Mechanics of Engineering Materials

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second edition

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Preface to the Second Edition

The historical development of this text was outlined in the Preface to the previous edition. During the nine years since its publication there have been no major changes in the theory of the subject but there have been advances in some of the manipulative tools which can be used to solve problems related to the strength of materials. In addition, with experience and hindsight one can always improve the presentation of material to make it more digestible to the reader.

In this new edition we have engaged the services of a new author Dr Cecil Armstrong to assist in a complete update of the text. New features of the book include the use of versatile solution techniques based on spreadsheets, the use of colour to enhance diagrams and provide emphasis of key concepts, an introduction to matrix methods as a powerful tool in engineering analysis, and the incorporation of additional practical problems to illustrate the application of the theory.

The essential structure and content of Chapters 1–9 has not been changed although regular users of the previous text will see subtle changes in the mode of presentation to make this important fundamental information more meaningful to students. Chapter 10 from the original text has been removed because it was felt that modern computer programs provide powerful tools for the solutions of problems in structures. In a general text of this type it is impossible to do justice to the wide range of methods available to analyse structures. Hence, we have moved the important fundamental information on tension coefficients, energy methods, etc. to other chapters and omitted the superficial and to some extent outdated solution methods based on scale drawings.

The remaining chapters have benefited from a complete overhaul and the chapter on Finite Elements has been almost totally re-written. Throughout the text the reader is introduced to the concept of creating a spreadsheet representation of a problem in strength of materials. Once such a solution has been formulated it is then a relatively simple matter to study the effects of changes in variables, to plot graphs of key variables and to optimize solutions. This is shown to be a very powerful solution methodology which provides a completely new insight to each problem and open up a whole new approach to problem solving in many subject areas.

The authors derived a great deal of pleasure in working on this new version of the text. However, although he participated fully in the revision of the book, regrettably Peter Benham died before he was able to see the new book in print. We know that the many young people who studied under him, and the readers all over the world who benefited from his books, were greatly saddened by his death. It is our hope that this new edition, about which he was very excited and proud, will be a fitting tribute to his memory.

R. J. CRAWFORD
C. G. ARMSTRONG
10 November 1995

Preface to the First Edition

The S.I. edition of *Mechanics of Solids and Structures* by P. P. Benham and F. V. Warnock was first published in 1973. It appears to have been very well received over the ensuing period. This preface is therefore written both for those who are familiar with the past text and also those who are approaching this subject for the first time. Although the subject matter is still basically the same today as it has been for decades, there are a few developing topics which have been introduced into undergraduate courses such as finite element analysis, fracture mechanics and fibre composite materials. In addition, style of presentation and illustrations in engineering texts have changed for the better and certain limitations of the previous edition, e.g. the number of problems and worked examples, needed to be rectified. Professor Warnock died in 1976 after a period of happy retirement and so it was left to the other author and the publisher to take the initiative to construct a new textbook.

In order to provide fresh thinking and reduce the time of rewriting Dr Roy Crawford, Reader in Mechanical Engineering at the Queen's University of Belfast, was invited, and kindly agreed, to join the project as a co-author. Although the present text might be regarded as a further edition of the original book the new authorship team preferred to make a completely fresh start. This is reflected in the change of title which is widely used as an alternative to *Mechanics of Solids*. The dropping of the reference to structures does not imply any reduction in that topic as will be seen in the contents.

In order that the book should not become any larger with the proposed expansion of material in some chapters, it was decided that the previous three chapters on experimental stress analysis should be omitted as there are several excellent texts in this field. The retention of that part of the book dealing with mechanical properties of materials for design was regarded as important even though there are also specialized texts in this area.

The main part (Ch. 1–18) of this new book of course still deals with the basic subject of Solid Mechanics, or Mechanics of Materials, whichever title one may prefer, being the study of equilibrium and displacement systems in engineering components and structures to enable designs to be effected in terms of stress and strain and the selection of materials. These eighteen chapters cover virtually all that is required in the three-year syllabus of a university or polytechnic degree course in engineering, or the examinations of the engineering Council, C.N.A.A. etc.

Although there is a fairly natural ordering of the material there is some scope for variation and lecturers will have their own particular detailed preferences.

As in the previous text, the first eleven chapters are concerned with forces and displacements in statically-determinate and indeterminate components and structures, and the analysis of uniaxial stress and strain due to various forms of loading such as bending, torsion, pressure and temperature change. The basic concepts of strain energy (Ch. 9) and elastic stability (Ch. 11) are also introduced. In Chapter 12 a study is made of two-dimensional states of stress and strain with special emphasis on principal stresses and the analysis of strain measurements using strain gauges. Chapter 13 combines two chapters of the previous book and brings together the topics of yield prediction and stress concentration which are of such importance in design.

Also included in these two chapters is an elementary introduction to the stress analysis and failure of fibre composite materials. These relatively new advanced structural materials are becoming increasingly used, particularly in the aerospace industry, and it is essential for engineers to receive a basic introduction to them. These thirteen chapters constitute the bulk of the syllabuses covered in first and second-year courses.

Four of the next five chapters appeared in the previous text and deal with more advanced or specialized topics such as thick-walled pressure vessels, rotors, thin plates and shells and post-yield or plastic behaviour, which will probably occupy part of final-year courses.

One essential new addition is an introductory chapter on finite element analysis. It may seem presumptuous even to attempt an introduction to such a broad subject in one chapter, but it is an attempt to provide initial encouragement and confidence to proceed to the complete texts on finite elements.

Chapters 19 to 22 cover much the same ground as in the previous text, but have been brought up to date particularly in relation to fracture mechanics. Since these chapters have such importance in relation to design, a number of worked examples have been introduced, together with problems at the end of each chapter. Bibliographies have still been included for further reading as required.

The first Appendix covers the essential material on properties of areas. The second deals with the simple principles of matrix algebra. A useful table of mechanical properties is provided in the third Appendix.

One of the recommendations of the Finniston Report to higher education was that theory should be backed up by more practical industrial applications. In this context the authors have attempted to incorporate into the worked examples and problems at the end of each chapter realistic engineering situations apart from the conventional examination-type applications of theory.

There had been a number of enquiries for a solutions manual for the previous text and this can be very helpful to both lecturer and student. Consequently this text is accompanied by another volume which contains worked solutions to nearly 300 problems. The manual should be used alongside the main text, so that steps in each solution can be referred back to the appropriate development in the relevant chapter. It is most important not to approach solutions on the basis of plucking the 'appropriate formula' out of the text, inserting the numbers, and manipulating a calculator!

Every effort has been made by the authors to ensure accuracy of text and solutions, but lengthy experience demonstrates human fallibility in this respect. When errors subsequently come to light they will be corrected at the next reprinting and readers' patience and comments will be appreciated!

Some use has been made of data and diagrams from other published literature and, in addition to the individual references, the authors wish to make grateful acknowledgement to all persons and organizations concerned.

P. P. BENHAM
R. J. CRAWFORD
1987

Notation

α	angle, coefficient of thermal expansion
β	angle
γ	shear strain, surface energy per unit area
δ	deflection, displacement
ϵ	direct strain
η	efficiency, viscosity
θ	angle, angle of twist, co-ordinate
λ	lack of fit
ν	Poisson's ratio
ρ	radius of curvature, density
σ	direct stress
τ	shear stress
ϕ	angle, co-ordinate, stress function
ω	angular velocity
A	area
C	complementary energy
D	diameter
E	Young's modulus of elasticity
F	force
G	shear or rigidity modulus of elasticity, strain energy release rate
H	force
I	second moment of area, product moment of area
J	polar second moment of area
K	bulk modulus of elasticity, fatigue strength factor, stress concentration factor, stress intensity factor
L	length
M	bending moment
N	number of stress cycles, speed of rotation
P	force
Q	shear force
R	force, radius of curvature, stress ratio
S	cyclic stress
T	temperature, torque
U	strain energy
V	volume
W	weight, load
X	body force
Y	body force
Z	body force, section modulus
a	area, distance, crack length
b	breadth, distance, crack length
c	distance
d	depth, diameter
e	eccentricity, base of Napierian logarithms
g	gravitational constant
h	distance

j	number of joints
k	diameter ratio of cylinder
l	length
m	mass, modular ratio, number of members
n	number
p	pressure
q	shear flow
r	co-ordinate, radius, radius of gyration
s	length
t	thickness, time
u	displacement in the x- or r-direction
v	deflection, displacement in the y- or θ-direction, velocity
w	displacement in the z-direction, load intensity
x	co-ordinate, distance
y	co-ordinate, distance
z	co-ordinate, distance

It should be noted that a number of these symbols have also been used to denote constants in various equations.

Contents

CHAPTER 1

Statically Determinate Force Systems

Structural and solid-body mechanics are concerned with analysing the effects of applied loads. These are *external* to the material of the structure or body and result in *internal* reacting forces, together with deformations and displacements, conforming to the principles of Newtonian mechanics. Hence a familiarity with the principles of statics, the cornerstone of which is the concept of *equilibrium of forces*, is essential.

A force system is said to be statically determinate if the internal forces can be calculated by considering only the forces acting on the system.

Forces result in four basic forms of deformation or displacement of structures or solid bodies and these are *tension, compression, bending,* and *twisting.*

The equilibrium conditions in these situations are discussed so that the forces may be determined for simple engineering examples.

1.1 Revision of statics

A particle is in a state of equilibrium if the resultant force and moment acting on it are zero. This hypothesis can be extended to clusters of particles that interact with each other with equal and opposite forces but have no overall resultant. Thus it is evident that solid bodies, structures, or any subdivided part, will be in equilibrium if the resultant of all external forces and the resultant of all moments are zero. This may be expressed mathematically in the following six equations which relate to Cartesian co-ordinate axes x, y and z.

$$\left.\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma F_z &= 0\end{aligned}\right\} \qquad [1.1]$$

where F_x, F_y and F_z represent the components of force vectors in the co-ordinate directions.

$$\left.\begin{aligned}\Sigma M_x &= 0\\ \Sigma M_y &= 0\\ \Sigma M_z &= 0\end{aligned}\right\} \qquad [1.2]$$

where M_x, M_y and M_z are components of moment vectors caused by the external forces acting about the axes x, y, z.

The above six equations are the necessary and sufficient conditions for equilibrium of a body.

If the forces all act in one plane, say $z = 0$, then

$$\Sigma F_z = \Sigma M_x = \Sigma M_y = 0$$

are automatically satisfied and the equilibrium conditions to be satisfied in a two-dimensional system are

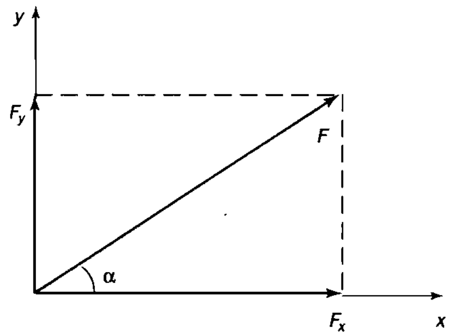
$$\left.\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma M_z &= 0\end{aligned}\right\} \qquad [1.3]$$

Forces and moments are vector quantities and may be resolved into components; that is to say, a force or a moment of a certain magnitude and direction may be replaced and exactly represented by two or more components of different magnitudes and in different directions.

Considering firstly the two-dimensional case shown in Fig. 1.1, the force F may be replaced by the two components F_x and F_y provided that

$$\left.\begin{aligned}F_x &= F\cos\alpha\\ F_y &= F\sin\alpha\end{aligned}\right\} \qquad [1.4]$$

Fig. 1.1



Note that throughout this book, externally applied forces will be shown coloured. Internal forces, that is those within a structural element, will be shown in black.

If the force F were arbitrarily oriented with respect to three axes x, y, z as in Fig. 1.2, then it could be replaced or represented by the following components:

$$\left.\begin{aligned}F_x &= F\cos\alpha\\ F_y &= F\cos\beta\\ F_z &= F\cos\gamma\end{aligned}\right\} \qquad [1.5]$$

Fig. 1.2

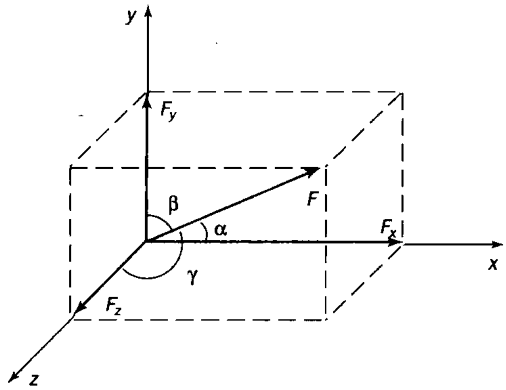
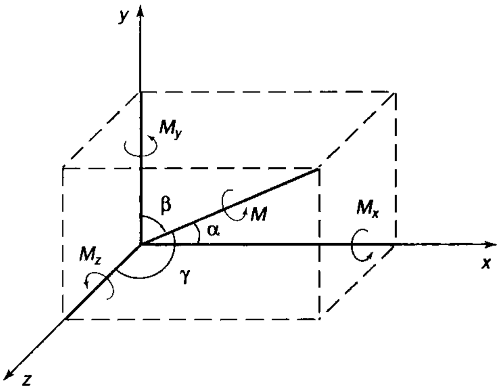


Fig. 1.3



A couple or moment vector about an axis can similarly be resolved into a representative system of component vectors about other axes, as shown in Fig. 1.3 and represented by the following equations:

$$\left.\begin{aligned} M_x &= M\cos\alpha \\ M_y &= M\cos\beta \\ M_z &= M\cos\gamma \end{aligned}\right\} \qquad [1.6]$$

Example 1.1

A set of concurrent forces F_i are defined by their components F_x, F_y, F_z in kN as

	F_x	F_y	F_z
F_1	80	−20	−40
F_2	−40	60	−80
F_3	100	−20	30
F_4	−30	10	40

Calculate the magnitude and direction of the resultant of this force system.

$$\bar{F}_x = \Sigma F_x = 80 - 40 + 100 - 30 = 110\,\text{kN}$$

$$\bar{F}_y = \Sigma F_y = -20 + 60 - 20 + 10 = 30\,\text{kN}$$

$$\bar{F}_z = \Sigma F_z = -40 - 80 + 30 + 40 = -50\,\text{kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(\bar{F}_x^2 + \bar{F}_y^2 + \bar{F}_z^2)} = \sqrt{(110^2 + 30^2 + (-50)^2)} = 124\,\text{kN}$$

to three significant figures. The angles between the resultant force and the axes shown in Fig. 1.2 can now be found using

$$\cos\alpha = \frac{\bar{F}_x}{F_R} = \frac{110}{124} = 0.884, \text{ so } \alpha = 27.9^\circ$$

$$\cos\beta = \frac{\bar{F}_y}{F_R} = \frac{30}{124} = 0.241, \text{ so } \beta = 76.1^\circ$$

$$\cos\gamma = \frac{\bar{F}_z}{F_R} = \frac{-50}{124} = -0.402, \text{ so } \gamma = -66.3°$$

Since all the forces are concurrent, none of them have any moment about the common point through which they all act and there is therefore no resultant moment about that point.

Spreadsheet solution

In engineering practice there are few investigations where the problem is as clearly defined and straightforward as that given above. There is usually a degree of uncertainty about the supplied information, some data may be missing and the numerical computations are significantly more complicated. Thus laborious calculations are needed and the sensitivity of the results to variations in the given or assumed information has to be evaluated. This has lead many engineers to develop computer programs in languages such as Fortran, Basic or C to automate the required calculations.

In this book, selected numerical problems will be solved using commercial programming and modelling tools called **spreadsheets**. Probably the best known of these are Lotus 1–2–3, Borland Quattro and Microsoft Excel. A computer shreadsheet can be considered as a grid of cells which can contain text labels, numbers or formulae which may refer to numbers or numerical results in other cells. If the cell contains a formula, and a number is changed in any cell to which that formula refers, then the formula is automatically recalculated and the updated result is displayed. A range of alternatives can be quickly evaluated by entering different numbers

Fig. 1.4

	A	B	C	D	E
1		Fx	Fy	Fz	Magnitude
2	F1	80	-20	-40	
3	F2	-40	60	-80	
4	F3	100	-20	30	
5	F4	-30	10	40	
6					
7	FR	+B2+B3+B4+B5	@SUM(C2..C5)	@SUM(D2..D5)	@SQRT(B7^2+C7^2+D7^2)
8					
9	Direction Cosines				
10		alpha	beta	gamma	
11		+B7/E7	+C7/E7	+D7/E7	

	A	B	C	D	E
1		Fx	Fy	Fz	Magnitude
2	F1	80	-20	-40	
3	F2	-40	60	-80	
4	F3	100	-20	30	
5	F4	-30	10	40	
6					
7	FR	110	30	-50	124.5
8					
9	Direction Cosines				
10		alpha	beta	gamma	
11		0.884	0.241	-0.402	

in a given cell. Modern packages have sophisticated facilities for the graphical display of results, 'What-if' evaluation of a range of alternatives, querying databases and optimization. Although these programs were initially developed for financial modelling, many engineers and students now use these packages as sophisticated programmable calculators.

Figure 1.4(*a*) shows the data and formulae required to solve this problem. These were entered in a Quattro spreadsheet but they will also work with Lotus 1–2–3 or Microsoft Excel. Figure 1.4(*b*) shows the resulting display in the spreadsheet. Normally the formulae will not be visible and only the result will be displayed. Throughout this book, the cells in which the user should input data have been highlighted. The other cells contain either formulae or text labels and should not be overwritten.

As can be seen in Fig. 1.4(*a*), the total force in the x-direction is found in cell B7 by adding up the contents of cells B2, B3, B4 and B5. An even more convenient technique for summing up the y-components is shown in cell C7, where the built-in function @SUM is used to total the contents of cells C2 to C5. Within the spreadsheet program the Edit, Copy and Paste commands can be used to copy the same formula to cell D7, which then sums the z-components. Individual cells or groups of cells can be identified with arrow keys or mouse clicks – it is not necessary to type the cell references. The resultant force is found using the three-dimensional equivalent of eqn. [1.7] in cell E7, using the built-in function @SQRT to calculate the square root and ^ to indicate that a number is to be raised to a power.

Once these formulae are entered, any change to any component of the four forces will cause an immediate recalculation of the magnitude of the resultant force and the directions. If only three forces are to be summed, deleting all the components of one force will give the correct answer. If a larger number of forces is to be summed, extra rows can be inserted above row 6 and the SUM formula in the current row 7 adjusted to the new range of cells.

1.2 Resultant force and moment

It is sometimes more convenient to replace a system of applied forces by a resultant which of course must have the same effect as those forces. Considering a two-dimensional case as illustrated in Fig. 1.5, then the most general solution is obtained by choosing any point A through which the resultant can act. Then the total force components in the co-ordinate directions are

$$\left.\begin{aligned}\bar{F}_x &= \Sigma F_x \\ \bar{F}_y &= \Sigma F_y\end{aligned}\right\} \qquad [1.7]$$

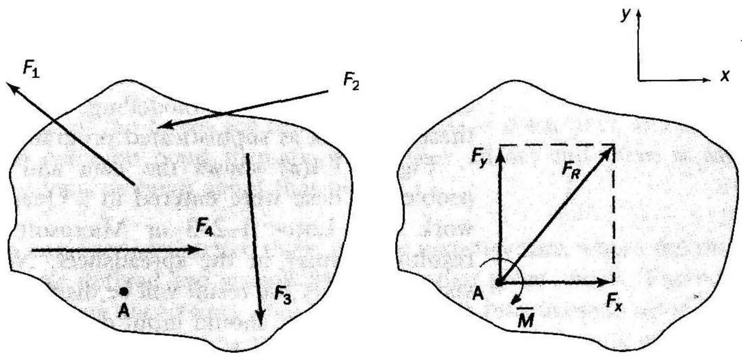
and the resultant force is given by

$$F_R = \sqrt{(\bar{F}_x^2 + \bar{F}_y^2)} \qquad [1.8]$$

However, this is not sufficient in itself since the moment due to the forces must be represented. This is done by having a couple acting about A such that

$$\bar{M} = \Sigma M_z \qquad [1.9]$$

Fig. 1.5



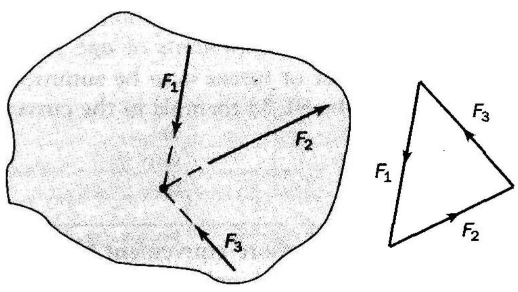
(a) (b)

In general, then, any system of forces can be replaced by a resultant force through, and a couple about, any chosen point.

The equivalent solution for a three-dimensional system of forces is similarly a couple and a resultant force whose direction is parallel to the axis of the couple.

One of the most useful constructions in force analysis is termed the *triangle of forces*. If a body is acted on by three forces then, for equilibrium to exist, these must act through a common point or else there will exist a couple about the point causing the body to rotate. In addition the magnitude and direction of the three force vectors must be such as to form a closed triangle as shown in Fig. 1.6.

Fig. 1.6



1.3 Types of structural and solid-body components

Structures are made up of a series of members of regular shape that have a particular function for load carrying. The shape and function are, through usage, implied in the name attached to the member. The first group is concerned with carrying loads parallel to a longitudinal axis. Examples are shown in Fig. 1.7. A member which prevents two parts of a structure from moving apart is subjected to a pull at each end, or tensile force, and is termed a *tie* (*a*). Conversely a slender member which prevents parts of a structure moving towards each other is under compressive force and is termed a *strut* (*b*). A vertical member which is perhaps not too slender and supports some of the mass of the structure is called a *column* (*c*). A *cable* (*d*) is a generally recognized term for a flexible string under tension which connects two bodies. It cannot supply resistance to bending action.

One of the most important of structural members is that which is frequently supported horizontally and carries transverse loading. This is