MATHEMATICS FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

S. T. TAN

Mathematics for the Managerial, Life, and Social Sciences

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Stonehill College

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PREFACE

Mathematics for the Managerial, Life, and Social Sciences is designed for students in business, management, economics, and the social and life sciences who have completed at least one year of high school algebra. The text is appropriate for use in mathematics courses that include topics from algebra, finite mathematics, and calculus. The author's objective in writing this book is to provide a textbook that is both readable by students and useful as a teaching tool for instructors. The text's approach is both problem-solving and applications-oriented. Wherever possible, mathematical concepts are presented intuitively and in the context of real-life settings to make them more meaningful to the students. Numerous examples and solved problems are used to amplify each new concept or result, in order to facilitate students' comprehension of the material. Graphs and pictures are used extensively to help students visualize the concepts and ideas presented.

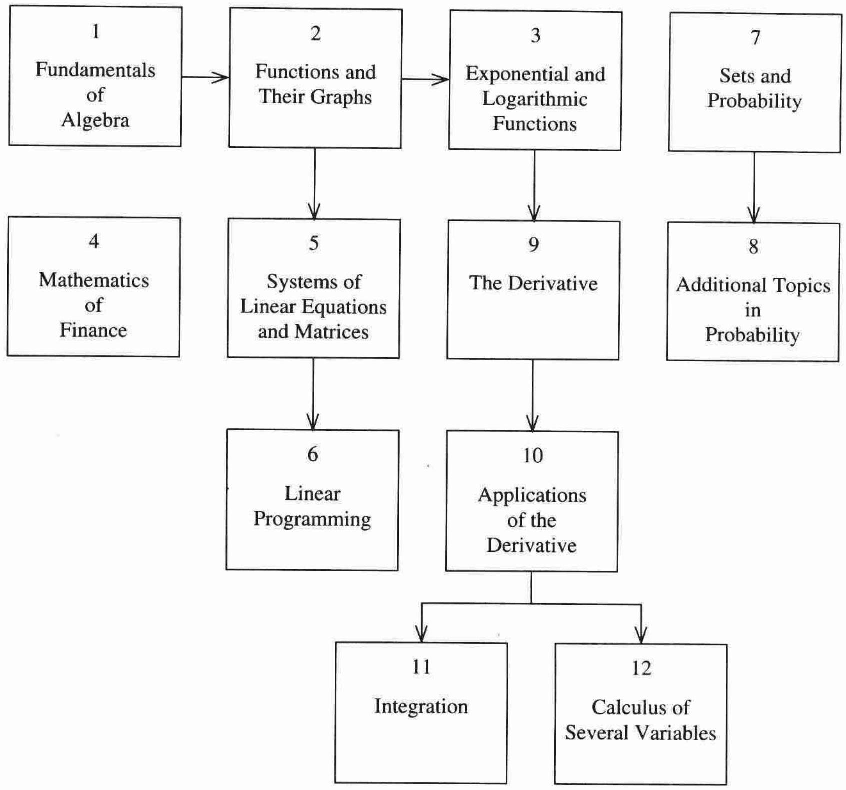
Text Features

Coverage of topics As the book contains more material than needed for a traditional two-semester or three-quarter course, the instructor may be flexible in choosing the topics most suitable for his or her course. The chart on page xiv on chapter dependency is provided to help the instructor design the most appropriate course.

Use of technology is optional. Discussion is placed at the end of appropriate sections, which are highlighted in the table of contents for easy reference. A technology index is also provided, written in a user-friendly "How Do I" format. Throughout the text, examples employing the use of technology are highlighted by an icon. Appendix A provides step-by-step instructions for the TI-85 graphing calculator for students who may need assistance. Although the TI-85 is used as a model, the technology is not limited to this calculator.

The presentation in this text is unique in that the use of technology is encouraged after the students have gained a firm grasp of concepts, much as grade-school students learn arithmetic before using calculators to do mathematical computations. Rather than simply teaching students to push buttons, the graphing calculator is presented as a tool for enhancing students' understanding of concepts and to facilitate computing. Whenever a choice of methods is available to solve a problem using a graphics calculator, the graphical method is emphasized. This approach gives the student an opportunity to "see" what he or she is doing.

The technology sections are designed to allow the instructor the flexibility of either using them in the classroom or allowing the students to work through them independently. These sections are written in the traditional example-exercise format (with answers provided at the back of the book),



allowing students to immediately test themselves to see if they are using the calculator correctly. The technology sections also contain real-life applications, often referenced by sources. The students can readily see how problems of a more complex nature can be solved using a graphing calculator. Freed of cumbersome computations, the students can easily interpret their results, further enhancing their understanding of the concepts involved.

Real-life applications and examples are drawn from fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest to students. These applications and examples are drawn from newspapers, magazines, and other weekly periodicals, and sources are noted. For ease in locating applications relevant to specific fields, they have been titled.

Self-tests are placed throughout the text to help students to monitor their own progress.

Remarks help clarify discussion and point out certain subtleties in the material being presented.

Caution symbols ⚠ alert students to common errors and pitfalls.

Portfolios contain accounts of professionals who relate their real-world experiences and how they use mathematics in their professions.

Extended problems and projects are found at the end of each chapter. Here students are presented with questions that require more in-depth thinking and in many instances the discovery of mathematical results related to the concepts presented in the chapter. Careful effort has been taken to keep these problems and projects at a level that is both doable by students and at the same time challenging to them.

Exercise sets contain an ample set of routine problems, computational in nature, to help students master newly learned techniques. These are followed by an extensive array of application-oriented problems that will test their mastery of the topics. Critical-thinking questions requiring the interpretation of results and graphs are also found in each section. The answers to selected exercises that require written answers are included in the answer section as a further aid to the student, as experience has taught the author that many students need help getting started with this type of question. Solutions to the self-check exercises are found at the end of each exercise set. Answers to both even and odd exercises in the review sections are included in the answer section.

Supplements

- *Student's Solutions Manual,* available to both students and instructors, includes the solutions to odd-numbered exercises.
- *Instructor's Complete Solutions Manual* includes solutions to all exercises.
- *Test Bank with Chapter Tests,* free to adopters of the book, contains sample tests for each chapter.
- *EXPTest 6.0* and *Chariot MicroTest III.* With these computerized testing systems for IBM PCs and Macintosh, respectively, instructors can select or modify questions from prepared test banks or add their own test items to create any number of tests. These tests can be viewed on screen and printed with typeset-quality mathematical symbols, notation, graphs, and diagrams.
- *Graphing Calculator Supplement,* by Robert E. Seaver of Lorain County Community College, is available to both students and instructors. The manual develops selected examples and exercises and also includes additional problems for reinforcement. It is specifically written for use with the TI line of programmable graphics calculators.

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Fundamentals of Algebra

This chapter contains a brief review of the algebra you will need for this course. In solving many practical problems, it is often necessary to simplify algebraic expressions before moving on to the next step. Thus, it is important that you reestablish familiarity with the process. This chapter also contains a short review of inequalities and absolute value, the uses of which range from describing the domains of functions to the formulation of practical problems.

1.1 Real Numbers

The Set of Real Numbers

We use *real numbers* everyday to describe quantities such as temperature, salary, annual percentage rate, shoe size, grade point average, and so on. Some of the symbols we use to represent real numbers are

$$3, \quad -17, \quad \sqrt{2}, \quad 0.666\ldots, \quad 113, \quad 3.9, \quad 0.12875$$

To construct the set of real numbers, we start with the set of **natural numbers** (also called counting numbers)

$$N = \{1, 2, 3, \ldots\}$$

and adjoin other numbers to it. The set

$$W = \{0, 1, 2, 3, \ldots\}$$

of **whole numbers** is obtained by adjoining the single number 0 to N. By adjoining the negatives of the natural numbers to the set W, we obtain the set of **integers**

$$I = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Next, we consider the set Q of **rational numbers,** numbers of the form a/b, where a and b are integers, with $b \neq 0$. Using set notation, we write

$$Q = \{a/b \mid a \text{ and } b \text{ are integers}, \quad b \neq 0\}$$

Observe that I is contained in Q since each integer may be written in the form a/b, with $b = 1$. For example, the integer 6 may be written in the form 6/1. Symbolically, we express the fact that I is contained in Q by writing

$$I \subset Q$$

However, Q is not contained in I since fractions such as 1/2 and 23/25 are not integers. To show the relationships of the sets N, W, I, and Q, we write

$$N \subset W \subset I \subset Q$$

which says that N is a proper subset of W, W is a proper subset of I, and so on.* (See Figure 1.)

Finally, we obtain the set of real numbers by adjoining the set of rational numbers to the set of **irrational numbers** (Ir)—numbers that cannot be expressed in the form a/b, where a, b are integers ($b \neq 0$). Examples

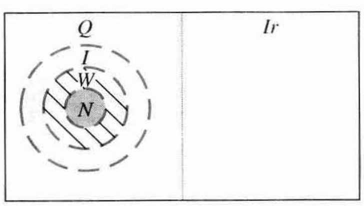

FIGURE 1
The set of all real numbers consists of the set of rational numbers plus the set of irrational numbers.

Q = Rationals
I = Integers
W = Whole numbers
N = Natural numbers
Ir = Irrationals

*A set A is a proper subset of a set B if every element of a set A is also an element of a set B and there exists at least one element in B that is not in A.

of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, π, and so on. Thus, the set

$$R = Q \cup Ir$$

comprising all rational numbers and irrational numbers is called the set of **real numbers.**

Representing Real Numbers as Decimals

Every real number can be written as a decimal. A rational number can be represented as either a repeating or terminating decimal. For example, 2/3 is represented by the repeating decimal

$$0.66666666\ldots \quad \text{Repeating decimal—note that the integer 6 repeats.}$$

which may also be written $0.\overline{6}$, where the bar above the 6 indicates that the 6 repeats indefinitely. The number 1/2 is represented by the terminating decimal

$$0.5 \quad \text{Terminating decimal}$$

When an irrational number is represented as a decimal, it neither terminates nor repeats. For example,

$$\sqrt{2} = 1.41421\ldots \quad \text{and} \quad \pi = 3.14159\ldots$$

Table 1 summarizes this classification of real numbers.

TABLE 1
The set of real numbers

Set	Description	Examples	Decimal Representation
Natural numbers	Counting numbers	$1, 2, 3, \ldots$	Terminating decimals
Whole numbers	Counting numbers and 0	$0, 1, 2, 3, \ldots$	Terminating decimals
Integers	Natural numbers, their negatives, and 0	$\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$	Terminating decimals
Rational numbers	Numbers that can be written in the form a/b, where a and b are integers and $b \neq 0$	$-3, -3/4, -0.22\overline{2}, 0, 5/6, 2, 4.311\overline{1}$	Terminating or repeating decimals
Irrational numbers	Numbers that cannot be written in the form a/b, where a and b are integers and $b \neq 0$	$\sqrt{2}, \sqrt{3}, \pi$ $1.414213\ldots,$ $1.732050\ldots,$	Nonterminating, nonrepeating decimals
Real numbers	Rational and irrational numbers	All of the above	All types of decimals

The Real Number Line

Real numbers may be represented geometrically by points on a line. This *real number,* or *coordinate, line* is constructed as follows: Arbitrarily select a point on a straight line to represent the number zero. This point is called the *origin.* Choose a point at a convenient distance to the right of the origin to represent the number 1. This determines the scale for the number line.

The point representing each positive real number x lies x units to the right of zero, and the point representing each negative real number x lies $-x$ units to the left of zero. Thus, real numbers may be represented by points on a line in such a way that corresponding to each real number there is exactly one point on a line, and vice versa. In this way, a *one-to-one correspondence* is set up between the set of real numbers and the set of points on the number line, with all the positive numbers lying to the right of the origin and all the negative numbers lying to the left of the origin (Figure 2).

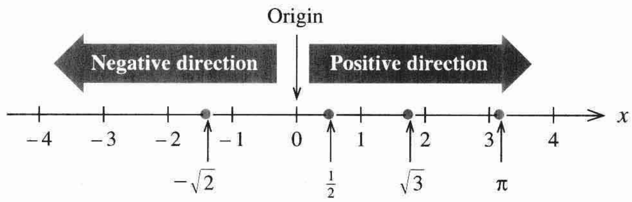

FIGURE 2
The real number line

Operations with Real Numbers

Two real numbers may be combined to obtain a real number. The operation of addition, written +, enables us to combine any two numbers a and b to obtain their *sum,* denoted by $a + b$. Another operation, multiplication, written $\cdot$, enables us to combine any two real numbers a and b to form their product, the number $a \cdot b$ (more simply written ab). These two operations are subject to the rules of operation given in Table 2.

TABLE 2
Rules of operation for real numbers

Rule		Illustration
I. Under addition		
1. $a + b = b + a$	Commutative law of addition	$2 + 3 = 3 + 2$
2. $a + (b + c) = (a + b) + c$	Associative law of addition	$4 + (2 + 3) = (4 + 2) + 3$
3. $a + 0 = a$	Identity law of addition	$6 + 0 = 6$
4. $a + (-a) = 0$	Inverse law of addition	$5 + (-5) = 0$
II. Under multiplication		
1. $ab = ba$	Commutative law of multiplication	$3 \cdot 2 = 2 \cdot 3$
2. $a(bc) = (ab)c$	Associative law of multiplication	$4(3 \cdot 2) = (4 \cdot 3)2$
3. $a \cdot 1 = 1 \cdot a$	Identity law of multiplication	$4 \cdot 1 = 1 \cdot 4$
4. $a(1/a) = 1 \quad (a \neq 0)$	Inverse law of multiplication	$3(\frac{1}{3}) = 1$
III. Under addition and multiplication		
1. $a(b + c) = ab + ac$	Distributive law for multiplication with respect to addition	$3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$

The operation of subtraction is defined in terms of addition. Thus,

$$a + (-b)$$

where $-b$ is the additive inverse of b, may be written in the more familiar form $a - b$, and we say that b is subtracted from a. Similarly, the operation of division is defined in terms of multiplication. Recall that the multiplicative inverse of a nonzero real number b is $1/b$, also written b^{-1}. Then,

$$a(1/b)$$

is written a/b, and we say that a is divided by b. Thus, $4(1/3) = 4/3$. Remember, 0 does not have a multiplicative inverse since division by zero is not defined.

Do the operations of associativity and commutativity hold for subtraction and division? Looking first at associativity, we see that the answer is no since

$$a - (b - c) \neq (a - b) - c \qquad 7 - (4 - 2) \neq (7 - 4) - 2, \text{ or } 5 \neq 1$$

and

$$a \div (b \div c) \neq (a \div b) \div c \qquad 8 \div (4 \div 2) \neq (8 \div 4) \div 2, \text{ or } 4 \neq 1$$

Similarly, commutativity does not hold because

$$a - b \neq b - a \qquad 7 - 4 \neq 4 - 7, \text{ or } 3 \neq -3$$

and

$$a \div b \neq b \div a \qquad 8 \div 4 \neq 4 \div 8, \text{ or } 2 \neq \tfrac{1}{2}$$

EXAMPLE 1 State the real number property that justifies each statement:

Statement	Property
a. $4 + (x - 2) = 4 + (-2 + x)$	The commutative law of addition
b. $(a + 2b) + c = a + (2b + c)$	The associative law of addition
c. $x(y - z + 2) = (y - z + 2)x$	The commutative law of multiplication
d. $4(xy^2) = (4x)y^2$	The associative law of multiplication
e. $x(y - 2) = xy - 2x$	The distributive law for multiplication under addition ■

Using the properties of real numbers listed earlier, we can derive all the other algebraic properties of real numbers. Some of the more important properties are given in Tables 3–5.

TABLE 3
Properties of negatives

Property	Illustration
1. $-(-a) = a$	$-(-6) = 6$
2. $(-a)b = -(ab) = a(-b)$	$(-3)4 = -(3 \cdot 4) = 3(-4)$
3. $(-a)(-b) = ab$	$(-3)(-4) = 3 \cdot 4$
4. $(-1)a = -a$	$(-1)5 = -5$