Theory of the Earth's Gravity Field

Miloš Pick
Jan Pícha
Vincenc Vyskočil

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PREFACE

This book is intended to be a textbook for university students studying geophysics and geodesy. However, it can also serve as a suitable aid to specialists and students of related branches of science (geology, astronomy, physics, etc.).

The contents were chosen with a view to the subject matter of university lectures in this field. In order to avoid excessive length, the contents of some of the chapters had to be restricted, and the book is particularly oriented to the problems of physical geodesy. Other trends in gravimetric research, e.g., the problems of applied gravimetry, are only outlined. However, the authors tried to give the reader a good general survey of gravimetry, the relevant methods of research, and of its applications. In order to help the reader who only wishes to acquire an overall picture of gravimetry, the more sophisticated sections of the book are marked with an asterisk. These sections may be omitted when the book is read for the first time.

As a background for some of the mathematically more difficult parts of the book, an appendix is included with a review of the necessary mathematical theorems and formulae. The appendix is intended only for this purpose and, therefore, only the relations used in this book are included. It was not the intention of the authors to present a comprehensive summary of the equations of mathematical physics.

Having studied the relevant chapters of the book, the reader should be able to draw on the monographs recommended in the list of references and other books and specialized papers, expanding his knowledge accordingly.

The book is divided into chapters, denoted by Roman numerals, followed by the appendix, and individual chapters are subdivided into sections. Equations are numbered consecutively in the whole book.

The authors are aware that like any book of this type, the present publication is not and cannot be perfect and complete. A constant stream of new scientific papers is being turned out, and it can easily happen that by the time the book is published, some of the data will be outdated, or some of the ideas and concepts disproved. Should the reader find any mistakes in the text, the authors would appreciate hearing about them.

The authors would like to thank the scientific editor of this book, Prof. T. Kolbenheyer, PhD., Academician of the Czechoslovak Academy of Sciences, and also the reviewer, Prof. E. Buchar, PhD., Corresponding Member of the Czechoslovak Academy of Sciences, for their efforts in reading and studying the manuscript, and for their valu-

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M. Pick, J. Pícha, V. Vyskočil

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I. INTRODUCTION

1. Gravimetry and Its Purpose

Geophysics, like geology, geodesy, geography and geochemistry, ranks among the sciences dealing with the earth. Broadly defined, geophysics deals with and investigates physical phenomena and properties of the whole earth or of its more extensive regions. Certain branches of geophysics, in its broader sense, have now become independent sciences, *e.g.*, the physics of the atmosphere (meteorology), the science of continental waters (hydrology), the physics of the seas and oceans (oceanography), and the science of glaciers (glaciology).

In this work the authors will consider geophysics in the narrower sense of the word, *i.e.*, the physics of the solid part of the earth's body, in particular the lithosphere. Geophysics narrowly defined, is concerned with bodies of water and the gaseous shell only to the extent that these influence problems pertaining to the whole earth.

Geophysics is further subdivided into such areas as gravimetry (the science of the earth's gravity field), geomagnetism (the science of the earth's magnetism), seismology (the science of earthquakes and related phenomena), geoelectricity (the science of the earth's electricity), radiometry (the science of the earth's radioactivity), and research into the ionosphere (*i.e.*, the region of the earth's atmosphere above about 60 km, in which the ions and electrons are of substantial physical significance). Gravimetry is thus a subdivision of geophysics.

Gravimetry as an independent scientific branch began to evolve only at the end of the last century. However, its roots go back to the times of those celebrated pioneers of classical mechanics, G. Galileo and I. Newton (16th and 18th centuries). A number of outstanding 18th and 19th century scientists (*e.g.*, A. C. Clairaut, G. Stokes, H. Bruns and F. R. Helmert) worked on various fundamental gravimetric problems concerned in particular with the application of the theory of potential to research into the earth's body. Nevertheless, the term gravimetry (from the Latin *gravis* = heavy and the Greek *metrein* = to measure) only became established in science in the 20th century. The name is not quite accurate and it does not fully express the tasks of this branch of geophysics. The gravimetrician not only must measure gravity, search for new methods of measurement and construct gravimetric instruments, but he must also solve many other fundamental gravimetric problems theoretically and practically.

Among the most important problems of contemporary gravimetry is the study of the figure and the dimensions of the earth's body (the geoid and its external gravity field). This is also related to determining the correct constants in the formula derived theoretically for the normal distribution of the acceleration of gravity over the earth's body, and to the explanation of the anomalies of the actual gravity field of the earth with respect to the theoretical field. Since the distribution of the real field of gravity over the surface of the earth is closely connected with the structure and composition of the whole earth's body, especially with its crust, the results of gravimetric surveys contribute to a great extent to knowledge in this area.

It is known that numerous geotectonic movements, which sometimes change the physical face of the earth's surface considerably, are generated and take place in the earth's crust. It follows that the earth's crust is not in equilibrium. Treatment of the important problem of the equilibrium of the earth's crust (the problem of isostasy) is another significant gravimetrical task.

The earth's crust is also acted upon by attracting forces due to the masses of celestial bodies, of which the moon and the sun have the largest effects. These disturbing forces result in periodical movements of the earth's crust called the tides of the solid crust. Similarly, the same forces act on the seas and oceans, creating sea tides (generally known as flood and ebb), and on the gaseous shell of the earth causing atmospheric tides. In this way, small changes in the magnitude and direction of the acceleration of gravity which can be observed by very sensitive instruments, take place.

Apart from these periodic changes there remains the unsolved problem of the secular changes of the acceleration of gravity which may be due to various geotectonic or geochemical processes in the earth or to a very slow variation of the flattening of the earth, etc.

Of great economic importance is the problem of applying gravimetry to surveying and investigating deposits of useful minerals and raw materials, *e.g.*, various ores, coal, oil, salt and ceramic materials. This requires a constant search for and thorough elaboration of suitable new methods of measuring and interpreting data.

From this list of some of the more important gravimetric problems it can be seen that gravimetry is closely related to geodesy and geology. As compared with geodesy which uses geometric methods to investigate the figure of the earth, gravimetry employs physical methods for the same problem. Geology uses applied gravimetry in the investigation of the geological structure of the upper parts of the earth's crust, and gravimetric methods are employed in prospecting. Besides being interrelated with other branches of geophysics, gravimetry is also intimately connected with physics, mathematics, astronomy and other related sciences.