HANDBOOK OF

Logic and Proof Techniques for Computer Science

STEVEN G. KRANTZ

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With 16 Figures

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Handbook of Logic and Proof Techniques for Computer Science

To little Hypatia, who doesn't seem to be very logical.

Preface

Logic is, and should be, the core subject area of modern mathematics. The blueprint for twentieth century mathematical thought, thanks to Hilbert and Bourbaki, is the axiomatic development of the subject. As a result, logic plays a central conceptual role. At the same time, mathematical logic has grown into one of the most recondite areas of mathematics. Most of modern logic is inaccessible to all but the specialist.

Yet there is a need for many mathematical scientists—not just those engaged in mathematical research—to become conversant with the key ideas of logic. *The Handbook of Mathematical Logic*, edited by Jon Barwise, is in point of fact a handbook written by logicians for other mathematicians. It was, at the time of its writing, encyclopedic, authoritative, and up-to-the-moment. But it was, and remains, a comprehensive and authoritative book for the *cognoscenti*. The encyclopedic *Handbook of Logic in Computer Science* by Abramsky, Gabbay, and Maibaum is a wonderful resource for the professional. But it is overwhelming for the casual user. There is need for a book that introduces important logic terminology and concepts to the working mathematical scientist who has only a passing acquaintance with logic. Thus the present work has a different target audience.

The intent of this handbook is to present the elements of modern logic, including many current topics, to the reader having only basic mathematical literacy. Certainly a college minor in mathematics is more than sufficient background to read this book. Specifically, courses in linear algebra, finite mathematics, and mathematical structures would be more than adequate preparation. From the computer science side, it would be good if the reader knew a programming language and had some exposure to issues of complexity and decidability. But all these prerequisites are primarily for motivation. This handbook is, to the extent possible, self-contained. It will be a compact and accessible reference.

This is not a textbook; it is a handbook. It contains very few proofs. What it does contain are definitions, examples, and discussion of all of the key ideas in basic logic. We also include cogent and self-contained introductions to critical advanced topics, including Gödel's completeness and incompleteness theorems, methods of proof, cardinal and ordinal numbers, the continuum hypothesis, the axiom of choice, model theory, number systems and their construction, multi-valued logics, category theory, universal algebra, proof theory, fuzzy set theory, recursive functions, **NP**-completeness, decision problems, Boolean algebra, semantics, decision problems, and the word problem.

This book is intended to be a resource for the working mathematical scientist. The computer scientist or engineer or system scientist who must have a quick sketch of a key idea from logic will find it here in self-contained form, accessible to a quick read. In addition to critical terminology, notation, and ideas, references for further reading are provided. There is a thorough index, a glossary, a lexicon of notation, a guide to the literature, and an extensive bibliography.

There is no other book like this one on the subject of logic and its allied areas. This book is both a reference and a portal to further study of topics in logic. A scientist reading a technical tract in another field and encountering an unfamiliar term or concept from logic can turn to this volume and find a rapid introduction to the key points. The book will be a useful reference both for working scientists and for students.

Certainly computer science is one of the most active areas of modern scientific activity that uses logic. This book has been written with computer scientists in mind. Important ideas, such as recursion theory, decidability, independence, completeness, consistency, model theory, **NP**-completeness, and axiomatics are presented here in a form that is particularly accessible to computer scientists. Examples from computer science are provided whenever possible. A special effort has been made to cut through the mathematical formalism, difficult notation, and esoteric terminology that are typical of modern mathematical logic. The treatment of any key topic is quick, incisive, and to the point.

Although this is a handbook of logic, it is not strictly logical in nature. By this we mean that it is virtually impossible to present all of the topics of this great tapestry in a strictly logical order. For that reason, topics have been repeated or presented in unusual sequence when convenient for local use.

Birkhäuser engaged a number of experts to review various versions of this project and to help prepare it for publication. These reviewers patiently guided me regarding both form and substance. Their criticisms and remarks have been invaluable. Of course, responsibility for all remaining errors and malapropisms resides entirely with the author.

It is hoped that this work will stimulate the communication between computer science and mathematics, and also be a useful handbook for both fields. It is safe to say that computer science is becoming ever more mathematical and also that mathematics is becoming increasingly comfortable with the use of computers. This book should serve as a catalyst in both activities. It is both a touchstone for a first look at topics in logic and an invitation to further reading. We hope that it will awaken in others the fascination with logic that we have experienced for more than a quarter century.

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