G}$ induced from ρ', where $\rho' = \rho$ on the $R^1 \times SL(2,C)$ subgroup but $\rho' = \mathrm{id}$ on the translation subgroup, is not equivalent to that induced from ρ. In fact, the set of special conformal weights is a finite set. This follows from the value of the suitably normalized second-order Casimir of $\tilde{G}$ acting on fields induced from an irreducible representation of $\underline{P}$ of spins (s_+,s_-), conformal weight w, and trivial on the translation subgroup, which value is $w(w-4) + 2s_+(s_+ +1) + 2s_-(s_- +1)$ [4, Part III].

For example, the spannor fields are induced from an indecomposable representation of $\underline{P}$ denoted $(w+1,(\frac{1}{2},0)) + (w,(0,\frac{1}{2}))$. Then evidently w is special when $(w+1)(w-3) = w(w-4)$, i.e. $w = 3/2$, recovering the conventional canonical dimensions 3/2 and 5/2 in a more rigid and structured context. For generic w the intertwining operator from Ind ρ' to Ind ρ is a differential operator, essentially the Dirac operator, which becomes singular when $w=3/2$. The special conformal weights for the 'wrenched' scalar and vector fields induced from the 5- and 6-dimensional indecomposable representations of $\underline{P}$ are familiar integral weights.

REFERENCES

1. I.E. Segal, "Mathematical Cosmology and Extragalactic Astronomy," Academic Press, New York, 1976.

2. I.E. Segal, Comment on paper of L. Wormald, J. Rel. Grav., in press.

3. I.E. Segal, A class of operator algebras determined by groups, Duke Math. J. 18 (1951), pp. 221-265.

4. S.M. Paneitz and I.E. Segal, Analysis in space-time bundles, Parts I and II, J. Func. Anal. 47 (1982) pp. 78-142 and 49 (1982) pp. 335-414; S.M. Paneitz, Part III, J. Func. Anal., in press.

5. I.E. Segal, Covariant Chronogeometry and Extreme Distances III, Int. J. Th. Phys. 21 (1982), pp. 851-869.

6. I.E. Segal, Chronometric cosmology and fundamental fermions, Proc. Nat. Acad. Sci. USA 79 (December 1982), pp. 7961-2.

7. S.M. Paneitz, All linear representations of the Poincare group up to dimension 8, Ann. Inst. H. Poincare (Th. Phys.), in press.

8. G. Mack and A. Salam, Finite-component field representations of the conformal group, Ann. Phys. 53 (1969), pp. 174-202.

Comment of the editors:

Steven M. PANEITZ considered two topics for his talk at this conference:

"Sharp Asymptotics of Solutions to the Yang-Mills Equations and the Conformal Connection"

and

"Indecomposable finite dimensional Representations of the Poincaré Group and associated Fields".

He decided to report on the first one. As we have only a written manuscript of the second one, we add this to the proceedings.