Edited by Heinz Georg Schuster

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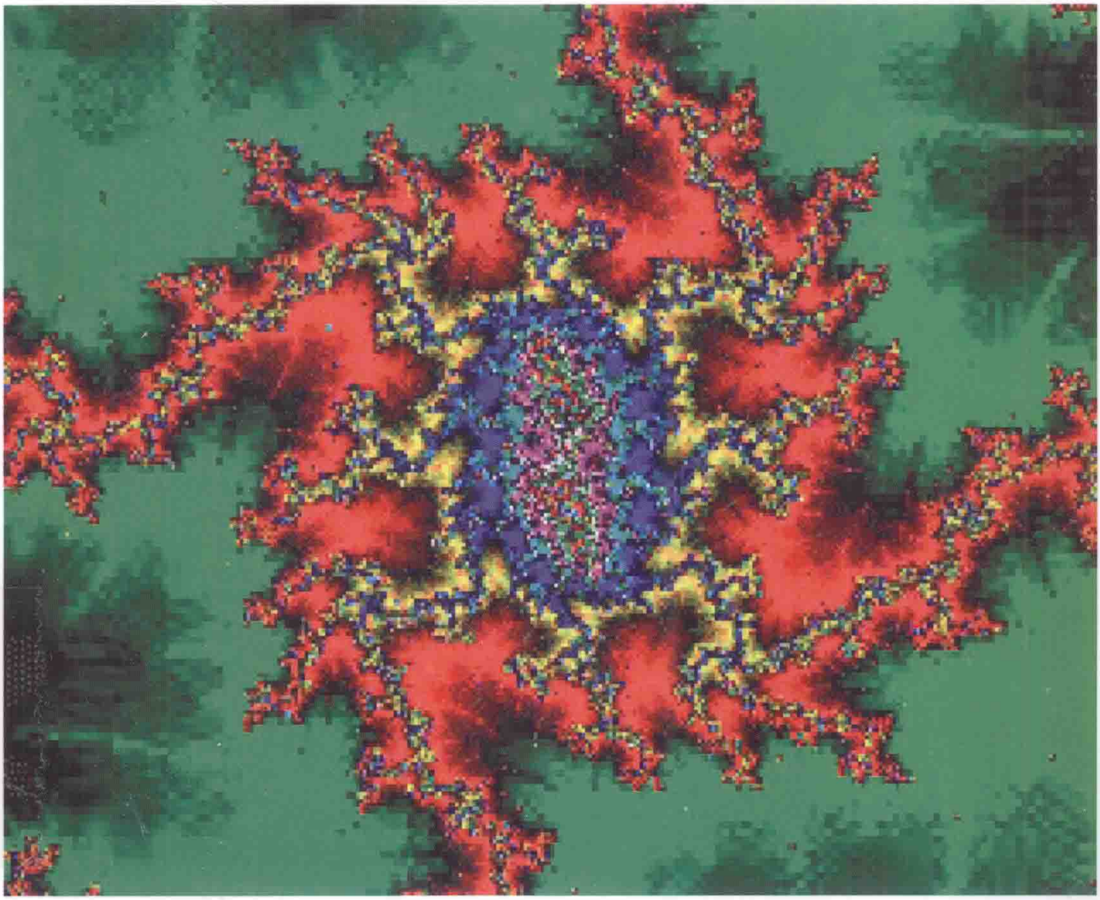
Volume 3



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Preface

Following the appearance of the first two very successful volumes of **Reviews of Nonlinear Dynamics and Complexity**, it is now my pleasure to introduce the third volume, beginning with an outline of the aims and purpose of this new series.

Nonlinear behavior is ubiquitous in nature and ranges from fluid dynamics, via neural and cell dynamics, to the dynamics of financial markets. The most prominent feature of nonlinear systems is that small external disturbances can induce large changes in behavior. This can and has been used for effective feedback control in many systems, from lasers to chemical reactions and the control of nerve cells and heartbeats. A new hot topic involves nonlinear effects that appear on the nanoscale. Nonlinear control of the atomic force microscope has improved its accuracy by orders of magnitude. The nonlinear electromechanical oscillations of nano-tubes, the turbulence and mixing of fluids in nano-arrays and the nonlinear effects in quantum dots are further examples.

Complex systems consist of large networks of coupled nonlinear devices. The observation that scale-free networks describe the behavior of the internet, cell metabolisms, financial markets and economic and ecological systems, has led to new discoveries concerning their behavior, such as damage control, optimal spread of information, or the detection of new functional modules that are pivotal for their description and control.

This shows that the field of **Nonlinear Dynamics and Complexity** consists of a large body of theoretical and experimental work with many applications, which is nevertheless governed and held together by some very basic principles, such as control, networks and optimization. The individual topics are definitely interdisciplinary, which makes it difficult for researchers to discover the new solutions – which

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could be most relevant for them – that have been found by their scientific neighbors. Therefore, its seems that there is an urgent need to provide **Reviews of Nonlinear Dynamics and Complexity** where researchers or newcomers to the field can find the most important recent results, described in a fashion which breaks down the barriers between the disciplines.

This third volume contains new topics ranging from chaotic computing, via random dice tossing and stochastic limit-cycle oscillators, to a number theoretic example of self-organized criticality, wave localization in complex networks and anomalous diffusion. I would like to thank all the authors for their excellent contributions. If readers take some inspiration for their further research from these interdisciplinary reviews, then this volume will have fully served its purpose.

I am grateful to all members of the Editorial Board, and the staff of Wiley-VCH, for their excellent help, and would like to invite my colleagues to contribute to the next volumes.

Kiel, January 2010 *Heinz Georg Schuster*

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The Chaos Computing Paradigm

William L. Ditto, Abraham Miliotis, K. Murali, and Sudeshna Sinha

1.1
Brief History of Computers

The timeline of the history of computing machines can probably be traced back to early calculation aids, varying in sophistication from pebbles or notches carved in sticks to the abacus, which was used as early as 500 B.C.! Throughout the centuries computing machines became more powerful, progressing from Napier's Bones and the slide rule, to mechanical adding machines and on to the modern day computer revolution.

The 'first generation' of modern computers, were based on wired circuits containing vacuum valves and used punched cards as the main storage medium. The next major step in the history of computing was the invention of the transistor, which replaced the inefficient valves with a much smaller and more reliable component. Transistorized (still bulky) computers, normally referred to as 'Second Generation', dominated the late 1950s and early 1960s.

The explosion in the use of computers began with 'Third Generation' computers. These relied on the integrated circuit or microchip. Large-scale integration of circuits led to the development of very small processing units. Fourth generation computers were developed, using a microprocessor to locate much of the computer's processing abilities on a single (small) chip, allowing the computers to be smaller and faster than ever before. Although processing power and storage capacities have increased beyond all recognition since the 1970s the underlying technology of LSI (large-scale integration) or VLSI (very-large-scale integration) microchips has remained basically the same, so it is widely regarded that most of today's computers still belong to the fourth generation.

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One common thread in the history of computers, be it the abacus or Charles Babbage's mechanical 'anlytical engine' or modern microprocessors, is this: *computing machines reflect the physics of the time and are driven by progress in the understanding of the physical world.*

1.2 The Conceptualization, Foundations, Design and Implementation of Current Computer Architectures

Computation can be actually defined as finding a solution to a problem from given inputs by means of an algorithm. This is what the theory of computation, a subfield of computer science and mathematics, deals with. For thousands of years computing was done with pen and paper, or chalk and slate, or mentally, sometimes with the aid of tables.

The theory of computation began early in the twentieth century, before modern electronic computers had been invented. One of the far-reaching ideas in the theory is the concept of a Turing machine, which stores characters on an infinitely long tape, with one square at any given time being scanned by a read/write head. Basically, a Turing machine is a device that can read input strings, write output strings and execute a set of stored instructions at a time. The Turing machine demonstrated both the theoretical limits and potential of computing systems and is a cornerstone of modern day digital computers.

The first computers were hardware-programmable. To change the function computed, one had to reconnect the wires or even build a new computer. John von Neumann suggested using Turing's Universal Algorithm. The function computed can then be specified by just giving its description (program) as part of the input rather than by changing the hardware. This was a radical idea which changed the course of computing.

Modern day computers still largely implement binary digital computing which is based on Boolean algebra; the logic of the true and false. Boolean algebra shows how you can calculate anything (within some epistemological limits) with a system of two discrete values. Boolean logic became a fundamental component of modern computer architecture, and is remarkable for its sheer conceptual simplicity. For instance, it can be rigorously shown that any logic gate can be obtained by adequate connection of NOR or NAND gates (i.e. any boolean circuit can be built using NOR/NAND gates alone). This implies that the capacity for universal computing can simply be demonstrated by the implementation of the fundamental NOR or NAND gates [1].

1.3
Limits of Binary Computers and Alternative Approaches to Computation: What Lies Beyond Moore's Law?

The operation of any computing machine is necessarily a physical process, and this crucially determines the possibilities and limitations of the computing device. For the past 20 years, the throughput of digital computers has increased at an exponential rate. Fuelled by (seemingly endless) improvements in integrated-circuit technology, the exponential growth predicted by Moore's law has held true. But Moore's Law will come to an end as chipmakers will hit a wall when it comes to shrinking the size of transistors, one of the chief methods of making chips that are smaller, more powerful and cheaper than their predecessors.

As conventional chip manufacturing technology runs into physical limits in the density of circuitry and signal speed, which sets limits to binary logic switch scaling, alternatives to semiconductor-based binary digital computers are emerging. Apart from analogue VLSI, these include bio-chips, which are based on materials found in living creatures; optical computers that live on pure light; and quantum computers that depend on the laws of quantum mechanics in order to perform, in theory, tasks that ordinary computers cannot.

Neurobiologically inspired computing, quantum computing and DNA computing differ in many respects, but they are similar in that their aim, unlike conventional digital computers, is to utilize at the basic level some of the computational capabilities inherent in the basic, analogue, laws of physics. Further, understanding of biological systems, has triggered the question: what lessons do the workings of the human mind offer for computationally hard problems? Thus the attempt is to create machines that benefit from the basic laws of physics and which are not just constrained by them.

Here we review another emerging computing paradigm: one which exploits the richness and complexity inherent in nonlinear dynamics. This endeavour also falls into the above class, as it seeks to extend the possibilities of computing machines by utilizing the physics of the device.

1.4 Exploiting Nonlinear Dynamics for Computations

We would now like to paraphrase the classic question 'What limits do the laws of classical physics place on computation' to read 'What opportunities do the laws of physics offer computation'.

It was proposed in 1998 that chaotic systems might be utilized to design computing devices [2]. In the early years the focus was on proof-of-principle schemes that demonstrated the capability of chaotic elements to do universal computing. The distinctive feature of this alternative computing paradigm was that it exploited the sensitivity and pattern formation features of chaotic systems.

In subsequent years there has been much research activity to develop this paradigm [3–17]. It was realized that one of the most promising directions of this computing paradigm was its ability to exploit a single chaotic element to reconfigure into different logic gates through a threshold-based morphing mechanism [3, 4]. In contrast to a conventional field programmable gate array element [18], where reconfiguration is achieved through switching between multiple single-purpose gates, reconfigurable chaotic logic gates (RCLGs) are comprised of chaotic elements that morph (or reconfigure) logic gates through the control of the pattern inherent in their nonlinear element. Two input RCLGs have recently been realized and shown to be capable of reconfiguring between all logic gates in discrete circuits [5–7]. Additionally, such RCLGs have been realized in prototype VLSI circuits (0.13 μm CMOS, 30 MHz clock cycles). Further, reconfigurable chaotic logic gates arrays (RCGA) which morph between higher-order functions such as those found in a typical arithmetic logic unit (ALU), have also been designed [17].

In this review we first recall the theoretical concept underlying the reconfigurable implementation of all fundamental logical operations utilizing nonlinear dynamics [3]. We also describe specific realizations of the theory in chaotic electrical circuits. Then we present recent results of a method for obtaining logic output from a nonlinear system using the time evolution of the state of the system. Finally we discuss a method for storing and processing information by exploiting nonlinear dynamics. We conclude with a brief discussion of some ongoing technological implementations of these ideas.

1.5 General Concept

We outline below a theoretical method for obtaining all basic logic gates with a single nonlinear system. The broad aim here is to use the rich temporal patterns embedded in a nonlinear time series in a controlled manner to obtain a computing device that is flexible and reconfigurable.

Consider a chaotic element (our *chaotic chip* or *chaotic processor*) whose state is represented by a value x. In our scheme all the basic logic gate operations (NAND, NOR, XOR, AND, OR, XNOR and NOT) involve the following steps:

1) Inputs:

   $x \rightarrow x_0 + X_1 + X_2$ for 2-input logic operations, such as the NAND, NOR, XOR, AND, OR and XNOR operations,

   and

   $x \rightarrow x_0 + X$ for 1-input operations, such as the NOT operation.

   Here x_0 is the initial state of the system, and

   $X = 0$ when $I = 0$

   and

   $X = V_{\text{in}}$ when $I = 1$

   where V_{in} is a positive constant.

2) Dynamical update, i.e. $x \rightarrow f(x)$

   where $f(x)$ is a nonlinear function.

3) Threshold mechanism to obtain output Z:

   $Z = 0 \quad \text{if } f(x) \leq E, \quad \text{and}$

   $Z = f(x) - E \quad \text{if } f(x) > E$

   where E is a monitoring threshold.