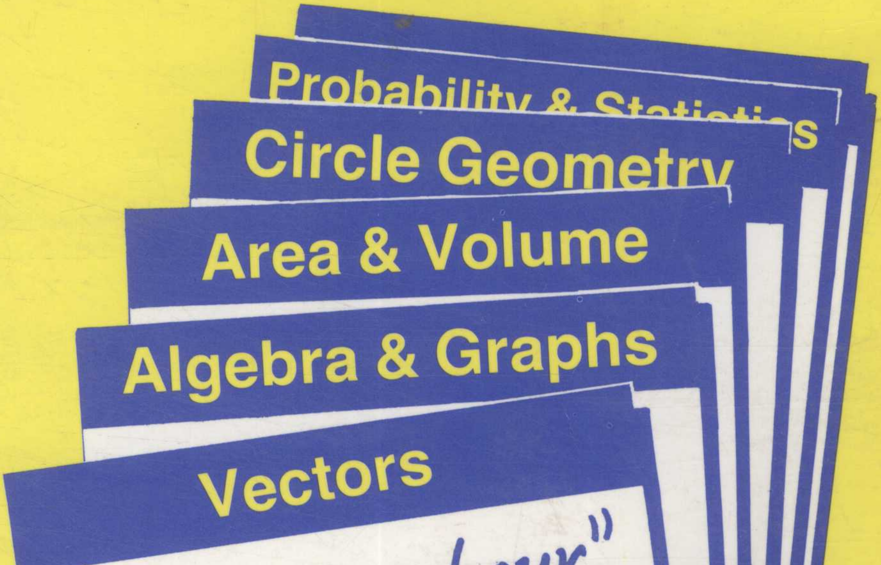
GCSE

MATHEMATICS (HIGHER LEVEL)

ONE HOUR PRACTICE PAPERS



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PREFACE

1. **Aim**

   The aim of this booklet is to provide *sufficient* GCSE Practice Papers to enable teachers, parents and pupils to know how well prepared the pupils are for their GCSE Mathematics examination at Higher Level.

2. **Need**

   The need was seen for a means of *testing* (both in the home and in the classroom) the *combination* of *breadth of knowledge* and the *speed needed* to convey that knowledge to the examiner.

3. **Approach**

   There are *two* sets of *one hour* Practice Papers for *each* topic or group of topics (Area and Volume: Matrices and Transformations etc.). One set is supplied with answers and is, therefore, ideal for self testing at home. The other, intended for classroom progress tests, has no asnwers in this booklet.

   In all, there are *ten* pairs of topic-based papers *plus* two *mixed* topics for mock examinations/final revision.

NOTES:-

(a) The GCSE Board's Syllabi, and their Sample Papers, have all been closely studied and their main requirements incorporated.

(b) One hour was chosen as feasible in a *double period* at school and as a *reasonable time* to set aside for home self testing.

(c) A teacher's guide is available, free of charge, to teachers using this booklet in their courses, providing answers to the classwork papers.

Table of Contents

PART I

GCSE PRACTICE PAPERS WITH ANSWERS

TIME ALLOWED
1 HOUR EACH PAPER

INTRODUCTION AND CONTENTS

Set aside an hour, *periodically*, to attempt each of the topic-based papers as you cover them in your studies. Check your answers *after* you have completed a paper.

In addition to the ten topic-based papers, there are two mixed example papers for final revision purposes. The first of the mixed papers has *short* questions and is designed to take you about 1½ hours. The second has a number of longer type questions for which around 12½ minutes should be allowed for each.

On the inside back cover is a formula sheet similar to one you will be provided with in your examination.

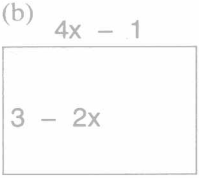
Paper

ALGEBRA AND GRAPHS

TIME 1 HOUR

1. (a) Solve the equation $5 - 3x = 2x - 10$.

(b)



The perimeter of this rectangle is 8 cm.
Calculate x.

(c) Solve the simultaneous equations: $x - 2y = 4$, and $5x + y = 9$.

(d) Solve the quadratic equation $4x^2 - 7x - 2 = 0$.

2. (a) The equation of a straight line is $4x + 5y = 20$. What is its gradient?

(b) What is the equation of the straight line which passes through

$(1, -3)$ and $(2, -5)$?

(c) Taking values of x as ± 3, ± 2, ± 1, $\pm \frac{1}{2}$, draw a sketch of the graph of

$$y = \frac{1}{x^2}$$

(d) Use your graph to solve the equation

$$x = \frac{1}{x^2}$$

3.

x	−2	−1	0	1	2	3
y			−5			−5

(a) Copy and complete the table for $y = x^2 - 3x - 5$.

(b) On a sheet of graph paper choose suitable axes and draw the graph of the function.

(c) Draw a tangent to the graph at the point $x = 2$, and find the gradient at this point.

(d) Write down the equation of the line of symmetry of the curve, and state the co-ordinates of the point where it intersects the curve.

4. (a) $x = m^2 - n^2$. Calculate x when $m = 2\frac{1}{2}$ and $n = -1\frac{1}{2}$.

(b) Factorise the expressions: (i) $4x^2 + 8xy$ (ii) $3x^2 - 5x - 2$.

(c) Rearrange the formula $n = v - mx^2$, to make x the subject.

(d) In the formula $k^2 = 100 - m^2$, what are the largest and smallest numerical values m can have if k is a real number?

5. (a) y varies inversely as the square of x; if $y = 10$ when $x = 3$, find x when $y = 360$

(b) Simplify $-3(2x - 4) + 5(6 - 3x)$

(c) Factorise $ax + 3y - ay - 3x$

(d) Expand $(3 - 2x)^2$, and state its value when $x = -1$.

6. (a) Simplify $x^{2/3}$ when $x = 64$.

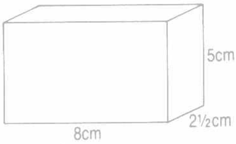
(b) The difference between two numbers is 6; their product is 16. Let the larger number be x, and form a quadratic equation in x. Hence find the two numbers.

(c) The gradient of a straight line is 3; it passes through (4,7). Calculate the intercept on the y–axis.

(d) The graph of a quadratic function intersects the x–axis at $x = 1$ and $x = 4$. Write down an equation of the form $y = ax^2 + bx + c$, where a, b, and c, are whole numbers, to satisfy the above.

AREA AND VOLUME
TIME 1 HOUR

1.



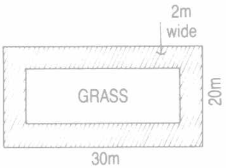
(a) What is the volume of the cuboid?

(b) Calculate the surface area of the cuboid.

(c) Another cuboid has measurements 16cm x 5cm x 10cm. What is the ratio of the surface area of the smaller cuboid to that of the larger?

(d) What is the least number of the original cuboids which must be stacked together to make a cube of side 40cm?

2. The diagram below shows a rectangular garden measuring 30m x 20m; there is a tiled path, 2m wide, all around the edge of the garden.



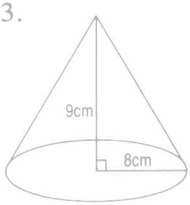
(a) Calculate the area of the grass portion of the garden.

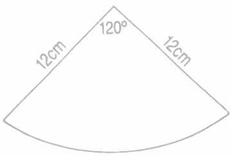
(b) If the path is covered with square tiles 20cm x 20cm, calculate the number of tiles needed to cover the path.

(c) The owner decides to plant a circular flower bed of radius 2m in the garden; calculate the area of the flower bed, taking π as 3.14.

(d) In order to get the flower bed ready the owner had to dig the soil to a depth of 20cm; calculate in cm^3 the volume of soil she moved, giving your answer to 2 significant figures. Take π as 3.14.

3.





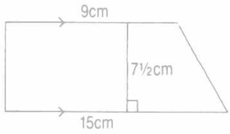
(a) Calculate the volume of the cone, giving your answer to the nearest whole number.

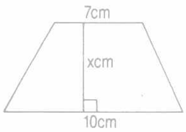
(b) Another cone has the same height, but its base radius is 6cm; what is the ratio of the volumes of the two cones?

(c) A cylinder of radius 8cm has the same volume as the cone; calculate the height of the cylinder.

(d) The sector of a circle (shown left) is to be folded to make a cone; calculate the radius of the base of this new cone. [You are advised not to substitute for π].

4.





(a) Calculate the area of the trapezium.

(b) A similar trapezium has dimensions ⅓ of this one; state the relationship between the areas of the two trapeziums and use it to write down the area of the smaller one.

(c) What is the definition of a prism?

(d) The trapezium (shown left) is the cross-section of a block of timber 300cm long, with volume $0.03825m^3$. Calculate the value of x.

5. A factory makes spherical balls of radius 2cm; taking $\pi = 3.14$ calculate

(a) the volume of one ball, giving your answer to 1 decimal place.

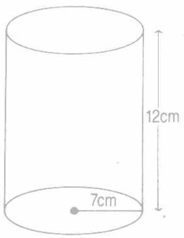
(b) to the nearest whole number the area of paper required to wrap one ball.

Each ball is enclosed in a cubical box of edge 4cm, calculate

(c) the minimum area of cardboard needed to make one box.

(d) what percentage of the volume of each box a ball takes up.

6.



(a) Calculate the volume of the cylinder, giving your answer to 3 significant figures.

(b) Calculate the curved surface area of the cylinder, giving your answer to the nearest whole number.

(c) A box (cuboid) is made which just holds the cylinder; write down the volume of the box.

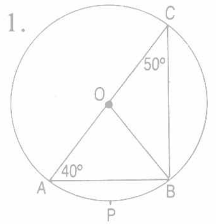
(d) The volume of a sphere is $70cm^3$; calculate its radius.

CIRCLE GEOMETRY
(including intersecting chords and the length of an arc)

TIME 1 HOUR

Note: In this paper the point marked O is the centre of the circle.

1.

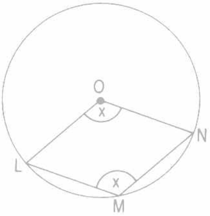


(i) In the diagram OA, OB, OC are radii; angle A = 40°, angle C = 50°.

(a) Calculate the size of angle AOB.

(b) Prove that A, O, C are in a straight line.

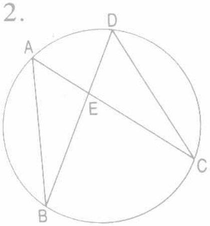
(c) If P is a point on the minor arc AB such that PB is parallel to AO, calculate angle PBO.



(ii) In this diagram L, M, N are three points on the circumference; if angle O = angle M calculate

(d) the value of x.

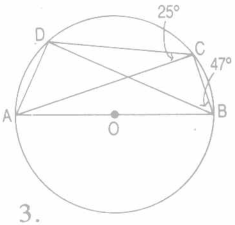
2.



(i) In the diagram 2 chords AC and BD intersect at E

(a) Prove that triangles ABE and DCE are similar.

(b) Given that AE = 6cm, EC = 3cm, and BE = 4cm, calculate ED.



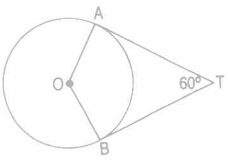
(ii) In the diagram AB is the diameter; angle ACD is 25°, and angle CBD is 47°. Calculate

(c) the size of angle ADC

(d) the size of angle CAB

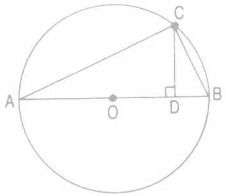
3.

(i) In the diagram TA and TB are tangents to the circle.



(a) What is the size of angle AOB?

(b) Explain why AOBT is a cyclic quadrilateral and state the centre of the circumscribing circle.

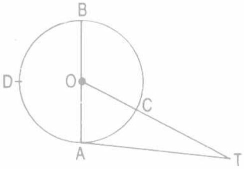


(ii) In this diagram AOB is a diameter and CD is perpendicular to AB.

(c) Prove that triangles CAD and CBD are similar.

(d) Given that AD = 6cm and DB = 4cm, calculate CD.

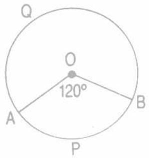
4.



(i) In this diagram AOB is the diameter and TA is a tangent.

(a) If AB = 10cm and TA = 12cm, calculate OT.

(b) D is a point on the circumference such that AD = DB; calculate the length of AD.



(ii) Take the radius of the circle to be 6cm.

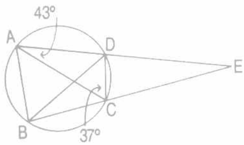
(c) Calculate the length of the arc APB – you may leave your answer in terms of π.

(d) Q is another point on the circumference such that the length of the arc AQ is

$$\frac{12\pi}{5}$$

Calculate the size of angle AOQ.

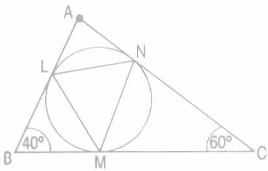
5.



(i) ABCD is a cyclic quadrilateral with AD and BC produced meeting at E.

(a) Calculate the size of angle ABC.

(b) If also EC = ED, calculate the size of angle E.



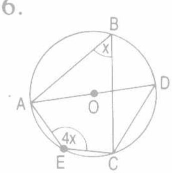
(ii) In the diagram the circle touches the 3 sides of $\triangle$ABC at L, M and N.

(c) Calculate the size of each angle in triangle LMN.

"You cannot have a cyclic parallelogram."

(d) Do you agree?

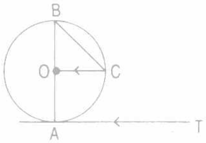
6.



(i) Given that AOD is a diameter

(a) calculate the size of angle B

(b) if also BA = BC, calculate the size of angle BCD.



(ii) AB is a diameter of the circle and TA is a tangent at A; if TA is parallel to the radius CO,

(c) calculate the size of angle B.

(d) if also BC produced meets at G, write down the size of each angle of triangle CAG.

CONSTRUCTIONS (including bearings and scales)

TIME 1 HOUR

1. (a) Construct a triangle with sides 6cm, 8cm and 10cm.

(b) Draw the circle that passes through each vertex.

The scale on a map is given as 1 : 25 000 000

(c) how many kilometres does 1mm on the map represent?

(d) how many square mm would represent an area of $2500km^2$?

2. A ship sailes 720km from a port on a bearing 040°, and then turns 90°, to the right; it continues on its new course until it is 840km from its starting point.

(a) Using a scale of 1cm = 120km, draw an accurate diagram of the first part of the journey.

(b) Draw on your diagram the locus of points 840km from the ship's starting point.

(c) Draw accurately the course of the ship for the second part of the journey.

(d) On what bearing must it now sail in order to return to port?

3. A children's playing area is a rhombus with diagonals 16m and 12m.

(a) Make an accurate scale drawing of the playing area, using a scale of your own, state the scale you have used.

(b) Measure the acute angle of the rhombus.

(c) A certain group of children are forbidden to be more than 6m from either of the narrow ends of the playing area. Show clearly on your diagram the part of the area where they can play.

(d) Calculate the area of the playground *forbidden* to these children (to nearest whole number).

4. Construct the parallelogram ABCD with AB = 9cm, AD = 6cm and angle B = 120°. X is a moveable point inside the parallelogram, nearer to BC than AB; indicate by shading the region where X moves.

P is another moving point inside the parallelogram such angle DPA is 90°; draw the locus of P.

Q is a point on AB such that QB = 2cm. Draw a line from Q to DC that divides the parallelogram into two congruent trapezia.

5. AB is a straight line 8cm long;

(a) Construct on one side of AB the locus of the point C, where the area of triangle ABC is $12cm^2$.

(b) Mark on the locus you have drawn the point D such that DA = DB; join DA and DB.

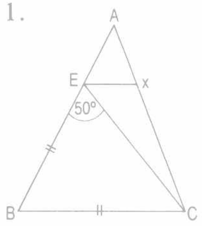
(c) Bisect angles DAB and DBA; let the bisectors meet at K; Use K to construct the circle that touches the three sides of triangle DAB.

(d) Construct on the other side of AB the locus of points M such that angle AMB is 70°.

GEOMETRY OF RECTILINEAR FIGURES

TIME 1 HOUR

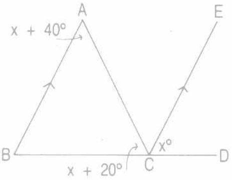
1.



(i) In this triangle BE = BC and AB = AC. Angle BEC is 50°.

(a) What is the size of angle ECA?

(b) Given that EX is parallel to BC, calculate the size of angle XEC.



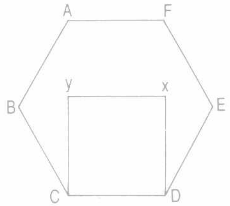
(ii) Given that EC is parallel to AB, calculate

(c) the value of x.

The interior angle of a regular polygon is 162°;

(d) how many sides has the polygon?

2.



(a) 4 of the angles of a hexagon are 100°, 120°, 140°, and 170°; the other two angles are equal. Calculate the size of each.

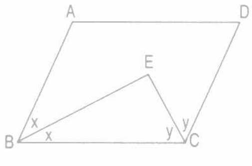
The diagram shows a square standing on the side CD of a regular hexagon.

(b) How many lines of symmetry has the complete shape?

(c) What is the size of angle DEX?

(d) Calculate the size of each angle of triangle FXE.

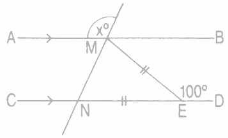
3.



(i) ABCD is a parallelogram; the bisectors of angles B and C meet at E.

(a) Prove that angle E is 90°

(b) Given that angle A is 110°, state the value of y.



(ii) In the diagram AB is parallel to CD and EN = EM.

(c) Given that angle MED is 100°, calculate the value of x.

(d) Suppose the size of angle MED is unknown, and that EM produced bisects the angle marked x; calculate the size of angle MED then.

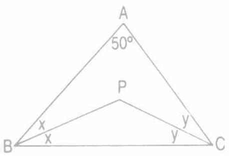
4. (a) Explain why each interior angle of a regular polygon cannot be 130°.

(b) ABCD is a parallelogram with AD produced to E so that AD = DE; if angle DBC is 30° and angle A is 130°, calculate the size of angle BCE.

(c) Which of these statements, if any, is true:
"Every rhombus is a square" or "Every square is a rhombus"?

(d) In a regular pentagon each line of symmetry divides the pentagon into 2 congruent trapezia. How does each line of symmetry divide a regular hexagon?

5.



(i) In the triangle ABC, the bisectors of angles B and C meet at P; angle A is 50°.

(a) Calculate the size of angle BPC.

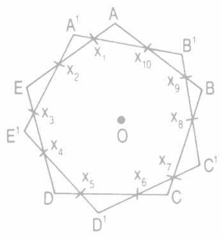
(b) If in general angle A = k°, express angle BPC in terms of k.

(ii) The ratio of the size of the interior angle to the exterior angle of a regular polygon is 7 : 1.

(c) Calculate how many sides the polygon has.

(d) What is the sum of its internal angles in degrees?

6. The regular pentagon ABCDE is rotated 40° anticlockwise to A′B′C′D′E′.



(a) Write down the size of angle A′.

(b) Prove that triangles OAA′ and OBB′ are congruent.

(c) Write down the size of each angle of triangle OAA′.

(d) What is the sum of the degrees of the decagon $X_1X_2 \ldots\ldots\ldots X_{10}$?

MATRICES AND TRANSFORMATIONS

TIME 1 HOUR

1. (a) On a piece of graph paper plot the points A(1,1), B(3,1) and C(1,3). Join the points to make a triangle and label it P.

   (b) Draw an enlargement of P with scale factor 3 and centre (0,0); label it Q. What is the ratio of the area of P to the area of Q?

   (c) Write down the matrix for the transformation in (b).

   (d) What is the inverse of the matrix you have used for (c)?

2.

$$M = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

   (a) Calculate M − N

   (b) What transformation does matrix M represent?

   (c) What transformation does matrix N represent? What is the inverse of N?

   (d) Calculate MN and state the transformation the product represents.

3.

$$K = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$$

   (a) For what value of a is the determinant of K equal to 2?

   (b) Calculate K^2, using the value of a you have found.

   (c) By inspection or otherwise complete the blank matrix:

$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ a & 2 \end{pmatrix}$$

(d) $N = \begin{pmatrix} x & 1 \\ 9 & x \end{pmatrix}$

If N is a singular matrix, calculate x.
(NB. A singular matrix has determinant 0.)

If a singular matrix is applied to the co-ordinates of a shape, what happens to the shape?

4. $Q = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $R = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

(a) Referring to the triangle in question 1, write down the co-ordinates of P under the translation Q.

(b) $m(Q) + n(R) = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$. Calculate m and n.

(c) S = (d, 5). Calculate QS.

(d) Show that (Q + R)S is singular.

5. The co-ordinates of a triangle are X(4,2), Y(8,2) and Z(6,6)

(a) Reflect the triangle in the y-axis, labelling the co-ordinates X′, Y′ and Z′ respectively.

(b) What matrix corresponds to the transformation in (a)?

(c) If the diagrams in (a) and (b) were unlabelled, the original triangle could have been mapped on to the second triangle by a different transformation. Describe fully what this transformation could have been.

(d) Calculate the area of triangle XYZ, and state what the area would have been under an enlargement with scale factor ½.