# Theory of the Earth's Gravity Field

Miloš Pick Jan Pícha Vincenc Vyskočil

ELSEVIER SCIENTIFIC PUBLISHING COMPANY AMSTERDAM / LONDON / NEW YORK 1973

SCIENTIFIC EDITOR

Academician Tibor Kolbenheyer

REVIEWER

Prof. Dr. Emil Buchar, DrSc.

## CZECHOSLOVAK ACADEMY OF SCIENCES

## THEORY OF THE EARTH'S GRAVITY FIELD

# Theory of the Earth's Gravity Field

Miloš Pick Jan Pícha Vincenc Vyskočil



ELSEVIER SCIENTIFIC PUBLISHING COMPANY
AMSTERDAM/LONDON/NEW YORK
1973

Published in co-edition with ACADEMIA, Publishing House of the Czechoslovak Academy of Sciences, Prague

Distribution of this book is being handled by the following publishers

for the U.S.A. and Canada American Elsevier Publishing Company, Inc. 52 Vanderbilt Avenue New York, New York 10017

for the East European Countries, China, Northern Korea, Cuba, Northern Vietnam and Mongolia Academia, Publishing House of the Czechoslovak Academy of Sciences, Prague

for all remaining areas
Elsevier Scientific Publishing Company
335 Jan van Galenstraat
P.O. Box 330, Amsterdam, The Netherlands

ISBN 0-444-40939-4 Library of Congress Card Number 72-142295 With 157 Illustrations and 13 Tables

© Miloš Pick, Jan Pícha, Vincenc Vyskočil, Prague 1973 Translation © Jaroslav Tauer, Prague 1973

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publishers

Printed in Czechoslovakia

### **CONTENTS**

Contents	. 5
I. Introduction	
by Jan Pícha	
<ol> <li>Gravimetry and Its Purpose.</li> <li>A Short History of Gravimetry</li> </ol>	
II. Newton's Theory of Potential	
by Miloš Pick	
1. Introduction	. 19
2. Attraction of Point Masses	. 20
3. Volume Potential	. 20
4. The First Derivative of the Volume Potential	. 21
5. The Second Derivative of the Volume Potential	. 22
6. Gauss's Formula	. 23
7. Volume Potential at Improper Points (Infinitely Distant)	. 24
8. Properties of the Volume Potential	
9. Surface Potentials	
10. Potential of a Double Layer	. 26
11. Continuity of the Potential of a Double Layer	
12. The Normal Derivative of the Double-Layer Potential	
13. Potential of a Single Layer	
14. The Normal Derivative of the Potential of a Single Layer	
15. Properties of the Potential of a Single and a Double Layer (Review of Results).	
16. Potential of the Centrifugal Force	
17. Chasles's Theorem	
*18. Physical Interpretation of Green's Second Theorem	. 36
III. Potential of Some Simple Formations, Approximate in Shape to the Figure of the Ea	rth
by Miloš Pick	
1. Potential of a Homogeneous Spherical Layer and Its Derivatives	. 37
2. The Homogeneous Sphere	
3. Potential of a Homogeneous Triaxial Ellipsoid and Its First Derivatives for	
Internal Point	. 39

4.	First Derivatives of the Potential and the Potential of a Homogeneous Triaxial	
	Ellipsoid for an External Point	2
5.	Potential of a Homogeneous Rotational Ellipsoid and Its First Derivatives 4	3
	Potential of a Rotating Equipotential Rotational Ellipsoid Using the Solution of	
	the First Boundary Problem	5
7.	Gravity of an Equipotential Ellipsoid	-
8.	Equilibrium Bodies	
	An Ellipsoid as an Equilibrium Body	
	Maclaurin's Ellipsoids	
11.	Jacobi's Ellipsoids	
12.	The Relation Between the Maclaurin and Jacobi Ellipsoids	
	Roche's Model	
	Clairaut's Problem	
	Further Treatment of the Clairaut Theorem	-
	Variation of the Attracting Force with Depth (Saigey's Theorem)	
	Gravity Potential of a Rotating Body, Approximate in Shape to the Figure of the Earth 5.	
18.	Clairaut's Theorem	
	Gravity Potential of a Rotating Body	
	Normal Spheroids	
	Normal Gravity for Some Reference Ellipsoids	
	Potential of a Homogeneous Spheroid	_
	Convergence of Series for the Potential of a Triaxial Ellipsoid 6	
	Development of the Potential Into a Series According to Wavre	
	Stokes's Theorem	
<b>*</b> 26.	Poincaré's Equation, Crudelli's Conditions	4
IV. Ear	ipotential Surfaces. Reductions	
-	Miloš Pick	
Оy	IVIIIOS I ICK	
1.	Equipotential Surfaces	7
	• •	0
		1
4		4
		5
	Old in the control of	7
	The Laye Account of the Control of t	7
	The Bougast Itematical Control of the Control of th	9
	Compared of the Grand, Topographic Control of the C	3
		6
	THE REGULAR TECCHOLOGY	8
	The Trey Reduction	8
	The Other Head to the Control (1997)	1
14.	Substitution of the External Gravity Field by the Potential of a Single Layer,	.~
<b>.</b> -	Spread Over the Surface of the Sea	2
15.	Determination of the Figure of the Earth from the First Derivatives of the Gravity	
	Potential	-
	Properties of the Disturbing Potential	

V. Absolute Gravity Measurements	
by Jan Picha	
·	
	10
2. Absolute Measurements of the Acceleration of Gravity	13
VI. Relative Measurements of the Acceleration of Gravity	
by Jan Pícha	
by Jan Ficha	
1. Relative Gravity Measurements with Pendulum Instruments	20
1.1. Measurement of the Oscillation Period	21
1.2. Corrections of the Oscillation Period	24
a) Correction for the Clock Rate	24
b) Correction for the Effect of the Medium Surrounding the Pendulum	
(Correction in Respect of Vacuum)	24
c) Temperature Corrections	26
d) Correction for an Infinitely Small Amplitude	28
e) Correction for the Co-oscillation of the Stand	<b>30</b>
f) Effect of the Pendulum Blade Camber	32
g) Effect of the Geomagnetic Field	34
1.3. Pendulum Instruments	36
2. Statical Methods of Relative Gravity Measurements	39
<b>2.1. Constant and the management of the second and the second a</b>	39
a) Calibration of Gravimeter by Adding a Weight	47
(0) 2110 01111111111111111111111111111111	47
c) Calibration of Gravimeter by Measuring at Points with Known Values of	
***** * * * * * * * * * * * * * * * *	48
( =-=- =	48
u) Clavination	49
5) <b>Quitable 1</b> 100 <b>1</b> 100 100 100 100 100 100 100 100 100 1	55
-,	60
<del>-,</del>	63
<b></b>	65
	68
2.5. Gravity Measurements Under Water and in Borholes	70
VII. Measurements of the Second Derivatives of the Gravity Potential	71
by Jan Pícha	
VIII. Some Comments on the Anomalous Gravity Field	
by Vincenc Vyskočil	
at an entire the second	179
	180
Ziz Ziziziy zimizimizini zizizizi zi giriyini zi giriyini	18( 187
	182
•••••••••••••••••••••••••••••••••••••••	185 187
4. Accuracy of Gravity Anomalies and of Gravity Maps	. 0

<ul> <li>4.1. Accuracy of Gravity Anomalies at Observation Points</li> <li>4.2. Theoretical Analysis of the Accuracy of Interpolated Values of Gravity Anomalies</li> </ul>	187
Anomalies	188
<ul><li>4.3. Accuracy of the Difference Between Gravity Anomalies at Two Points</li><li>4.4. Estimate of the Accuracy of a Gravity Map</li></ul>	191 192
IX. Gravimetry and the Internal Structure of the Earth	
by Vincenc Vyskočil	
1. The Internal Structure of the Earth	195
2. Some Physical Properties of the Earth's Body	197
2.1. Density Inside the Earth	197
2.2. Gravity, Pressure and Flattening of Equipotential Surfaces Inside the Earth.	201
3. Density of Rocks of the Earth's Crust	203
*3.1. Fundamental Terms	203
*3.2. Determination of Density by Weighing Rock Samples	204
*3.3. Density Derived from Gravity Measurements	206
*3.4. Densities of Most Common Rocks	209
4. The Earth's Crust	212
5. Correlation Between Gravity Anomalies, Elevations Above Sea Level and Depths	
of the Mohorovičić Discontinuity	214
5.1. Fundamental Terms	214
5.2. Correlation Between Gravity Anomalies $\Delta g$ and Elevations $h$	217
5.3. Isostasy and Correlation Between Gravity Anomalies $\Delta g$ , Elevations h and	
Depths of the Mohorovičić Discontinuity $H$	219
6. General Principles of the Geological Interpretation of Gravity Anomalies	223
7. Application of the Gravimetric Method to Surveying Deposits of Usable Raw Materials	224
X. The Geoid	
by Miloš Pick	
A. The Geoid of a Regularized Earth	226
1. A Regularized Earth	226
2. Theory of the Regularized Geoid	228
3. Stokes's Formula for an External Point	230
*4. Pizzetti's Formula	233
*5. Stokes's Formula for an External Point (cont.)	234
*6. Stokes's Formula Derived by Solving the Third Boundary Problem	234
*7. Malkin's Integral Equation	236
*8. Determination of the Height of the Geoid (Summary of Results)	
*9. Idelson's Integral Equation	239 241
*10. Mologenskii s, Malkin s and Monin s Integral Equations	241
*11. Poincare's Series	247
*12. Stokes's Formula without Using the Normal Gravity	470
the First Order	251
*14. Limiting the Effect of the Influence of Distant Zones on the Height of the Geoid.	252
*14. Elimiting the Effect of the Inducate of Distant Zones on the Height of the Geold	254
415. I otential and wass of the George	257

	Practical Application of the Stokes's Formula	255
17.	Deviations of the Plumb Line	257
18.	Vening Meinesz's Formulae	258
	Adjustment of Vening Meinesz's Formulae for Near Zones	262
20.	Practical Application of Vening Meinesz's Formulae	264
	Auxiliary Functions Necessary for Determining the Regularized Geoid	268
	Determination of the Position of the Reference Body Relative to the Real Earth	271
	Influence of the Choice of the Normal Body on the Height of the Geoid and on the	
4.25.	Deviations of the Plumb Line	272
<b>4.7</b> 4		212
<b>₹</b> 24.	Influence of the Choice of the Geodetic System on the Height of the Geoid and on	27.
2.5	the Deviations of the Plumb Line	274
	The Mean Value of Normal Gravity	277
<b>*</b> 26.	The Figure of the Regularized Geoid Computed from Horizontal Gradients	
	of Gravity	279
27.	Determination of the Mean Value of a Gravity Anomaly	280
B. The	eory of the Unregularized Geoid	282
	The Unregularized Geoid	282
	Moiseev's Determination of the Shape of the Unregularized Geoid	284
<b>*</b> 30.	Malkin's Determination of the Shape of the Unregularized Geoid	286
<b>*</b> 31.	Molodenskii's Determination of the Shape of the Unregularized Geoid	288
<b>*</b> 32.	General Integral Equation for the Disturbing Potential of the Regularized and the	
	Unregularized Earth	289
	ermination of the Figure of the Earth Without Considering Hypotheses About Its Into mposition	ernal
Cor	ermination of the Figure of the Earth Without Considering Hypotheses About Its Into nposition Miloš Pick	ernal
Cor by	nposition Miloš Pick	ernal 293
Cor by 1.	nposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293
Cor by 1. 2.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294
Cor by 1. 2. *3.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293
Cor by 1. 2. *3.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294 299
Cor by 1. 2. *3. 4.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294 299 303
Cor by 1. 2. *3. 4.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294 299 303 308
Cor by 1. 2. *3. 4. *5.	Miloš Pick         Determining the Figure of the Real Earth Generally	293 294 299 303 308 309
Cor by 1. 2. *3. 4. *5. *6.	Miloš Pick         Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311
Corr by  1. 2. *3. 4.  *5. *6. *7. *8.	mposition         Miloš Pick         Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312
Corr by  1. 2. *3. 4.  *5. *6. *7. *8.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312 313
Corr by  1. 2. *3. 4.  *5. *6. *7. *8. *9.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312 313 315
Corr by  1. 2. *3. 4.  *5. *6. *7. *8. *9. *10. *11.	mposition         Miloš Pick         Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312 313
Corr by  1. 2. *3. 4.  *5. *6. *7. *8. *9. *10. *11. *12.	mposition         Miloš Pick         Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312 313 317
Corr by  1. 2. *3. 4.  *5. *6. *7. *8. *9. *10. *11. *12.	mposition         Miloš Pick         Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312 313 315
Cor by  1. 2. *3. 4.  *5. *6. *7. *8. *9. *10. *11. *12.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312 313 317
Cor by  1. 2. *3. 4. *5. *6. *7. *8. *9. *10. *11. *12.	Determining the Figure of the Real Earth Generally  Heights Above Sea Level  Molodenskii's Quasigeoid  Solution of the Molodenskii's Equation by Successive Approximations (Second Method)  Modification of the Molodenskii's Method  The Integro-differential Equation of Molodenskii for ζ  The Linear Integral Equation for T  The Mean Square Value of the Deflection of the Vertical for the Earth  Computation of the Vertical Gradient of a Gravity Anomaly  Bjerhammar's Method  Generalization of the Theory of Normal Heights  Determination of the Figure of the Earth by Means of the Second Vertical Gradients of the Disturbing Potential  me Variations of the Gravity Field	293 294 299 303 308 309 311 312 313 317
**Cor by  1. 2. **3. 4. **5. **6. **7. **8. **9. **10. **11. **12.  ***XII. Ti by	Determining the Figure of the Real Earth Generally  Heights Above Sea Level  Molodenskii's Quasigeoid  Solution of the Molodenskii's Equation by Successive Approximations (Second Method)  Modification of the Molodenskii's Method  The Integro-differential Equation of Molodenskii for ζ  The Linear Integral Equation for T  The Mean Square Value of the Deflection of the Vertical for the Earth  Computation of the Vertical Gradient of a Gravity Anomaly  Bjerhammar's Method  Generalization of the Theory of Normal Heights  Determination of the Figure of the Earth by Means of the Second Vertical Gradients of the Disturbing Potential  me Variations of the Gravity Field  Jan Pícha	293 294 299 303 308 309 311 312 313 317
**Cor by  1. 2. **3. 4. **5. **6. **7. **8. **9. **10. **11. **12.  ***XII. Ti by	Determining the Figure of the Real Earth Generally  Heights Above Sea Level  Molodenskii's Quasigeoid  Solution of the Molodenskii's Equation by Successive Approximations (Second Method)  Modification of the Molodenskii's Method  The Integro-differential Equation of Molodenskii for ζ  The Linear Integral Equation for T  The Mean Square Value of the Deflection of the Vertical for the Earth  Computation of the Vertical Gradient of a Gravity Anomaly  Bjerhammar's Method  Generalization of the Theory of Normal Heights  Determination of the Figure of the Earth by Means of the Second Vertical Gradients of the Disturbing Potential  me Variations of the Gravity Field	293 294 299 303 308 309 311 312 313 317
**Cor by  1. 2. **3. 4.  **5. **6. **7. **8. **9. **10. **11. **12.  ***XII. Till by  1.	Determining the Figure of the Real Earth Generally  Heights Above Sea Level  Molodenskii's Quasigeoid  Solution of the Molodenskii's Equation by Successive Approximations (Second Method)  Modification of the Molodenskii's Method  The Integro-differential Equation of Molodenskii for ζ  The Linear Integral Equation for T  The Mean Square Value of the Deflection of the Vertical for the Earth  Computation of the Vertical Gradient of a Gravity Anomaly  Bjerhammar's Method  Generalization of the Theory of Normal Heights  Determination of the Figure of the Earth by Means of the Second Vertical Gradients of the Disturbing Potential  me Variations of the Gravity Field  Jan Pícha	293 294 299 303 308 309 311 312 313 315 317
**Cor by  1. 2. **3. 4. **5. **6. **7. **8. **9. **10. **11. **12.  ***XII. Ti by  1. 2.	mposition  Miloš Pick  Determining the Figure of the Real Earth Generally	293 294 299 303 308 309 311 312 313 315 321

Absolutely Solid Forth	221
Absolutely Solid Earth	331
5. Notation of Tidal Waves	336
XIII. Observations of Tides of the Earth's Crust	
by Jan Pícha	
1. Introductory Comments and the Theory of the Horizontal Pendulum	338
1.1. Direct, Indirect and Secondary Effects. Love's Constants	343
2. Instruments for Tidal Measurements	345
2.1. The Quartz Horizontal Pendulum of Verbaandert and Melchior	345
2.2. The Photoelectric Ostrovskii Tiltmeter	346
2.3. The Lettau Horizontal Double Pendulum	348
2.4. The Vertical Pendulum for Tidal Measurements	348
3. Installation of Tiltmeter Sets	350
4. Measurements of Azimuths	352
5. Determination of the Sensitivity of Horizontal Pendula	358
6. Errors due to Instruments and to the Recording Equipment	366
7. Processing of Tidal Observations	369
XIV. Fundamental Mathematical Principles of Gravimetric Interpretation	
by Vincenc Vyskočil	
1. General Comments	372
2. Separation of the "Regional" and "Local" Components of the Anomalous Gravity	
Field	373
2.1. Review of Methods	373
2.2. The "Mean Value" Method	374
*2.3. Expression of the Regional Field by a Mathematically Definable Surface	375
*2.4. Transformation of the Anomalous Gravity Field to a Certain Height Above	
the Earth's Surface	376
*2.5. Stressing the Effect of Anomalous Masses at Small Depths by Means of the	277
Vertical Gravity Gradient	377
*2.6. The "Second Derivative" Method	378
2.7. Concluding Remarks	382
3. Initial Formulae for Gravimetric Interpretation	383 387
4. Direct and Inverse Gravimetric Problems for Simple Gravitating Bodies 4.1. A Sphere	387
4.1. A Sphere	390
	320
*4.3. "Two-dimensional" Prisms, Oblique and Vertical Steps	207
+4.4 An Infinitely Long Horizontal Material Plane Pelt	392
*4.4. An Infinitely Long, Horizontal, Material Plane Belt	395
*4.5. A Right Parallelepiped	395 398
*4.5. A Right Parallelepiped	395 398 400
*4.5. A Right Parallelepiped	395 398
*4.5. A Right Parallelepiped	395 398 400 400
*4.5. A Right Parallelepiped	395 398 400

6. Shape of the Density Interface Determined from the Anomalous Gravity Field . 6.1. Shape of the Interface Determined Along a Profile with Boreholes	410 410
6.2. Investigation of the Shape of the Bed of a Sedimentary Basin Without Reference Boreholes	411
6.3. Condensation of Anomalous Masses Bound to a Density Interface Into	411
a Horizontal Plane	411 414
7. Computation of Subsurface Anomalies (Analytical Continuation Downward)	414
7.1. Principle of the Method	415
7.2. Computation of a Substitute Attornary by Means of a Taylor Development 7.3. The Method of Finite Differences (Method of Nets)	416
*7.4. The Method of Computing Subsurface Anomalies by Means of a Development	
Into a Fourier Series	417
*8. Shape of the Mohorovičić Discontinuity Determined from the Various Characte-	
ristics of the Outer Gravity Field (by M. Pick)	422
9. Some Comments on the Quantitative Interpretation of Gravity Anomalies	424
XV. Substitution of the Earth's Body by a Reference Surface	
by Miloš Pick	
1. Representation of a Part of an Ellipsoid on Another by the Translation Method	425
2. Transformation from One Reference Surface to Another by Projection Along the	126
Normal to the Former Surface (Projection Method)	426 429
3. Longitudinal Distortion in the Projection Method	430
4. Deflections of the Vertical and the Laplace Equation	434
<ul><li>5. Relations Between Deflections of the Vertical and the Geographic Coordinates</li><li>6. Determination of Six Degrees of Freedom of a Reference Body</li></ul>	436
7. Transformation of Cartesian Coordinates into Geodetic Coordinates	437
XVI. Astronomical and Astrogravimetric Levelling by Miloš Pick	
1. Astronomical Levelling (a General Treatment)	439
2. Astrogravimetric Levelling (a General Treatment)	441
3. Molodenskii's Astrogravimetric Levelling	443
4. Molodenskii's Elliptical Transparency	447
5. Other Methods of Evaluating the Fundamental Equation of Astrogravimetric	450
Levelling	450
6. Astronomical and Astrogravimetric Levelling in Czechoslovakia	452
Appendix	
by Miloš Pick	
and the state of all Deletions	453
Auxiliary Mathematical Relations	454
2. On Some Types of Improper Integrals	455
4. Spherical Inversion of a Function	457
5. The Harmonic Function	458
6. Gauss's Theorem	. 459
V. Gauss o Theorem	

7.	Gauss's Integral
<b>*</b> 8.	Curvilinear Orthogonal Coordinate Systems in Space
	Molodenskii's Operators
10.	Laplace's Expression
11.	Green's Theorem
	Orthogonality of Functions
	Spherical Functions
14.	Series for the Function $1/r$
	Development of the Function $Y(\Theta, \varphi)$ Into a Series of Spherical Functions 47.
	Solution of the Laplace's Equation for a Sphere
	Sum of Series of Spherical Functions
	Sums of Certain Series of Spherical Functions (Review of Results) 470
	Boundary Problems in General
	Green's Function
	Properties of Green's Function $G_1$
	Green's Function for a Sphere
<b>*23.</b>	Potential of an Inhomogeneous Spherical Layer
<b>*</b> 24.	Potential of an Inhomogeneous Spherical Double Layer
<b>*</b> 25.	Solution of the Dirichlet's Problem for a Sphere Using Surface Integrals 48
<b>*</b> 26.	The Second Boundary Problem (Neumann's Problem)
<b>*27.</b>	Neumann's Functions for a Sphere (Internal Problem)
<b>*</b> 28.	Neumann's Function (External Problem)
<b>*</b> 29.	Solution of Neumann's Problem for a Sphere Using Spherical Functions 49
	The Third Boundary Problem for a Sphere
<b>*31.</b>	Integral Equations
	The Application of Integral Equations to the Solution of Boundary Problems
	(Summary of Results)
n	50
	ferences
	thor Index
Sul	oject Index

#### **PREFACE**

This book is intended to be a textbook for university students studying geophysics and geodesy. However, it can also serve as a suitable aid to specialists and students of related branches of science (geology, astronomy, physics, etc.).

The contents were chosen with a view to the subject matter of university lectures in this field. In order to avoid excessive length, the contents of some of the chapters had to be restricted, and the book is particularly oriented to the problems of physical geodesy. Other trends in gravimetric research, e.g., the problems of applied gravimetry, are only outlined. However, the authors tried to give the reader a good general survey of gravimetry, the relevant methods of research, and of its applications. In order to help the reader who only wishes to acquire an overall picture of gravimetry, the more sophisticated sections of the book are marked with an asterisk. These sections may be omitted when the book is read for the first time.

As a background for some of the mathematically more difficult parts of the book, an appendix is included with a review of the necessary mathematical theorems and formulae. The appendix is intended only for this purpose and, therefore, only the relations used in this book are included. It was not the intention of the authors to present a comprehensive summary of the equations of mathematical physics.

Having studied the relevant chapters of the book, the reader should be able to draw on the monographs recommended in the list of references and other books and specialized papers, expanding his knowledge accordingly.

The book is divided into chapters, denoted by Roman numerals, followed by the appendix, and individual chapters are subdivided into sections. Equations are numbered consecutively in the whole book.

The authors are aware that like any book of this type, the present publication is not and cannot be perfect and complete. A constant stream of new scientific papers is being turned out, and it can easily happen that by the time the book is published, some of the data will be outdated, or some of the ideas and concepts disproved. Should the reader find any mistakes in the text, the authors would appreciate hearing about them.

The authors would like to thank the scientific editor of this book, Prof. T. Kolbenheyer, PhD., Academician of the Czechoslovak Academy of Sciences, and also the reviewer, Prof. E. Buchar, PhD., Corresponding Member of the Czechoslovak Academy of Sciences, for their efforts in reading and studying the manuscript, and for their valu-

able comments. They would also like to express their gratitude to K. Pěč, PhD., J. Kašpar, PhD., and P. Velkoborský, PhD., for carefully reading the manuscript. Some parts of the book were also discussed with M. I. Yurkina and V. F. Eremeev.

M. Pick, J. Pícha, V. Vyskočil

Prague, August 1970

#### I. INTRODUCTION

### 1. Gravimetry and Its Purpose

Geophysics, like geology, geodesy, geography and geochemistry, ranks among the sciences dealing with the earth. Broadly defined, geophysics deals with and investigates physical phenomena and properties of the whole earth or of its more extensive regions. Certain branches of geophysics, in its broader sense, have now become independent sciences, e.g., the physics of the atmosphere (meteorology), the science of continental waters (hydrology), the physics of the seas and oceans (oceanography), and the science of glaciers (glaciology).

In this work the authors will consider geophysics in the narrower sense of the word, *i.e.*, the physics of the solid part of the earth's body, in particular the lithosphere. Geophysics narrowly defined, is concerned with bodies of water and the gaseous shell only to the extent that these influence problems pertaining to the whole earth.

Geophysics is further subdivided into such areas as gravimetry (the science of the earth's gravity field), geomagnetism (the science of the earth's magnetism), seismology (the science of earthquakes and related phenomena), geoelectricity (the science of the earth's electricity), radiometry (the science of the earth's radioactivity), and research into the ionosphere (i.e., the region of the earth's atmosphere above about 60 km, in which the ions and electrons are of substantial physical significance). Gravimetry is thus a subdivision of geophysics.

Gravimetry as an independent scientific branch began to evolve only at the end of the last century. However, its roots go back to the times of those celebrated pioneers of classical mechanics, G. Galileo and I. Newton (16th and 18th centuries). A number of outstanding 18th and 19th century scientists (e.g., A. C. Clairaut, G. Stokes, H. Bruns and F. R. Helmert) worked on various fundamental gravimetric problems concerned in particular with the application of the theory of potential to research into the earth's body. Nevertheless, the term gravimetry (from the Latin gravis = heavy and the Greek metrein = to measure) only became established in science in the 20th century. The name is not quite accurate and it does not fully express the tasks of this branch of geophysics. The gravimetrician not only must measure gravity, search for new methods of measurement and construct gravimetric instruments, but he must also solve many other fundamental gravimetric problems theoretically and practically.

Among the most important problems of contemporary gravimetry is the study of the figure and the dimensions of the earth's body (the geoid and its external gravity field). This is also related to determining the correct constants in the formula derived theoretically for the normal distribution of the acceleration of gravity over the earth's body, and to the explanation of the anomalies of the actual gravity field of the earth with respect to the theoretical field. Since the distribution of the real field of gravity over the surface of the earth is closely connected with the structure and composition of the whole earth's body, especially with its crust, the results of gravimetric surveys contribute to a great extent to knowledge in this area.

It is known that numerous geotectonic movements, which sometimes change the physical face of the earth's surface considerably, are generated and take place in the earth's crust. It follows that the earth's crust is not in equilibrium. Treatment of the important problem of the equilibrium of the earth's crust (the problem of isostasy) is another significant gravimetrical task.

The earth's crust is also acted upon by attracting forces due to the masses of celestial bodies, of which the moon and the sun have the largest effects. These disturbing forces result in periodical movements of the earth's crust called the tides of the solid crust. Similarly, the same forces act on the seas and oceans, creating sea tides (generally known as flood and ebb), and on the gaseous shell of the earth causing atmospheric tides. In this way, small changes in the magnitude and direction of the acceleration of gravity which can be observed by very sensitive instruments, take place.

Apart from these periodic changes there remains the unsolved problem of the secular changes of the acceleration of gravity which may be due to various geotectonic or geochemical processes in the earth or to a very slow variation of the flattening of the earth, etc.

Of great economic importance is the problem of applying gravimetry to surveying and investigating deposits of useful minerals and raw materials, e.g., various ores, coal, oil, salt and ceramic materials. This requires a constant search for and thorough elaboration of suitable new methods of measuring and interpreting data.

From this list of some of the more important gravimetric problems it can be seen that gravimetry is closely related to geodesy and geology. As compared with geodesy which uses geometric methods to investigate the figure of the earth, gravimetry employs physical methods for the same problem. Geology uses applied gravimetry in the investigation of the geological structure of the upper parts of the earth's crust, and gravimetric methods are employed in prospecting. Besides being interrelated with other branches of geophysics, gravimetry is also intimately connected with physics, mathematics, astronomy and other related sciences.