Methods in

RELATIVISTIC NUCLEAR PHYSICS

Michael DANOS

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Preface

When Yukawa introduced the mesons in order to explain the short range character of the nuclear forces, the field of nuclear physics split into two parts: nuclear structure and nuclear forces. Nuclear structure developed into non-relativistic nuclear physics and led to the creation of various models to describe the emerging wealth of nuclear data. The field of nuclear forces developed into high-energy particle physics with its own immense body of phenomena and data.

For most nuclear phenomena the non-relativistic framework is fully adequate. However, this framework is too narrow in phenomena associated with exchange currents and with high-momentum transfers studied with intermediate energy nuclear accelerators. In such cases the presence of particles other than protons and neutrons in the nuclei must be explicitly accounted for. Since the concept of nuclear forces is a strictly non-relativistic construct, it must be abandoned and the forces must be replaced explicitly by their physical origin, i.e., by the interaction between baryons and mesons or, at a deeper level, between quarks and gluons.

When actually attempting to follow this program, namely the description of the nucleus as a system of interacting quantized fields, one is immediately confronted by the difficulties related to combining relativistic invariance with the fundamental characteristics of bound many-body systems such as: spatial localization, conservation and coupling of angular momenta, center-of-mass motion, etc. In order to handle these problems new tools and methods had to be devised. They are the subject of this book. Thus, in this work, the description of a bound many-body system has been formulated as a problem of relativistic quantum field theory which is solved by nuclear physics methods. Even though these tools and methods have been developed in the nuclear context, they are fully general and applicable to the description of any bound quantum systems composed both of relativistic and non-relativistic particles.

We are indebted to many colleagues for useful exchange of views on this new subject. In particular L.C. Biedenharn always has been helpful in elucidating subtle points of mathematical physics. Our special thanks go to R. Hayward for teaching us the treatment of higher spin fields and for the permission to use his material, some of which is still unpublished. We also thank T. Kohmura and T. Suzuki for useful comments and discussions, and for their contribution to the Appendix.

The authors are very grateful to Mrs. Paulette Gugenberger, head of the documentation group of the Department of Nuclear Physics of Saclay, for her dedicated assistance in preparing the manuscript. We owe a special gratitude to Mrs. Eliane Thureau, a member of this group, who performed miracles in typing the manuscript from the sometimes almost unintelligible handwritten pages provided her by the authors. Her patience and competence was essential in allowing us to keep within the Editor's schedule. We thank also Mrs. Marie-Odile Reuter for bringing her personal touch and esthetic feeling to the otherwise rather arid figures.

We have done our best to eliminate errors from the formulae. It is not likely that all have been found. Consequently our thanks also go to the Reader for finding and communicating them to us.

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CHAPTER 1

Introduction

1.1. SCOPE OF THE WORK

The present work is intended to provide the methods and tools for performing actual calculations for finite many-body systems of bound relativistic constituent particles. Of course, an extensive literature exists on the subject of the many-body problem. For finite systems all these treatments are given in a non-relativistic framework. Meanwhile the need has been growing for extending these treatments to relativistic dynamics, i.e. to quantum field theory. However, that subject is always treated in the covariant framework which is well suited for exhibiting the underlying principles and for the treatment of few-body reactions. It is not easily applicable to many-body bound states.

Our aim is to cover thoroughly the methodological aspects of the relativistic many-body problem for bound states while avoiding the presentation of specific models. The many examples contained in the later part of the work are meant to give concrete illustrations of how to actually apply the methods which are given in the first part. The choice of the applications was governed by the aim to provide specific examples for the problems which arise in the context of relativistic nuclear physics with obvious extension to sub-nuclear as well as to atomic physics. Thus we shall often name, but only for convenience, the diverse fields by the mesonic or nucleonic degrees of freedom they may represent. For example, the interaction between a vector field and a spinor field here is called the vector meson-nucleon interaction. However, the methods and results are fully general and they can, of course, be applied to other situations, for example, to the gluon-quark interaction upon the introduction of the needed SU(3) recoupling coefficients.

The difficulties which have hampered the development of relativistic calculations for bound finite many-body systems, besides the problem of divergences, were of several kinds. They belong to the following families: the relativistic center of mass, the discretization of fields, and the technical difficulties of relativistic angular momentum calculus. The first two difficulties are related to the lack of a central potential as it is provided in atomic physics by the Coulomb field of a massive non-relativistic source, the nucleus. Likewise in non-relativistic nuclear physics the shell-model potential is used to provide a discrete representation. Of course in a relativistic treatment no potential can be used as a starting point of the problem. Concerning the center-of-mass motion, its elimination is linked to the restoration of translational invariance and in relativistic kinematics this leads to non-trivial many-body operators. One conventional technique is that of the Bethe-Salpeter equation, which is hard to apply to systems containing more than two fermions. Finally, the handling of angular momentum is also a formidable stumbling block in relativistic calculations. In particular, relativistic angular momentum is frequently formulated in the helicity representation. Helicity is well suited for two-body scattering problems, but it does not lend itself simply to many-body angular momentum coupling.

This work addresses itself to providing the methods and tools for treating the above problems. In it the reader will find within the framework of quantum field theory:

(a) a representation of the fields in terms of multipoles and discretized states;

(b) a consistent angular momentum technique, which may as well be used in the non-relativistic case, but whose power makes it particularly appropriate in the more difficult relativistic case;

(c) the evaluation and a tabulation of the elementary vertices of the usual interactions between bosons and fermions;

(d) the rules for systematically generating relativistic many-body matrix elements, applied for illustration purposes to systems containing up to two fermions with a meson cloud of several pions and of one heavy vector or scalar meson;

(e) the treatment of the coupling of many-body systems to an external electromagnetic field, with explicit applications to the cases above;

(f) the method for performing relativistic boosts of the non-covariant wave functions;

(g) the method for removing the center-of-mass energy and for calculating the associated matrix elements of the center-of-mass many-body operator, and again a tabulation of matrix elements derived to provide the reader with concrete examples.

1.2. ORGANIZATION AND CONTENT OF THE CHAPTERS

The basic framework of our approach is the lagrangian field theory solved in the time-independent Schrödinger picture. The opening Chapter 2 recalls the main definitions and results of quantum field theory and the forms of the basic free field solutions for the cases of 0, $\frac{1}{2}$, 1 and higher spins. The general form of the simplest interaction terms between the basic fields is also presented as well as the minimal coupling, the anomalous moment coupling, the vector-dominance coupling with an external electromagnetic field. These interactions are those which most commonly appear in nuclear physics problems. The extension to other interactions follows along similar lines. Some examples of such extensions are discussed, namely interactions involving the spin-$\frac{3}{2}$ field and interactions between Yang-Mills fields allowing application to quantum chromodynamics.

The secular problem of field theory in the Schrödinger picture is treated in Chapter 3. To actually solve this problem one requires a discretized representation of the free fields. Here, a discretization cannot be performed by the introduction of an external central potential as it is done in non-relativistic shell-model calculations, or as it exists in the relativistic hydrogen atom. The usual shell-model potential is not a relativistic concept. In a potential a system is accelerated and its different parts experience different accelerations because of retardation effects. They arise as a consequence of the transformation of the time coordinate from the laboratory to the rest system. This leads to deformations of the system, i.e. to admixture of different eigenstates, for example baryon resonances if the system is a baryon. No such effect occurs in non-relativistic mechanics for a harmonic potential. There the system can be accelerated as a whole. Analogous difficulties beset the classical relativistic model of an extended electron. Therefore, the discretization here is carried out by a unitary transformation applied to the plane wave solutions of the free field equations of motion.

All quantities can be expressed in terms of the discretized fields. In particular, the solutions of the full hamiltonian are expanded on configurations made up of these discretized fields. Therefore the secular problem is now represented by a discrete matrix equation. These equations can be solved numerically after giving a suitable truncation prescription for the Hilbert space.

Of course the solutions of the resulting finite hamiltonian matrices are unitary and non-divergent. They are, however, functions both of the input parameters, i.e., masses and coupling constants, and of the size of the retained Hilbert space. Divergences appear in this approach as the size of the retained Hilbert space is increased. This point is also discussed in Chapter 3. There a comparison is made with the time-dependent perturbation method. Finally, Chapter 3 describes a method for treating the relativistic center-of-mass problem. This method consists in introducing an auxiliary harmonic central potential for the c.m. motion, in analogy with a well-known method of the non-relativistic shell model. However, because of the reasons discussed above special care must be exercised in applying this method in the relativistic context to avoid the mixing up of eigenstates when removing the c.m. motion.

Most of the technical tools are given in Chapter 4. They include a graphical method for angular momentum recoupling, phase conventions which are uniform for all tensorial quantities, extension of the tensorial formalism to the creation and annihilation operators in the Fock space, treatment of symmetrization and anti-symmetrization of coupled tensors, etc. These techniques are central for performing the involved calculations required for relativistic bound systems. They are employed in all the examples developed in this book. Of course, they are applicable to a much wider range of problems. For example, they can be used to greatly simplify the usual non-relativistic many-body shell-model calculations. We also show how they can be extended to the treatment of SU(n) tensors, for example SU(3) of chromodynamics.

The main reason for the new phase definitions introduced in Chapter 4 is to simplify phase factors in the expressions associated with angular momentum calculus, such as in the Wigner-Eckart theorem or in the reduction and recoupling of tensorial products. It is this property of the adopted phase convention which permits the introduction of a remarkably simple graphical method for handling angular momentum problems. Thus throughout the book diagrams will replace all intermediate algebraic calculational steps and will allow the reader to obtain directly from the graphs the final expressions given in the text. The many examples presented in this book will allow the reader to acquire familiarity with the graph method.

The phase convention for creation and annihilation operators has always been a delicate matter because of the interrelationship of time reversal and hermitian conjugation. This is easily taken care of within our adopted phase convention. Consequently the graphical technique is directly applicable also to field operators and state vectors. This way the explicit symmetrization and antisymmetrization of coupled fields is no more difficult than that of the usual *c*-number wave functions.

Chapter 5 contains the expressions for the discretized basis fields in invariant tensorial form for spins 0, $\frac{1}{2}$, 1 and $\frac{3}{2}$. It also briefly presents the definition of models. The discretization of the fields involves the expansion of the fields into multipoles of good total angular momentum and it is performed by introducing harmonic oscillator wave packets in momentum space. The derivation requires substantial angular momentum computation which is given in detail using the graphical method. These basis fields are used in later chapters for constructing the many-body configurations. As examples models for light hadron systems are given together with specific tables, listing the many-body configurational states consisting of one or two nucleons with a meson cloud of several pions and one heavy meson. Of course the domains of application are much wider than these examples. For example, the present methods can be used directly for the study of the effects of mesonic degrees of freedom in all the usual aspects of the microscopic theory of nuclei, such as Hartree-Fock or BCS descriptions, Tamm-Dancoff or random-phase approximations, particle-hole or quasi-particle configuration mixing models, etc. The extension to the quark-gluon picture has been discussed in Chapters 2 and 4 and is also immediate.

Chapter 6 contains the full expressions for the matrix elements of the elementary vertices of the interactions chosen as examples in Chapter 2, except for the electromagnetic interactions which are treated in Chapter 9. Again all derivations are given in full detail. These elementary matrix elements enter the many-body hamiltonian matrix which is described in Chapter 7. In that chapter, first a systematic method for constructing properly the many-body matrix elements is presented both in Fock space and Minkowski space. Then as an example, we give the hamiltonian matrix for systems consisting of pions, nucleons and heavy mesons covering the model spaces of Chapter 5.

The details concerning the treatment of the center-of-mass problem along the lines of Chapter 3 are given in Chapter 8. The difficulties of the many-body nature of the c.m. position operator become manifest in this chapter. Here again, as an example, we generate the expressions of the c.m. pseudo-hamiltonian for the model spaces of Chapter 5.

The many-body aspects of the interaction of a relativistic system with an external electromagnetic field are developed in Chapter 9. In particular, the electron elastic scattering form factor, the magnetic dipole and electric quadrupole moments are given in detail. As already mentioned, minimal coupling, vector dominance, as well as anomalous moment interactions are included.

1.3. REFERENCES

In this book we have tried to achieve a self-contained presentation in that all needed expressions are developed from stated basic definitions. Of course, our treatment builds on the fundamental work of field theory and group theory. A minimal familiarity with these subjects is required from the reader. In order to assist in finding the background material and elucidation on the deeper points, we here give a short selection of references from the literature.

Relativistic quantum field theory has been discussed in a large number of books, practically all in the framework of the covariant renormalization theory. They differ in their relative emphasis between the presentation of concepts and the development of techniques. We mention here specifically the work of Hayward on fields of arbitrary spin, which we adopt as the basis for the treatment of spin-1 and -$\frac{3}{2}$ fields. Hayward's formulation contains a minimal number of auxiliary (ghost) fields, to be determined by gauge conditions. In this way no supplementary conditions have to be imposed and no unphysical effects arise when introducing interactions as is the case, e.g., for the Rarita-Schwinger formulation.

The mathematical basis of the angular momentum treatment is found in many books. Most of them are limited to SU(2). We use in particular the notation of Fano and Racah. Some of the references include also the treatment of SU(n), which is required in the treatment of QCD; we refer in particular to Biedenharn and Lauck.

This book is an outgrowth of our previous publication, NBS Monograph 147. The present development is much more complete and supersedes that work.

BIEDENHARN, L.C. and J.D. LAUCK, 1981, *Angular momentum in quantum physics*, vol. 8 and 9, Encycl. of Math. and its applications, ed. G.-C. Rota (Addison-Wesley, Reading, Mass.).

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CHAPTER 2

Fields and interactions

2.1. INTRODUCTION

The general framework of quantum field theory is briefly recalled. We give the basic definitions needed in the book and the main expressions for the energy and the interactions which will be evaluated later in detail. The treatment presented here is strictly that of quantized fields and not that of classical fields. Quantum fields arise from quantization of classical fields and have a much richer structure. However, we do not attempt a complete treatment of the underlying field theory which can be found in any standard textbook on this subject.

We treat first the spin-0, spin-$\frac{1}{2}$, spin-1 fields and some of their interactions. We also discuss their interactions with the electromagnetic field for the minimal, the anomalous moment, and the vector dominance couplings. For the higher spin fields we review the comprehensive formalism of Hayward and apply it specifically to the case of the spin-$\frac{3}{2}$ field. Finally, the extension to quantum chromodynamics is discussed.

The hamiltonian of the system is a functional of the basic free fields described in sect. 2.2. It is divided into two parts

$$H = H_0 + H_\mathrm{I},$$

where H_0 describes the free fields and H_I represents the various interactions between the free fields. The free hamiltonian H_0 describes non-interacting particles and is the sum of their free field energies given in sect. 2.3. The forms of the strong interactions contained in H_I are given in sect. 2.4. while the electromagnetic interactions are described in sect. 2.5. The treatment of higher spins is given in sect. 2.6., and the extension to Yang-Mills theories is sketched in sect. 2.7.

Throughout we work with the metric*

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

* This metric is employed for example by Lee, Hayward, Lurie, and Wentzel. It is more convenient for the angular momentum calculus than the metric $g_{ii} = -1$, $g_{00} = 1$ found for example in Bjorken and Drell, Bogoliubov and Shirkov, or Roman.

Hence the four-vector scalar product ab will be denoted by

$$ab = \boldsymbol{a}\cdot\boldsymbol{b} + a_4 b_4 = \boldsymbol{a}\cdot\boldsymbol{b} - a_0 b_0$$

with $a_4 = ia_0$ and $b_4 = ib_0$. For example

$$(x_1, x_2, x_3, x_4) \equiv (x, y, z, it).$$

Furthermore we use the conventions that repeated greek indices, $\mu, \nu \dots$, are summed over 1, 2, 3, 4 while the latin indices, $i, j \dots$, are summed over 1, 2, 3.

Also we use throughout units such that $\hbar = c = 1$.

2.2. FRAMEWORK OF FIELD THEORY

2.2.1. Free fields and state vectors

We denote by $\varphi, \phi, \psi, \Phi$ the basic single-particle free field solutions. We use the notation φ for the spin-0 free field, ψ for the spin-$\frac{1}{2}$ field, ϕ for the spin-1 field and Φ for the spin-$\frac{3}{2}$ field.

The free fields are solutions of the free-particle equations of motion, namely the Klein-Gordon equation for spin 0, sect. 2.3.1, the Dirac equation for spin $\frac{1}{2}$, sect. 2.3.2, and the Hayward equations for higher spin, sects. 2.3.3 and 2.6.

For quantization the free fields are expanded into suitable orthonormal bases

$$\varphi = \sum_i a_i \varphi_i ,$$

$$\psi = \sum_i b_i \psi_i ,$$

$$\phi = \sum_i A_i \phi_i ,$$

$$\Phi = \sum_i B_i \Phi_i .$$

Quantization is achieved by ascribing the meaning of annihilation operators to the amplitudes of these expansions which together with their hermitian conjugates are postulated to fulfill commutation relations for bosons and anticommutation relations for fermions:

$$[a_i, a_j^+]_- = \delta_{ij}, \qquad [a_i^+, a_j^+]_- = 0, \qquad [a_i, a_j]_- = 0,$$

$$[b_i, b_j^+]_+ = \delta_{ij}, \qquad [b_i^+, b_j^+]_+ = 0, \qquad [b_i, b_j]_+ = 0, \qquad \text{etc.}$$