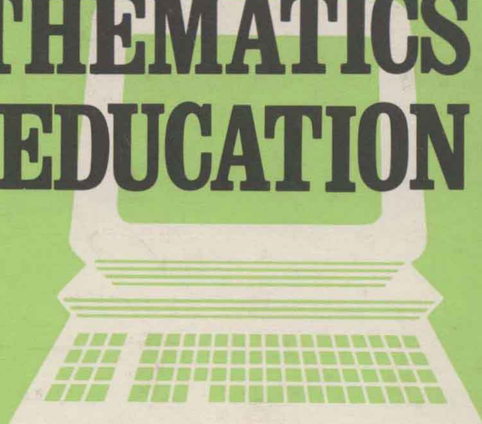
COMPUTERS IN MATHEMATICS EDUCATION



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NCTM

Computers in Mathematics Education

1984 Yearbook

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Second printing 1985

Library of Congress Cataloging in Publication Data:

Main entry under title:

Computers in mathematics education.

(Yearbook ; 1984)
Bibliography: p.
1. Mathematics—Computer-assisted instruction—Addresses, essays, lectures. I. Hansen, Viggo P. II. Zweng, Marilyn J. III. Series: Yearbook (National Council of Teachers of Mathematics) ; 1984.
QA3.N3 1984 [QA20.C65] 510.7s [510'.7'8] 84-2037
ISBN 0-87353-210-4

Printed in the United States of America

Preface

The title of this NCTM 1984 Yearbook, *Computers in Mathematics Education,* reflects a most paradoxical situation. Certainly no branch of human knowledge has withstood the test of time better than mathematics. The Pythagorean theorem is as immutable today as it was the very day it first became apparent. However, the role of the computer in mathematics education is so mutable as to make this book about it obsolete even as it is being written. Mathematics teachers have witnessed in the span of a few years an incomprehensible number of suggestions on how to teach mathematics with computers. We might add that not only was the number of suggestions incomprehensible but so were many of the suggestions themselves. If we may borrow from marketing terminology, we might say that the bottom line is extremely exciting but not clearly in focus at this time. Perhaps this is exactly the same situation we see in general for this much heralded year of 1984.

It was into this maelstrom of paradoxes that many noble NCTM authors took up their battle stations. The range and depth of ideas submitted was indicative of the impact the computer is having on many of our teaching practices. New philosophical positions began to emerge; new language, more interactive teaching strategies, and different mathematics topics arose; and most gratifying of all was the inherent enthusiasm each author projected in his or her manuscript.

As we read and studied the many submitted articles, certain generalized assumptions began to appear. A few of these can be summarized as follows: (1) The computer is definitely here to stay in education, both in the classroom and in the home; (2) to date there is still a shortage of good software; (3) we are desperately in need of more research (this is critical and will be costly); (4) the integration of the computer into the curriculum is at best in its infancy; (5) cost factors, though coming down, are still a major factor; and (6) we are painfully reminded of the need for better teacher training in this area.

Lurking in these and other, similar assumptions are additional social concerns. Who will best be able to capitalize on this technology? What is going to be the role of industry in delivering hardware and software and cooperating with educators? Finally, who will be able to afford these sophisticated devices, especially if they can be used effectively in the home? For the most part these kinds of questions are not discussed in depth in this yearbook but are crucial issues for all of us to consider when viewing the total impact the computer will have on mathematics education.

Your 1984 Yearbook Committee, which was held to a very restricted number of pages, had to select and then heavily edit submitted material to make it fit. It was with great humility and regret that we could not include all the material given us. But we were refreshed by the abundance of ideas and enthusiasm all authors exhibited. NCTM can be proud of its membership. It was recently projected that Orwell's *Nineteen Eighty-Four* will be this year's best-seller. However, our survey forecasts that this yearbook, *Computers in Mathematics Education,* will capture that honor.

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There were simply too many people involved in the preparation of this book to list all of them. But you can be certain this committee knows who they are, and we are forever indebted to them for their assistance.

VIGGO P. HANSEN
1984 Yearbook Editor

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Toward Comprehensive Instructional Computing in Mathematics

Larry L. Hatfield

THROUGHOUT the world educators are exploring the potential of the computer in instruction. Indeed, during more than twenty years of innovative efforts, we have seen considerably varied computer applications in instruction. Mathematics education has been among the most extensively explored areas of instruction, and there exists a broad base of experience and materials for mathematics instructional computing.

Let us examine a vision toward which the emerging emphasis on computers in instruction might progress. The vision may stimulate your conceptions of what might be feasible or desirable. It may suggest ideas for how to think about instructional computing in mathematics and how to consider the selection or creation of computing activities in your own classroom. Of course, it is not intended to be prescriptive, but rather provocative and suggestive of a mature approach to instructional computing in the mathematics classroom.

What Is Mathematics Instructional Computing?

Most of us realize that the computer has already assumed a ubiquitous role in our society. At first glance, it may seem obvious that the term *instructional computing* means any application of the computer that serves the goals and functions of instruction. But here we shall consider instructional computing in mathematics to include any application that gives either the student or the teacher direct contact with the computer and serves the goals and functions of mathematics instruction.

The past two decades have witnessed considerable exploratory innovations of computing in classrooms. You may be surprised at the wide range of applications that have been tried. But amid the growing flood of computer programs for instruction, one may identify some distinguishing characteristics. For the purposes of this article, we shall divide the domain of mathematics instructional computing into three broad categories: *student programming, computer-based instruction,* and *teacher utilities.*

Student programming

When mathematics learners construct their own computer programs, powerful learning experiences can occur. Indeed, many mathematics teachers have realized the apparent connections between the thinking involved in building, testing, correcting, and refining one's own computer algorithm and many aspects of mathematical thought. In addition, many mathematics educators have recognized the potential effects that solving a curriculum-oriented computer programming task can have on stimulating or enhancing understandings of a mathematical concept, problem, or procedure. It has seemed almost a natural relationship for computer programming to be taught within the mathematics curriculum; this may have led to serious distortions of our goals and objectives for a sound mathematical education. We know that students can learn to write computer programs; we must now consider the more critical questions about why our mathematics students should become engaged in such tasks.

Computer-based instruction

The distinguishing feature of *computer-based instruction* is the execution of a prepared computer program by the student or the teacher for the purposes of instructing. For the student the focus is on the *output* of the program, not on the program's code or logic. The student does not become involved with the program as an object, only as a generator of tasks, text, graphics, sounds, or stored records. Within this category we include eight types of uses: practicing, tutoring, simulating, gaming, demonstrating, testing, informing, and communicating. It should be noted that precise classification is difficult; these terms only suggest a working framework. Each use is described briefly.

Practicing. Computer-based practicing involves the student in rehearsing different elements of thinking and behaving to bring about more effective performance. Most educators believe that a student should engage in practice after certain meanings and understandings about a situation have been constructed. Thus, practice sessions at a computer should be predicated on a readiness to rehearse based on a background of meaning and motivation. This usually includes no attempt to present underlying explanations, although some computer-based practice programs attempt to incorporate brief reviews or diagnostic comments to be used when the student errs.

Tutoring. What is usually sought in computer-based tutoring is a high-fidelity simulation of a knowledgeable, sensitive, stimulating teacher's live instruction. Obviously, most examples of such computer programs fall considerably short of this intent. Yet a tutoring program may attempt to approximate a flow of activity similar to ordinary classroom discourse: an *introduc-*

tion, or overview of the session; *explanations* with examples and characteristics of the ideas being taught; *questions* aimed at checking the student's comprehension of the idea at different points in the development; differing *teaching branches* to be followed, depending on the student's responses; correct *feedback* and appropriate *reinforcement* following responses; and *records* of what and how the student performed during the session.

Simulating. Computer-based simulating involves the student in experiences with certain aspects of some true-to-life environment or phenomenon in ways that approximate the real events. The program incorporates certain features of the phenomenon through a model embodied in the program and its execution. It is usually possible to manipulate, interactively, various parameters or variables in order to examine the effects of choices or random factors. Mathematics instruction that incorporates simulating may include demonstrations of the phenomenon, explorations of the functional aspects of the underlying model, or applications of ideas being studied.

Gaming. Computer-based gaming for mathematics instruction is among the most stimulating uses for students. These computer programs, which are essentially a special kind of simulation, offer competitive situations in which one or more persons can play, score, and win. Our society abounds with electronic games; our focus on gaming should feature worthwhile learning activities related to the objectives, contents, and processes deemed integral to our mathematics curriculum. One unique focus could be on learning about a game as a system or structure.

Demonstrating. The mathematics teacher can execute a computer program to *demonstrate* an idea or a process. The computer can augment a presentation by generating examples, instances, summaries, counterexamples, illustrations, or questions. The machine can be made to produce such displays rapidly, accurately, and flexibly. The teacher needs to integrate such demonstrations into the overall development of instruction by stimulating, guiding, questioning, pacing, and directing.

Testing. In computer-based testing, the student interacts with a computer program that presents test items, accepts responses, and records and reports scores, all for the purpose of student evaluation. Such programs may simply display a single, stored test; more complex testing programs can generate multiple forms of a test from item pools or item forms, using prescribed or random criteria given by the teacher.

Informing. Students can access and query a stored computer-based reference system to gain information. In combination with video (for example, video cassettes and videodiscs), it is possible to provide high-speed access to textual as well as animated displays of information. Of course, the significant elements of instruction will still focus on how to use such information in the situations and challenges of mathematics.

Communicating. In computer-based instruction, communicating involves the use of a computer as a word-processing system by the student. Appropriate examples of such word-processing programs will be student-oriented, embodying not only the "forgiving" ways that compositions can be easily corrected and changed but also instructional helps with spelling, vocabulary, and grammar. Mathematics students involved in significant problematic projects could be expected to write and refine substantial reports of their efforts and results. Additionally, classroom "bulletin boards" that foster shared notices, both private and public, would be possible.

Teacher utilities

The third category of mathematics instructional computing employs computer programs executed by the teacher that serves as an electronic teacher's aide. The teacher's *utility* focus is on the output of the program. Within this category we include six types of uses: *generating tests, generating curriculum materials, scoring and analyzing tests, grading, managing,* and *communicating*. In general, these applications can enhance aspects of a teacher's responsibilities in planning, managing, evaluating, and reporting on instruction and learning. The computer programs usually feature interactive sessions in which stored files are retrieved, created, or updated.

What Are We Doing?

The computer has proved to be a provocative, motivating device in instruction. Students and teachers alike become excited and enthusiastic about the computer as a new tool in the classroom. However, we should briefly identify certain limitations or flaws in what has been attempted, as a basis for considering more mature manifestations for the future.

Rationales. Often, it appears, eager educators have acquired and used classroom computers simply to become part of the exciting computer age. Too little attention is given to the potential impacts of computing technology in relation to what we conceptualize education to be. We must now rethink the question, What can mathematics education become? Our rationales must acknowledge our goals and beliefs and consider heretofore unattainable purposes and procedures for educating. We must focus attention on learning as a person-centered, constructive process whose purpose is to build up and refine knowledge. Mathematics instruction must be in a context and include a process for involving the learner in problematic approaches that stimulate the development of a concept, a generalization, a procedure, or a solution. Accepting a challenge, constructing an understanding, and reflecting on the processes of thinking that might be used are all aspects of the problematic approach. A classroom computer can foster mathematical learning based on such constructivist, process-oriented, problem-solving approaches.

Roles. The changing roles of the teacher and the learner have not been carefully considered in our past efforts with instructional computing. Too often, mathematics teachers have been informers and students have been receivers. Indeed, the computer may be used as an interactive source of information. But it may also be used as a constructive context in which a learner solves, generalizes, abstracts, conjectures, searches, observes, simplifies, experiments, deduces, or remembers. Whether we mathematics teachers stimulate such processes through instructional computing will depend on the activities and tasks in which we involve our students. The teaching act may be changed, and it is to be hoped it will be changed in ways that capitalize on the computer as an instructional tool and that put the mathematics teachers into deep, personal interactions with the student. For example, in the domain of student programming, the teacher can become much more involved in setting *algorithmic tasks as problems to be solved* rather than modeling or dictating a single, inflexible procedure to be imitated by all students. The learning act may also be changed, and we hope it will be changed in ways that allow firsthand *experiences* with ideas significant to mathematical knowledge, personal *activity and responsibility* in building and refining one's own mathematics, and *individuality* in the ways and results of coming to know mathematics.

Development and research. Though our past efforts have been largely innovative, our approaches in developing classroom computing have been *single-use approaches.* That is, innovators have singled out one type of use, such as practicing or testing or student programming, to the exclusion of other kinds of instructional computing. Once a particular strategy for using the machine was identified, many of the pioneering developments sought to draw choice situations from school curricula that would permit the exhibition of that kind of computing use. In other words, school mathematics has been used to enhance the visibility of the computing application instead of the computer being used systematically to enhance mathematics curriculum. This has resulted in a sporadic, helter-skelter employment of the computer in perspectives too narrow to reflect the broad potentials of instructional computing.

Criticisms related to particular uses can be more specific. For example, mathematics educators have widely adopted student programming in the mathematics curriculum. Too often we have witnessed the isolated computer programming course as a mistaken approach:

1. As a "computer science" course, its primary objectives involve the learning of programming rather than mathematics.
2. As an upper-level elective, often with a prerequisite of algebra or an

even higher course, it delays computing and restricts it to an intellectually elite subset.

3. As a course taught only by a specially prepared teacher, it fosters the attitude that computing has no place in any other part of the curriculum, thus obscuring its potential impact on all levels of mathematics instruction.
4. Such a narrow implementation of programming portrays computing as an end rather than a means, and results in minimal impact on students.

Educators have also failed to analyze carefully the pedagogical and psychological bases for including programming tasks: the focus must be on stimulating and guiding students as they construct the ideas and processes we call mathematical knowledge. Curriculum developers have included computer-programming activities as optional enrichment rather than as significant situations embodying mathematical thinking essential to the goals of the curriculum.

Developing effective computer-based instruction can be a complex, time-consuming endeavor. We have all experienced productions that present poorly conceptualized and weakly operationalized forms of teaching and learning. When the capabilities and enhancements of the computer are not capitalized on, the machine becomes a shallow, electronic exposition. Artistic, sensitive, knowledgeable mathematics teachers must be involved in direct ways with the construction of effective computer-based mathematics instruction.

Little research has been undertaken to investigate the design of instructional computing in mathematics or the effects of using diversified computing in the classroom. A few studies have examined the relative effects of student programming on the learning of mathematics. Also, some evaluation studies have sought to identify the effectiveness of some forms of computer-based instruction, such as practicing or gaming. Overall, our knowledge base for developing or implementing instructional computing in mathematics is extremely thin, relying primarily on intuitive notions of "good" teaching.

Implementation. Teachers, students, principals, school boards, and parents eagerly desire the "computer age" for their schools. In seeking to begin, we can easily err. One serious mistake that can result is the absence of thoughtful *planning*. In an eagerness to get something started, little or no thought is given to how or why we should be using a classroom computer. Sometimes a top-down decision comes from an administrator; or the "isolated enthusiast" may be a teacher who tries to get something started. In any event, without comprehensive planning that attempts to build rationales and perspectives for integrating instructional computing throughout the mathematics curriculum, efforts for implementation have narrow, limited impact. Few teachers, few students, few curricular goals are affected.

Too often, educators have tunnel vision in viewing the implementation of instructional computing. Mathematics teachers have primarily accepted computer-programming, or perhaps practicing, usages. Students are typically allowed to use the computer in one way. These experiences can bear little or no relationship in the mind of the child to what might be going on in the learning activities of the regular classroom instruction of the day (e.g., when students go to a practicing session "down the hall" apart from the current mathematical studies, or when students solve computer-programming tasks that have no current relationship to the mathematics being studied). Thus, the *multiple uses* of the computer, *embedded* in the mainstream mathematical curriculum, have rarely been achieved.

Lastly, we must be sensitive to potential misuses and abuses of the computer. It is wrong to expect the computer to take over. Computers do not make good replacements for effective mathematics teachers. When using student programming, we must employ a constructive, problematic pedagogy; it violates the interactive, solving context to give completed computer programs to our students. When conducting any computer-based instruction session, we must realize that students will need preparation and follow-up if the session is to be an effective, integrated experience. In our classroom implementations we must avoid sporadic, hit-or-miss approaches and, instead, base our use of the computer on thoughtful, careful planning that embeds and integrates computing applications into the curricular goals and teaching strategies.

Multiusage Approaches

Goals

Instructional computing in mathematics must be determined through a careful construction of our planning for instruction. Classroom computing is a context of activity for learning and teaching; computers are tools to aid in the construction of knowledge of mathematics. You are well aware that any teaching context or aid must be fitted into plans for teaching *after* many thoughtful decisions have been made regarding the educational goals, objectives, and subject matter. To avoid the possible distortions to a sound mathematics curriculum that such an exciting, eye-catching technology as the microcomputer may produce, we must first determine what sorts of knowledges (ideas and processes) we seek to have students construct. Of course, we must be alert to the impacts of the computer on the discipline of mathematics and the myriad domains of mathematical applications in our culture. We must keep abreast of how the computing and informational technologies are altering the problems or tasks, as well as the solution strategies, that producers and consumers of mathematics employ. But any implementation of instructional computing must be preceded by careful decisions about the purposes and contents of our mathematics curricula.

Presuming that a thorough analysis of the aims and contents of the mathematics curriculum has been completed, then it is appropriate to consider the full range of the computer's potential for our instruction. The perspective of a *multi-use approach* is embodied in the following question to be posed for each instructional objective or topic:

> Of *all* the known ways that a computer can be used for instruction, which may be effective for the learning experiences we wish to provide in teaching a particular topic?

That is, for each objective or topic, we would consider each of the approaches within instructional computing (student programming, eight types of computer-based instruction, six types of teacher utilities) as potential applications of the computer.

Obviously, not all uses of instructional computing by either student or teacher can occur within all units of instruction. Indeed, it would be a serious distortion to force usages where they are not appropriate. But there are sensible and varied applications that can contribute substantively to the different aspects of learning and teaching mathematics. What is sought through the concept of *multiuse approaches* is a recognition of the rich, varied potential that instructional computing can offer as a context and a tool for furthering the goals and objectives of mathematics instruction.

Strategies

What are the appropriate tactics or strategies for developing a multiuse approach? The following advice may suggest ways to proceed:

1. Construct a comprehensive view of instructional computing, especially acknowledging a perspective of multiuse approaches. You will need to learn some computer programming and how to build student programming tasks appropriate to school mathematics learning. You will also need to learn to "play student" and "play teacher" to computer-based instruction programs.

2. In terms of your own commitments and classroom computing resources, set some sensible long-term goals. Ask, "What might mathematics learning and teaching in my classroom look like in three years?" To set goals, you will need to build visions.

3. In terms of your visions and goals, you will need to set some priorities. Ask, "What can I reasonably attempt first? Where would I like to be with my instructional computing in one year?" You might focus on certain topics and certain types of computing. You might decide to develop fairly extensive multiuse approaches for one or two topics or units of instruction during the first year. Or you might want to focus on developing one mode of computer usage for several topics.

4. Engage in mature curriculum planning in order to specify the approach. Decide on the goals and content of the curriculum. Only after these

decisions have been made should the computer decisions be formulated. Avoid the computer as a "tail wagging" distorter of the curriculum.

5. Construct a detailed list describing the computer programs that will be needed during the instruction of the unit. Can you select and acquire needed programs, or will you write them yourself?

6. Plan for the episodes involving computing sessions to occur within your overall lesson plans.

7. Reflect on, and evaluate, your instructional computing accomplishments in terms of your goals. Ask, "In what ways might my conduct of this instructional unit be improved next time through an enhanced application of computing?" Such self-evaluation is important; the complex innovations that comprehensive computing may make possible will probably evolve through a repeated "successive approximation" in our classroom.

Mathematics teachers stand at the threshold of a new era in education. Imagine how you might have felt if, as a teacher, you had learned of a technological breakthrough called a printing press, which would become an easy, inexpensive, reliable, comprehensive, graphical, dynamic instrument for transmitting information and stimulating learning! Computing technology constitutes such a revolution.

Meanwhile, it is *we* who face the challenge and excitement of exploring the potentials of the microcomputer in the classroom. What strategies are possible for implementing instructional computing? What steps are advisable? What cautions might we observe? How might instructional computing change mathematics education, allowing us to approach teaching in new ways? What traditional ideas and processes will the microcomputer make irrelevant? What new content emphases, such as algorithmics, might be amplified in our curricula? These are complex questions requiring our thoughtful, deliberate attention. Perhaps the most important step is simply to begin.

Yet during the pressure to "get something started," we must pause to realize the necessity to proceed rationally. Comprehensive planning at all levels, including the classroom, school system, state, and nation, is needed. Our vision of what mathematics educating *might be,* given the computational, literal, graphical, and analytical power of a classroom microcomputer, must be open and constructive. Mathematics teachers need to study state-of-the-art instructional computing—from the point of view of both the student and the teacher; staff development must be included in our comprehensive planning. And throughout our explorations we must avoid computing activities whose only basis may be "doing computing activities." Computing activities must fit into our mathematical instruction in terms of the goals and objectives, the mathematical knowledge to be built, the overall classroom framework as a social activity involving students and teacher, and the beliefs we hold regarding mathematics learning and teaching.