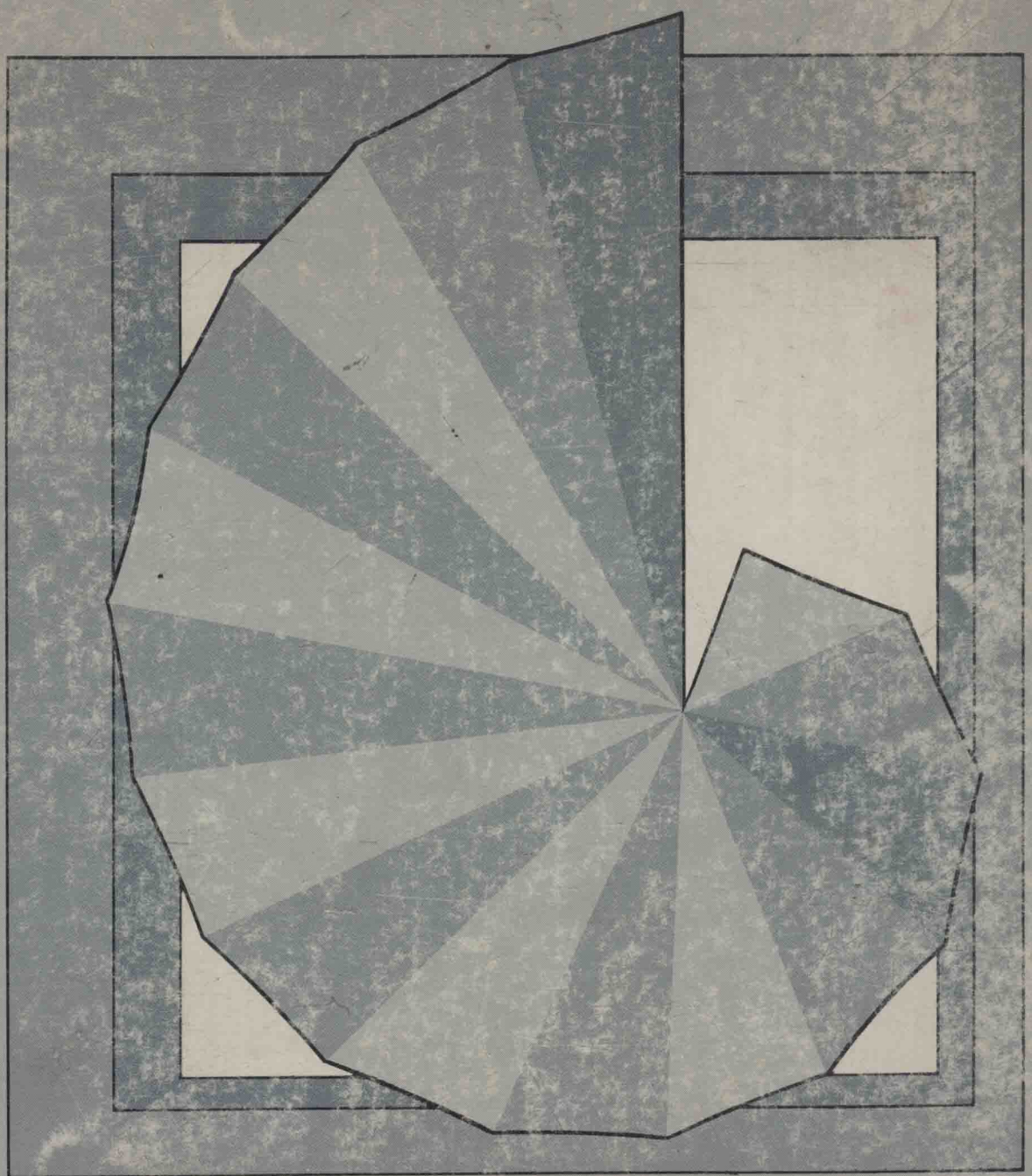
derek i. bloomfield

introductory algebra



Introductory Algebra

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To my mother.

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Preface

This book is written for anyone who needs to learn the fundamental concepts of algebra. It assumes little or no previous knowledge of algebra. The main objective of this book is to provide a clear explanation of the basic ideas of algebra with an emphasis on *understanding* rather than formality or abstraction. In addition to these much-needed algebraic skills, the student will also develop confidence in his ability to learn mathematics —perhaps for the first time in his life.

The book is intended for use in any of three instructional modes: (1) a conventional lecture-type class; (2) a mathematics laboratory along with video tapes, film loops, or slide materials; or (3) a self-study program in which the student works at his own pace.

The book consists of 12 chapters, and material in each chapter is presented in the following manner: A concise explanation of the fundamental concepts for the particular topic is given; examples illustrating these fundamental concepts are worked out step by step; and an ample number of similar problems are given in the exercise sets enabling the student to master the concepts involved. The exercises progress from simple to more difficult in an effort to give the student confidence in his ability to solve problems. The answers to odd-numbered exercises are given at the end of the text.

The major goal of this method of presentation is a correct understanding of the topics and maximum skill in performing the mathematical opera-

tions. An achievement test is given at the end of each chapter with answers to all questions given in the answer section.

A summary of arithmetic is given in the Appendix for those who need an intensive review of the basic fundamentals.

Outstanding Features

- The material is presented in measured amounts so that the student can complete the topic before moving on to the next concept.
- Explanations are short, direct, and written to be understood.
- Rules are highlighted in boxes.
- Over 600 worked-out examples with step-by-step explanations are presented with important steps highlighted with boxes.
- Over 3,000 exercises have been carefully chosen to clarify explanations and provide necessary drill.
- (STOP) warnings are given to students about common errors.
- Sufficient space is provided for working out the exercises. This also provides a good reference when it comes time to review.
- Word problems covering a wide variety of applications are found throughout the text. These show the power of algebra in real-world situations.
- Chapter summaries are provided with definitions and rules restated, along with an example of each concept for quick review.
- Achievement tests at the end of each chapter examine the student's mastery of the material in that chapter.
- Answers to odd-numbered exercises and all achievement test questions are given in the answer section. Page numbers for the answers are given after each exercise set for easy reference.
- An arithmetic summary is given in the appendix for those who need a refresher.

An Instructor's Manual Is Available Containing:

- An arithmetic diagnostic test to be given before starting Chapter 1.
- Alternate forms of achievement tests for each chapter.
- A bank of test questions that may be tailored to the instructor's needs to make up a comprehensive final exam.
- Answers to the even-numbered exercises.

ACKNOWLEDGMENTS

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My wife, Marcella, and my children, Jennifer, Max, Derek, Jr., and David, gave up many hours of their time. I thank them for their encouragement, their understanding, and their love.

A Note From the Author to the Student

I have written this book with the belief that every student can learn algebra if he wants to. If you have decided that you would like to learn algebra, here are some suggestions to help you attain success.

1. Attend all of your classes. You can't hope to learn what goes on in class unless you are there. Even if you don't understand everything that's being taught, you'll pick up a lot.
2. Work lots of problems. Even if you don't have a thorough understanding of what you're doing, if you do enough problems, the concepts eventually filter through.
3. Use your book. The worked-out examples are there to help you do the exercises. Go over the examples several times until you can repeat the procedures. Understanding will eventually follow.
4. Work on a regular basis. Try not to get behind. Solving today's problems usually depends on yesterday's results, so it's very hard to catch up once you're behind.
5. Try to remain confident. Even if you've never had success at algebra before, I'm convinced that you can succeed if you keep trying.

Contents

1
Signed Numbers

1.1 INTRODUCTION TO SIGNED NUMBERS

The first numbers seen in arithmetic are the counting numbers 1, 2, 3, 4, 5, 6, et cetera. When we add two counting numbers we always get a counting number for an answer. For example, $6 + 2 = 8$, $15 + 12 = 27$, and so on.

Now let's see what happens when we subtract one whole number from another.

$15 - 7 = 8$ a whole number

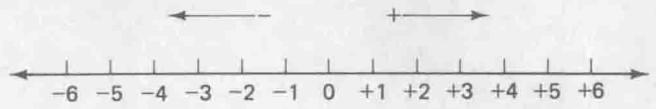
$10 - 6 = 4$ a whole number

$3 - 8 = ?$

Can such a subtraction be performed and, if so, what does the answer look like? To answer the question, consider the case where the temperature is 3 degrees above zero and then drops 8 degrees. The new temperature is 5 degrees below zero, or negative 5 degrees.

This example of temperature above and below zero nicely illustrates the concept of positive and negative numbers. These positive and negative numbers, along with zero, are called the **signed numbers,** or **integers.** This set, or group, of signed numbers is often looked at in terms of the number line. To make a number line, select a point on a line and call it zero. Then

place the positive numbers equally spaced to the right of zero and place a positive sign (+) in front of each of them. The negative numbers are equally spaced to the left of zero and a negative sign (−) is placed in front of each.



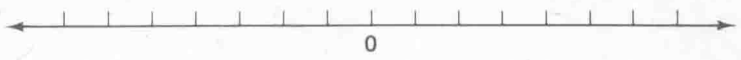
It is common practice to omit the + sign in front of positive numbers; thus a number with neither a + sign nor a − sign in front of it is positive. For example, 7 is + 7 and 34 is +34.

Signed numbers are the language of algebra, and we must become fluent in their use.

EXERCISE 1.1

Locate each of the following numbers on the number line:

1. −3
2. −1
3. +5
4. +6
5. −4
6. 5



Represent each quantity by a signed number:

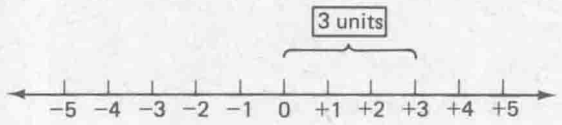
7. A bet in which you lose $5.00.
8. A bill for $23.50.
9. A mountain 7,288 feet above sea level.
10. Death Valley lies 280 feet below sea level.
11. The year 1990 A.D.
12. Euclid lived around 300 B.C.
13. A football team is penalized 15 yards.
14. You get a bonus of $500.
15. You are penalized $25 for a late payment.
16. The boiling point of water is 100° C above zero.
17. A stock goes down $2\frac{1}{4}$ points.

Answers to odd-numbered exercises on page 453.

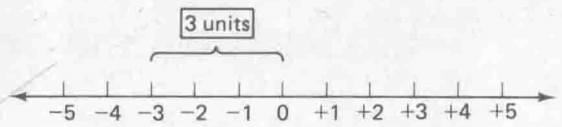
1.2 ABSOLUTE VALUE

Before we add signed numbers we must first learn a concept called **absolute value.**

The absolute value of a signed number is the number of units that it is away from zero on the number line. For example, +3 is 3 units away from zero on the number line, so the absolute value of +3 is 3, and it is written as $|+3| = 3$.



What about the absolute value of −3? Since it is also 3 units away from zero, its absolute value is also 3, which is written as $|-3| = 3$.



Example 1:

(a) $|+3| = 3$ is read "the absolute value of +3 is 3"
(b) $|-3| = 3$ is read "the absolute value of −3 is 3"
(c) $|-14| = 14$ is read "the absolute value of −14 is 14"
(d) $|0| = 0$ is read "the absolute value of zero is zero"

EXERCISE 1.2

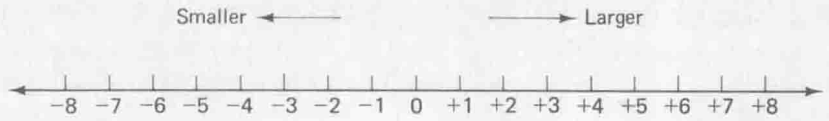
Find each of the following absolute values:

1. $|-8|$
2. $|+5|$
3. $|-4|$
4. $|-6|$
5. $|-1462|$
6. $|+46|$
7. $|0|$
8. $|+34|$
9. $|+6|$
10. $|-62|$

Answers to odd-numbered exercises on page 453.

1.3 GREATER THAN AND LESS THAN

On the number line, as we move to the right the numbers get larger, continuing on and on in such a way that there is no largest integer. Similarly, as we move to the left of zero on the number line the negative numbers have smaller values. Therefore, there is also no smallest negative integer.



From this we can see that when any two numbers on the number line are compared, the one lying to the right of the other must always be larger. For example, +6 is greater than −2 since +6 lies to the right of −2 on the number line. This is usually written as

$$+6 > -2$$

and is read "+6 is greater than −2."

We can also write

$$-2 < +6$$

which is read "−2 is less than +6."

Example 1:

(a) $6 < 18$ is read "6 is less than 18"

(b) $-3 > -8$ is read "−3 is greater than −8"

(c) $4 > -2$ is read "4 is greater than −2"

(d) $-7 < 4 < 6$ is read "−7 is less than 4 and 4 is less than 6"

Note that $4 < 8$ and $8 > 4$ really mean the same thing even though we read them differently.

The symbol

$>$ is read "is greater than"

and

$<$ is read "is less than."

You can remember which way the symbol goes by noticing that the small side of the symbol is on the side of the smaller number and the larger side of the symbol is found on the side of the larger number.

small side
smaller number $<$ large side
larger number

EXERCISE 1.3

Indicate which of the following are true and which are false:

1. $-6 < -5$
2. $3 > -4$
3. $7 > 14$
4. $6 < -7$
5. $3 > -4$
6. $-17 > -6$

7. $4 < -8.6$ 8. $3.6 < -1.4$ 9. $-784 < -326$

10. $-\frac{7}{8} < -\frac{1}{4}$ 11. $-7\frac{1}{2} > \frac{2}{3}$ 12. $\frac{5}{16} > -\frac{1}{8}$

Insert $>$ or $<$ between each pair of numbers to make a true statement:

13. 17 14 14. -4 12 15. 23 -5

16. -7 -4 17. 46 49 18. $13\frac{3}{4}$ $-11\frac{1}{2}$

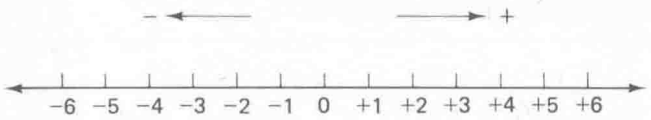
19. 0 14 20. $\frac{5}{8}$ $-2\frac{1}{3}$ 21. 586 -12

22. 0 -4 23. 314 -8.9 24. $-24\frac{2}{3}$ -25

Answers to odd-numbered exercises on page 453.

1.4 ADDING SIGNED NUMBERS

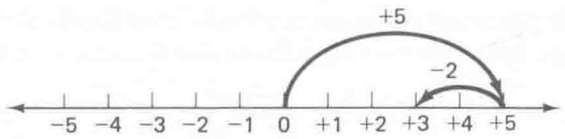
To add signed numbers we again use the number line. To add a positive number move to the right; to add a negative integer, move to the left. Always start at zero.



Let's start with an example that we already know the answer to.

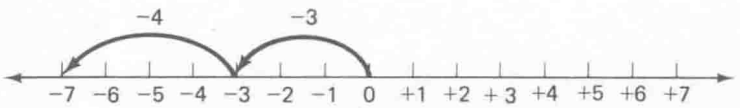
Example 1: On the number line 4 + 2 is added by starting at zero, moving 4 spaces to the right, +4, and then moving 2 more spaces to the right, +2. We end at +6, which is the answer. $4 + 2 = 6$

Example 2: $5 + (-2)$ Starting at zero, first move 5 units to the right and then 2 units to the left. The answer is +3 or 3. $5 + (-2) = 3$

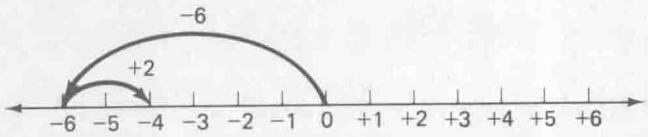


Example 3: $-3 + (-4)$

A move of 3 units to the left plus another move of 4 units to the left leaves us at -7, the answer. $-3 + (-4) = -7$



Example 4: $-6 + 2 = -4$



The sign + or − tells us which direction to move in, and the size of the number (its absolute value) tells us how far to move.

The number line provides a good visual explanation of what is taking place when we add signed numbers. However, it is cumbersome to draw a number line everytime we are asked to add two signed numbers, particularly if the numbers are very large. Therefore, we must establish a rule that provides us with the same result as using the number line.

To add signed numbers:

1. If both numbers have the same sign, add the absolute values and keep the common sign for the answer.
2. If the numbers have different signs, find the difference in their absolute values and take the sign of the number with the larger absolute value for the answer.

Example 5: $3 + 4 = 7$
3 and 4 have the same sign so we add their absolute values and keep the same sign (+).

Example 6: $-4 + 1 = -3$
−4 and +1 have different signs so we find the difference between their absolute values, 4 and 1, giving us 3, and give it the sign of the larger absolute value.

Example 7: $-6 + (-5) = -11$
−6 and −5 have the same sign so add their absolute values ($6 + 5 = 11$) and keep the common sign (−).

Example 8: $2 + (-6) = -4$
2 and −6 have different signs so we find the difference between 2 and 6, giving us 4, and take the (−) sign from the 6 since it is larger than the 2.

EXERCISE 1.4

Add the following:

1. $6 + 8$	2. $4 + 9$	3. $3 + (-6)$
4. $4 + (-7)$	5. $7 + (-5)$	6. $6 + (-8)$
7. $13 + (-6)$	8. $15 + (-3)$	9. $-6 + 5$

10. $-8 + 4$
11. $-13 + 7$
12. $-11 + 6$
13. $-26 + 16$
14. $-28 + 12$
15. $-3 + (-5)$
16. $-6 + (-3)$
17. $-8 + (-9)$
18. $-7 + (-6)$
19. $-12 + (-18)$
20. $-16 + (-24)$
21. $-\frac{3}{8} + \left(-\frac{1}{8}\right)$
22. $-\frac{2}{9} + \left(-\frac{4}{9}\right)$
23. $\frac{1}{7} + \left(-\frac{3}{7}\right)$
24. $\frac{1}{5} + \left(-\frac{2}{5}\right)$

25. In two successive nights of playing poker you lost \$2.58 the first night and won \$1.76 the second night. What were your total winnings or losses?

26. Starting with a balance of zero in his checking account a financier writes checks for \$72.05, \$31.16, \$24.17, and \$12.12. The next day he rushes to the bank and makes a deposit of \$125.00 to cover them. What is the status of his account?

27. On Monday your stock in Hexagon Oil Co. goes up $2\frac{1}{3}$ points. On Tuesday it falls $6\frac{2}{3}$ points. What is the net amount of the change for the two days and in what direction?

28. Death Valley, the lowest point in North America, is 282 feet below sea level. Mount McKinley in Alaska is 20,320 feet above sea level, which is the highest point in North America. What is the difference in altitude between these two places?

29. Place one of the signed numbers -5, -4, -3, -2, -1, 0, 1, 2, and 3 in each circle to make the sum of each side of the triangle equal to -3.