

# *A modern course in aeroelasticity*

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# *Preface*

A reader who achieves a substantial command of the material contained in this book should be able to read with understanding most of the literature in the field. Possible exceptions may be certain special aspects of the subject such as the aeroelasticity of plates and shells or the use of electronic feedback control to modify aeroelastic behavior. The first author has considered the former topic in a separate volume. The latter topic is also deserving of a separate volume.

In the first portion of the book the basic physical phenomena of divergence, control surface effectiveness, flutter and gust response of aeronautical vehicles are treated. As an indication of the expanding scope of the field, representative examples are also drawn from the non-aeronautical literature. To aid the student who is encountering these phenomena for the first time, each is introduced in the context of a simple physical model and then reconsidered systematically in more complicated models using more sophisticated mathematics.

Beyond the introductory portion of the book, there are several special features of the text. One is the treatment of unsteady aerodynamics. This crucial part of aeroelasticity is usually the most difficult for the experienced practitioner as well as the student. The discussion is developed from the basic fluid mechanics and includes a comprehensive review of the fundamental theory underlying numerical lifting surface analysis. Not only the well known results for subsonic and supersonic flow are covered; but also some of the recent developments for transonic flow, which hold promise of bringing effective solution techniques to this important regime.

Professor Sisto's chapter on Stall Flutter is an authoritative account of this important topic. A difficult and still incompletely understood phenomenon, stall flutter is discussed in terms of its fundamental aspects as well as its significance in applications. The reader will find this chapter particularly helpful as an introduction to this complex subject.

## Preface

Another special feature is a series of chapters on three areas of advanced application of the fundamentals of aeroelasticity. The first of these is a discussion of Aeroelastic Problems of Civil Engineering Structures by Professor Scanlan. The next is a discussion of Aeroelasticity of Helicopters and V/STOL aircraft by Professor Curtiss. The final chapter in this series treats Aeroelasticity in Turbomachines and is by Professor Sisto. This series of chapters is unique in the aeroelasticity literature and the first author feels particularly fortunate to have the contributions of these eminent experts.

The emphasis in this book is on fundamentals because no single volume can hope to be comprehensive in terms of applications. However, the above three chapters should give the reader an appreciation for the relationship between theory and practice. One of the continual fascinations of aeroelasticity is this close interplay between fundamentals and applications. If one is to deal successfully with applications, a solid grounding in the fundamentals is essential.

For the beginning student, a first course in aeroelasticity could cover Chapters 1–3 and selected portions of 4. For a second course and the advanced student or research worker, the remaining Chapters would be appropriate. In the latter portions of the book, more comprehensive literature citations are given to permit ready access to the current literature.

The reader familiar with the standard texts by Scanlan and Rosenbaum, Fung, Bisplinghoff, Ashley and Halfman and Bisplinghoff and Ashley will appreciate readily the debt the authors owe to them. Recent books by Petre\* and Forsching† should also be mentioned though these are less accessible to an english speaking audience. It is hoped the reader will find this volume a worthy successor.

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\*Petre, A., *Theory of Aeroelasticity. Vol. I Statics, Vol. II Dynamics*. In Romanian. Publishing House of the Academy of the Socialist Republic of Romania, Bucharest, 1966.

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Landahl, M. T. and Stark, V. J. E., 'Numerical Lifting Surface Theory-Problems and Progress', *AIAA Journal*, No. 6, No. 11, November 1968, pp. 2049-2060.

Many Authors, 'Symposium on Fluid-Solid Interaction', *ASME Annual Winter Meeting*, November 1967.

### *Journals*

AIAA Journal

ASCE Transactions, Engineering Mechanics Division

ASME Transactions, Journal of Applied Mechanics

International Journal of Solids and Structures

Journal of Aircraft

Journal of Sound and Vibration

Other journals will have aeroelasticity articles, of course, but these are among those with the most consistent coverage.

The impact of aeroelasticity on design is not discussed in any detail in this book. For insight into this important area the reader may consult the following volumes prepared by the National Aeronautics and Space Administration in its series on SPACE VEHICLE DESIGN CRITERIA. Although these documents focus on space vehicle applications, much of the material is relevant to aircraft as well. The depth and breadth of coverage varies considerably from one volume to the next, but each contains at least a brief State-of-the-Art review of its topic as well as a discussion of Recommended Design Practices. Further some important topics are included which have not been treated at all in the present book. These include, as already mentioned in the Preface,

Aeroelasticity of plates and shells (panel flutter) (NASA SP-8004)

Aeroelastic effects on control system dynamics (NASA SP-8016, NASA SP-8036 NASA SP-8079)

as well as

Structural response to time-dependent separated fluid flows (buffeting) (NASA SP-8001)

Fluid motions inside elastic containers (fuel sloshing) (NASA SP-8009, NASA SP-8031)

Coupled structural—propulsion instability (POGO) (NASA SP-8055)

It is intended to revise these volumes periodically to keep them up-to-date.

NASA SP-8001 1970

Buffeting During Atmospheric Ascent

NASA SP-8002 1964

Flight Loads Measurements During Launch and Exit

NASA SP-8003 1964

Flutter, Buzz, and Divergence

NASA SP-8004 1972

Panel Flutter

NASA SP-8006 1965

Local Steady Aerodynamic Loads During Launch and Exit

NASA SP-8008 1965

Prelaunch Ground Wind Loads

NASA SP-8012 1968

Natural Vibration Modal Analysis

## *Short bibliography*

NASA SP-8016 1969

Effects of Structural Flexibility on Spacecraft Control Systems

NASA SP-8009 1968

Propellant Slosh Loads

NASA SP-8031 1969

Slosh Suppression

NASA SP-8035 1970

Wind Loads During Ascent

NASA SP-8036 1970

Effects of Structural Flexibility on Launch Vehicle Control Systems

NASA SP-8050 1970

Structural Vibration Prediction

NASA SP-8055 1970

Prevention of Coupled Structure-Propulsion Instability (POGO)

NASA SP-8079 1971

Structural Interaction with Control Systems.

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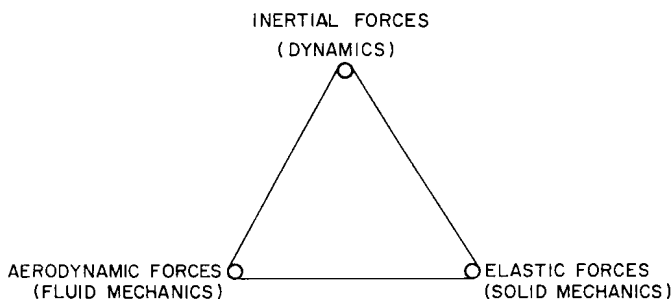
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## Introduction

Several years ago, Collar suggested that aeroelasticity could be usefully visualized as forming a triangle of disciplines.



Aeroelasticity is concerned with those physical phenomena which involve significant mutual interaction among inertial, elastic and aerodynamic forces. Other important technical fields can be identified by pairing the several points of the triangle. For example,

Stability and control (flight mechanics) = dynamics + aerodynamics

Structural vibrations = dynamics + solid mechanics

Static aeroelasticity = fluid mechanics + solid mechanics

Conceptually, each of these technical fields may be thought of as a special aspect of aeroelasticity. For historical reasons only the last topic, static aeroelasticity, is normally so considered. However, the impact of aeroelasticity on stability and control (flight mechanics) has increased substantially in recent years, for example.

In modern aerospace vehicles, life can be even more complicated. For example, stresses induced by high temperature environments can be important in aeroelastic problems, hence the term

‘aerothermoelasticity’

## 1 *Introduction*

In other applications, the dynamics of the guidance and control system may significantly affect aeroelastic problems or vice versa, hence the term 'aeroservoelasticity'

For a historical discussion of aeroelasticity including its impact on aerospace vehicle design, consult Chapter I of Bisplinghoff and Ashley (BA) and AGARD C.P. No. 46, 'Aeroelastic Effects from a Flight Mechanics Standpoint'.

We shall first concentrate on the dynamics and solid mechanics aspects of aeroelasticity with the aerodynamic forces taken as given. Subsequently, the aerodynamic aspects of aeroelasticity shall be treated from first principles. Theoretical methods will be emphasized, although these will be related to experimental methods and results where this will add to our understanding of the theory and its limitations. For simplicity, we shall begin with the special case of static aeroelasticity.

Although the technological cutting edge of the field of aeroelasticity has centered in the past on aeronautical applications, applications are found at an increasing rate in civil engineering, e.g., flows about bridges and tall buildings; mechanical engineering, e.g., flows around turbomachinery blades and fluid flows in flexible pipes; and nuclear engineering; e.g., flows about fuel elements and heat exchanger vanes. It may well be that such applications will increase in both absolute and relative number as the technology in these areas demands lighter weight structures under more severe flow conditions. Much of the fundamental theoretical and experimental developments can be applied to these areas as well and indeed it is hoped that a common language can be used in these several areas of technology. To further this hope we shall discuss subsequently in some detail several nonairfoil examples, even though our principal focus shall be on aeronautical problems. Separate chapters on civil engineering, turbomachinery and helicopter (rotor systems) applications will introduce the reader to the fascinating phenomena which arise in these fields.

Since most aeroelastic phenomena are of an undesirable character, leading to loss of design effectiveness or even sometimes spectacular structural failure as in the case of aircraft wing flutter or the Tacoma Narrows Bridge disaster, the spreading importance of aeroelastic effects will not be warmly welcomed by most design engineers. However, the mastery of the material to be discussed here will permit these effects to be better understood and dealt with if not completely overcome.

# Static aeroelasticity

## 2.1 Typical section model of an airfoil

We shall find a simple, somewhat contrived, physical system useful for introducing several aeroelastic problems. This is the so-called ‘typical section’ which is a popular pedagogical device.\* This simplified aeroelastic system consists of a rigid, flat plate airfoil mounted on a torsional spring attached to a wind tunnel wall. See Figure 2.1; the airflow over the airfoil is from left to right.

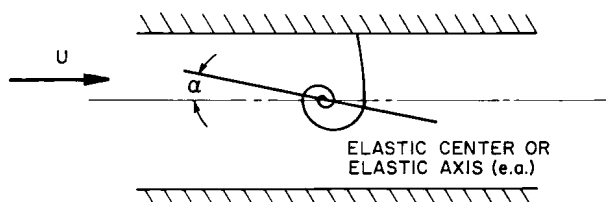


Figure 2.1 Geometry of typical section airfoil.

The principal interest in this model for the aeroelastician is the rotation of the plate (and consequent twisting of the spring),  $\alpha$ , as a function of airspeed. If the spring were very stiff or airspeed were very slow, the rotation would be rather small; however, for flexible springs or high flow velocities the rotation may twist the spring beyond its ultimate strength and lead to structural failure. A typical plot of elastic twist,  $\alpha_e$ , vs airspeed,  $U$ , is given in Figure 2.2. The airspeed at which the elastic twist increases rapidly to the point of failure is called the ‘divergence airspeed’,  $U_D$ . A major aim of any theoretical model is to accurately predict  $U_D$ . It should be emphasized that the above curve is representative not only of our typical section model but also of real aircraft wings. Indeed the

\* See Chapter 6, BA, especially pp. 189–200.



## 2 Static aeroelasticity

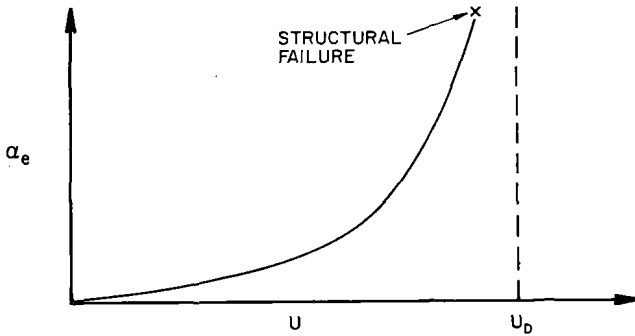


Figure 2.2 Elastic twist vs airspeed.

primary difference is not in the basic physical phenomenon of divergence but rather in the elaborateness of the theoretical analysis required to predict accurately  $U_D$  for an aircraft wing versus that required for our simple typical section model.

To determine  $U_D$  theoretically we proceed as follows. The equation of static equilibrium simply states that the sum of aerodynamic plus elastic moments about any point on the airfoil is zero. By convention, we take the point about which moments are summed as the point of spring attachment, the so-called 'elastic center' or 'elastic axis' of the airfoil.

The total aerodynamic angle of attack,  $\alpha$ , is taken as the sum of some initial angle of attack,  $\alpha_0$  (with the spring untwisted), plus an additional increment due to elastic twist of the spring,  $\alpha_e$ .

$$\alpha = \alpha_0 + \alpha_e \quad (2.1.1)$$

In addition, we define a point on the airfoil known as the 'aerodynamic center'.\* This is the point on the airfoil about which the aerodynamic moment is independent of angle of attack,  $\alpha$ . Thus, we may write the moment about the elastic axis as

$$M_y = M_{AC} + Le \quad (2.1.2)$$

where  $M_y$  moment about elastic axis or center  
 $M_{AC}$  moment about aerodynamic center,  
 both moments are positive nose up  
 $L$  lift, net vertical force positive up  
 $e$  distance from aerodynamic center to  
 elastic axis, positive aft.

\* For two dimensional, incompressible flow this is at the airfoil quarter-chord; for supersonic flow it moves back to the half-chord. See Ashley and Landahl [1]. References are given at the end of each chapter.