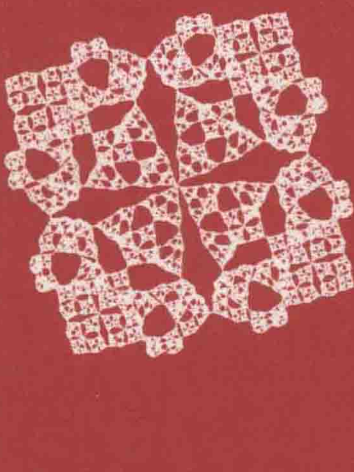
János Pach (Ed.)

LNCS 3383

Graph Drawing

12th International Symposium, GD 2004
New York, NY, USA, September/October 2004
Revised Selected Papers



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Volume Editor

János Pach
City University of New York
City College, NY, USA
and
Hungarian Academy of Sciences
Rényi Institute, Budapest, Hungary
E-mail: pach@cs.nyu.edu

Library of Congress Control Number: 2005920703

CR Subject Classification (1998): G.2, F.2, I.3, E.1

ISSN 0302-9743
ISBN 3-540-24528-6 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media

springeronline.com

Printed in Germany

Typesetting: Camera-ready by author, data conversion by Olgun Computergrafik
Printed on acid-free paper SPIN: 11384861 06/3142 5 4 3 2 1 0

Preface

The 12th International Symposium on Graph Drawing (GD 2004) was held during September 29–October 2, 2004, at City College, CUNY, in the heart of Harlem, New York City. GD 2004 attracted 94 participants from 19 countries.

In response to the call for papers, the program committee received 86 regular submissions describing original research and/or system demonstrations. Each submission was reviewed by at least three program committee members and comments were returned to the authors. Following extensive e-mail discussions, the program committee accepted 39 long papers (11 pages each in the proceedings) and 12 short papers (6 pages each). In addition, 4 posters were displayed and discussed in the conference exhibition room (2 pages each in the proceedings).

The program committee of GD 2004 invited two distinguished lecturers. Professor Paul Seymour from Princeton University presented a new characterization of claw-free graphs (joint work with Maria Chudnovsky). Professor Erik Demaine from MIT reported on his joint work with Fedor Fomin, MohammadTaghi Hajiaghayi and Dimitrios Thilikos, concerning fast (often subexponential) fixed-parameter algorithms and polynomial approximation schemes for broad classes of NP-hard problems in topological graph theory. A survey of the subject by Professors Demaine and Hajiaghayi is included in this volume.

As usual, the annual graph drawing contest was held during the conference. This time the contest had two distinct tracks: the graph drawing challenge and the freestyle contest. A report is included in the proceedings.

Many people in the graph drawing community contributed to the success of GD 2004. First of all, special thanks are due to the authors of submitted papers, demos, and posters, and to the members of the program committee as well as to the external referees. Many thanks to organizing committee members Gary Bloom, Peter Brass, Stephen Kobourov, and Farhad Shahrokhi. My very special thanks go to Hanna Seifu who was in charge of all local arrangements, Robert Gatti who developed the software used for registration and paper submission, and John Weber and Eric Lim who designed the logo, the webpage, and the brochures of the conference. I am very much indebted to Dr. Joseph Barba and Dr. Mohammad Karim, present and former Deans of the School of Engineering, and to Dr. Gregory H. Williams, President of the City College of New York, for their continuing support.

Thanks are due to our "gold" sponsors, the City College of New York, the University of North Texas at Denton, and Tom Sawyer Software, and to our "silver" sponsors, ILOG, the DIMACS Center for Discrete Mathematics and Theoretical Computer Science, and the Computer Science Program at the CUNY Graduate Center. Springer and World Scientific Publishing contributed to the success of GD 2004 by sending selections of their recent publications in the subject.

The 13th International Symposium on Graph Drawing (GD 2005) will be held in Limerick, Ireland, 12–14 September, 2005, with Peter Eades and Patrick Healy as conference co-chairs.

December 2004

János Pach
New York and Budapest

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Reconfiguring Triangulations with Edge Flips and Point Moves*

Greg Aloupis[1], Prosenjit Bose[2], and Pat Morin[2]

[1] School of Computer Science, McGill University
athens@cs.mcgill.ca
[2] School of Computer Science, Carleton University
{jit,morin}@scs.carleton.ca

Abstract. We examine reconfigurations between triangulations and near-triangulations of point sets, and give new bounds on the number of *point moves* and *edge flips* sufficient for any reconfiguration. We show that with $O(n \log n)$ edge flips and point moves, we can transform any geometric near-triangulation on n points to any other geometric near-triangulation on n possibly different points. This improves the previously known bound of $O(n^2)$ edge flips and point moves.

1 Introduction

An *edge flip* is a graph operation that is defined on (near)-triangulations[1]. An edge flip on a triangulation is simply the deletion of an edge, followed by the insertion of another edge such that the resulting graph remains a triangulation. The definition of an edge flip gives rise to several natural questions: Does there always exist a sequence of flips that reconfigures a given triangulation to any other triangulation? Are there bounds on the lengths of such sequences if they exist? Can these sequences be computed? These questions have been studied in the literature in many different settings. In particular, Wagner [19] proved that given any two n-vertex triangulations G_1 and G_2, there always exists a finite sequence of edge flips that reconfigures G_1 into a graph isomorphic to G_2. Subsequently, Komuro [10] showed that in fact $O(n)$ edge flips suffice. Recently, Bose et al. [2] showed that $O(\log n)$ simultaneous edge flips suffice and are sometimes necessary. This setting of the problem is referred to as the combinatorial setting since the triangulations are only embedded combinatorially, i.e. only the cyclic order of edges around each vertex is defined.

In the geometric setting, the graphs are embedded in the plane with edges represented by straight line segments. Pairs of edges can only intersect at their endpoints. Edge flips are still valid operations in this setting, except that now the edge that is added must be a line segment that cannot properly intersect any of the existing edges of the graph. This implies that there are valid edge flips

* Research supported in part by the Natural Science and Engineering Council of Canada.

[1] A triangulation is a plane graph where every face is a triangle. In a *near-triangulation*, the outer face may not be a triangle.

J. Pach (Ed.): GD 2004, LNCS 3383, pp. 1–11, 2004.

in the combinatorial setting that are no longer valid in the geometric setting. Lawson [12] showed that given any two geometric near-triangulations N_1 and N_2 embedded on the same n points in the plane, there always exists a finite sequence of edge flips that transforms the edge set of N_1 to the edge set of N_2. Hurtado, Noy and Urrutia [9] showed that $O(n^2)$ flips are always sufficient and that $\Omega(n^2)$ flips are sometimes necessary.

Note that in the geometric setting, only the near-triangulations that are defined on the *specified point set* can be attained via edge flips. For example, no planar K_4 can be drawn on a convex set of four points without introducing a crossing.

In order to resolve the discrepancy between the combinatorial and geometric settings, Abellanas et al. [1] introduced a geometric operation called a *point move.* A point move on a geometric triangulation is simply the modification of the coordinates of one vertex such that after the modification the graph remains a geometric triangulation. That is, the move is valid provided that after moving the vertex to a new position, no edge crossings are introduced. They also showed that with $O(n^2)$ edge flips and $O(n)$ point moves, any geometric triangulation on n points can be transformed to any other geometric triangulation on n possibly different points.

The question which initiated our investigation is whether or not $O(n^2)$ edge flips are necessary. In this paper, we show that with $O(n \log n)$ edge flips and point moves, we can transform any geometric near-triangulation on n points to any other geometric near-triangulation on n possibly different points. Next, we show that if we restrict our attention to geometric near-triangulations defined on a fixed point set of size n, the problem is just as difficult even with the use of point moves. Finally, we show that with a slightly more general point move, we can remove the extra log factor from our main result.

2 Results

In the remainder of the paper, all triangulations and near-triangulations are geometric. It is assumed that the outer face any given near-triangulation is convex, and that any two near-triangulations involved in a reconfiguration have the same number of points on the convex hull.

We assume that the n vertices of any given triangulation are in general position. It is not difficult to see that $O(n)$ point moves can reconfigure a triangulation to this form. We begin with some basic building blocks that will allow us to prove the main theorems.

Lemma 1. [2] *A reconfiguration between two triangulations of the same point set that is in convex position can be done with $O(n)$ edge flips.*

Lemma 2. [9] *Let v_1, v_2 and v_3 be three consecutive vertices on the outer face of a near-triangulation T_1. Let C be the path from v_1 to v_3 on the convex hull of all vertices but v_2. A near-triangulation T_2 containing all edges of C may be constructed from T_1 with t edge flips, where t is the number of edges initially intersecting C in T_1.*

Lemma 3. *Given a near-triangulation T, any vertex $p \in T$ with degree $d > 3$ that is inside the convex hull of the vertices of T can have its degree reduced to 3 with $d - 3$ edge flips.*

Proof. Let P be the polygon that is the union of all triangles incident to p. By Meister's *two-ears theorem* [13], if P has more than three vertices, then it has at least two disjoint ears[2]. At most one of them can contain p. Therefore p and one of the ears form a convex quadrilateral. We may flip the edge from p to the tip of the ear, effectively cutting the ear from P and reducing the number of vertices of P by one. This process may be continued until P is reduced to a triangle that contains p as desired. □

Lemma 4. *Given a near-triangulation T, any vertex $p \in T$ with degree 3 that is inside the convex hull of the vertices of T can be moved to a new position in the triangulation along a straight path crossing t edges, using at most $2t$ edge flips and $2t + 1$ point moves, assuming the path does not cross through any vertices.*

Proof. Suppose that p is joined by edges to vertices v_1, v_2 and v_3. Without loss of generality, let edge v_2v_3 intersect the path that p must follow, and let this path continue into triangle $v_2v_3v_4$, as shown in Figure 1.

Clearly p can be moved anywhere within triangle $v_1v_2v_3$ without the need of any edge flips. Then it can be moved along its path, as close to edge v_2v_3 as necessary, so that the quadrilateral $pv_2v_3v_4$ becomes convex. This allows edge v_2v_3 to be flipped into edge pv_4. Now p may continue along its path. As soon as it enters $v_2v_3v_4$, edge pv_1 may be flipped into v_2v_3. Now, with two edge flips and two point moves, p has crossed through the first edge intersecting its path, and still has degree 3. By the same argument, p may traverse its entire path with two edge flips and two point moves for each intersecting edge. One additional point move is required in the last triangle. Note that only three edges in the original and final triangulations will be different. □

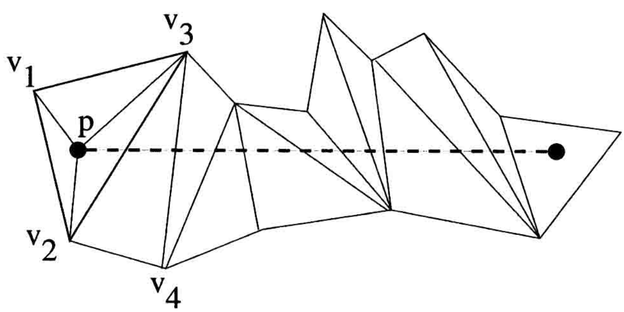

Fig. 1. A vertex p and a straight path that it must move along (dashed). p can pass through any edge with two edge flips.

[2] A triangle, defined by three consecutive vertices of a polygon, is an ear if it is empty and the vertices form a convex angle. The second vertex is the *tip* of the ear.

Lemmata 3 and 4 imply the following result:

Lemma 5. *Given a near-triangulation T, any vertex in the interior of the convex hull of the vertices of T with degree d can be moved to a new position in the triangulation along a path crossing t edges, using $O(d+t)$ edge flips and point moves.*

Lemma 6. *An edge can be constructed between a convex hull vertex and any other vertex in a triangulation using $O(n)$ edge flips, with the aid of one moving point that is moved $O(n)$ times.*

Proof. Let v_1 be the hull vertex. First suppose that the second vertex is an interior point. Then it will play the role of the moving point, and we will label it p. We can move p directly towards v_1, until it is located within a triangle that has v_1 as a vertex. Now v_1 and p must be joined with an edge. Next we move p back along the same line to its original position, always maintaining edge v_1p. To do this, we consider the set of triangles that intersect p's path, as in Lemma 4. The point p can always enter a triangle intersecting the path back to its original location. The difference is that once it has crossed an intersecting edge, we do not restore the edge. This means that p will accumulate edge degree. An issue that needs to be taken care of is that of maintaining a triangulation when p is about to lose visibility to another vertex. This occurs when one of its incident edges is about to overlap with another edge in the triangulation, as shown in Figure 2.

Suppose that edge pv_3 is about to overlap with edge v_3v_4. Vertices v_3 and v_4 cannot be on opposite sides of the remaining path that p must traverse, otherwise v_3v_4 may be flipped. The point p must share an edge with v_4 in this configuration. Points p and v_3 are also part of another triangle, along with some vertex v^* which may be anywhere on the path from v_1 to v_3. These two triangles must form a convex quadrilateral $pv^*v_3v_4$, otherwise p would have already lost visibility to v^*. Thus pv_3 may be flipped into v_4v^*, which means that v_3 is removed from the polygon that intersects p's path. The result is that when p reaches its original position, it leaves a *fan*[3] behind it, which includes edge v_1p.

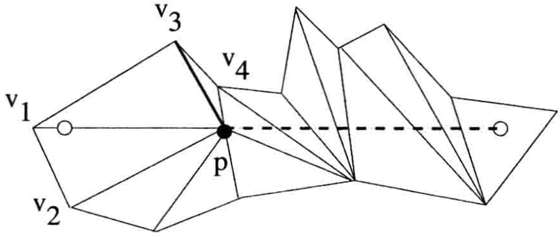

Fig. 2. Maintaining a triangulation while extending edge v_1p: p has moved from a position close to v_1 (shown white), and still has to traverse the dashed segment to its original position. Edge pv_3 causes a problem if p is to continue.

[3] A fan is a star-shaped polygon with a vertex as its kernel.

Overall one edge flip is used when p enters a new triangle, and at most one flip is used for every edge that attaches to p.

If both vertices of the edge that we wish to construct are on the hull, then we can take any point p within the hull and move it close to v_1 and onto the segment between the two hull vertices. p can then move along this segment to the second hull vertex until it is connected to both. At this moment, p may be perturbed so that the three vertices form a triangle. This triangle might contain other edges incident to p. Lemma 2 implies that these edges may be removed so that the desired edge can be constructed with $O(n)$ edge flips. □

2.1 Triangulations

With the basic building blocks in place, we now prove one of our main results.

Theorem 1. *With $O(n \log n)$ edge flips and point moves, we can transform any geometric triangulation on n points to any other geometric triangulation on n possibly different points.*

Proof. We transform one triangulation to another via a canonical configuration. As shown in Figure 3, the interior vertices form a *backbone* (i.e. their induced subgraph is a path). The top of the backbone is joined to the topmost hull vertex v_1, and all interior vertices are joined to the other two hull vertices, v_L and v_R.

The canonical configuration is constructed in a divide-and-conquer manner. We perform a radial sweep from v_1, to find the median vertex interior to the convex hull, v_M. After constructing edge $v_1 v_M$ we move v_M directly away from v_1 towards the base $v_L v_R$, maintaining $v_1 v_M$ until triangle $v_M v_L v_R$ contains no interior points. By Lemma 6, we use $O(n)$ operations to accomplish this. Now, we transform $v_1 v_M v_L$ and $v_1 v_M v_R$ into backbone configurations by induction since they are smaller instances of the same problem. The resulting configuration is shown in Figure 4.

We now show that the two sides may be merged using $O(n)$ operations. As shown in Figure 5a, we first move the lowest vertex of a backbone into a position that is close to the base and is along the extension of edge $v_1 v_M$. This requires one edge flip. The vertices on the left/right backbones are processed in

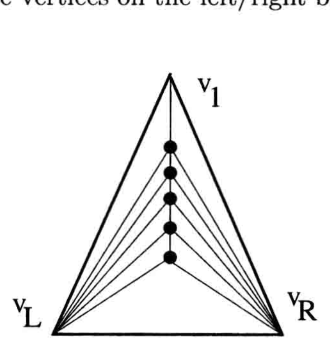

Fig. 3. The canonical configuration used for triangulations.

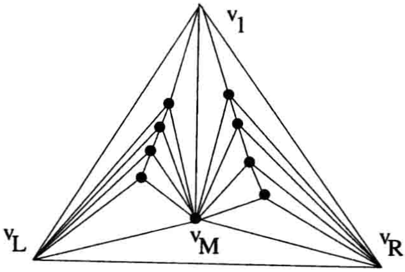

Fig. 4. The configuration of a triangulation prior to merging the backbones on each side of the median vertex v_M.

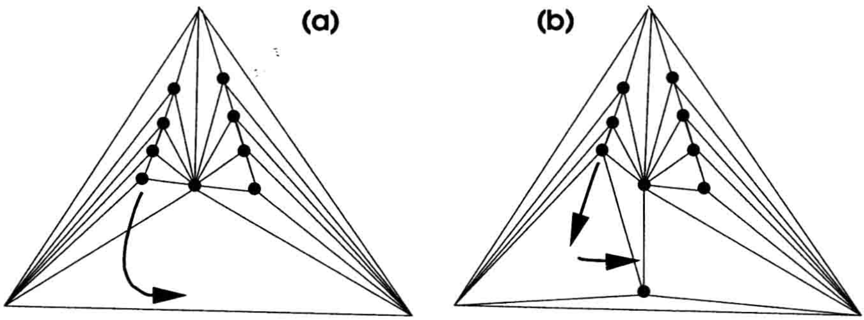

Fig. 5. Merging two backbones into one.

ascending order, and are always moved just above the previous processed vertex, as shown in Figure 5b. Each vertex will require two point moves and one edge flip. Thus $v_1 v_L v_R$ is reconfigured into canonical form, and by a simple recurrence the number of edge flips and point moves used is $O(n \log n)$. It is trivial to move a canonical triangulation to specific coordinates using n point moves. Thus the transformation between any two triangulations may be completed. □

2.2 Near-Triangulations

If the initial graph is a near-triangulation, Theorem 1 does not directly apply. Some care must be taken to handle a non-triangular outer face. Details are given in the proof of the following theorem:

Theorem 2. *With $O(n \log n)$ edge flips and point moves, we can transform any geometric near-triangulation on n points to any other geometric near-triangulation on n possibly different points.*

Proof. As in the case with triangulations, we transform one near-triangulation to another via a canonical configuration. In the primary canonical configuration, shown in Figure 6, one chosen hull vertex (v_1) is joined by chords to all other