

B. Wayne Bequette

Process Dynamics

Modeling, Analysis, and Simulation

**Prentice Hall International Series
in the Physical and Chemical
Engineering Sciences**



PROCESS DYNAMICS

Modeling, Analysis, and Simulation

B. Wayne Bequette
Rensselaer Polytechnic Institute



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Prentice Hall PTR
Upper Saddle River, New Jersey 07458

Library of Congress Cataloging-in-Publication Data

Bequette, B. Wayne.

Process dynamics : modeling, analysis, and simulation / B. Wayne

Bequette.

p. cm.

Includes bibliographical references and index.

ISBN 0-13-206889-3

1. Chemical processes. I. Title.

TP155.7.B45 1998

660'.284'01185—dc21

97-36053

CIP

Acquisitions editor: Bernard M. Goodwin

Cover design director: Jerry Votta

Manufacturing manager: Alexis R. Heydt

Marketing manager: Miles Williams

Compositor/Production services: Pine Tree Composition, Inc.



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Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN: 0-13-206889-3

Prentice-Hall International (UK) Limited, *London*

Prentice-Hall of Australia Pty. Limited, *Sydney*

Prentice-Hall Canada Inc., *Toronto*

Prentice-Hall Hispanoamericana, S.A., *Mexico*

Prentice-Hall of India Private Limited, *New Delhi*

Prentice-Hall of Japan, Inc., *Tokyo*

Simon & Schuster Asia Pte. Ltd., *Singapore*

Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

Process Dynamics

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To Pat and Brendan

PREFACE

An understanding of the dynamic behavior of chemical processes is important from both process design and process control perspectives. It is easy to design a chemical process, based on steady-state considerations, which is practically uncontrollable when the process dynamics are considered. The current status of computational hardware and software has made it easy to interactively simulate the dynamic behavior of chemical processes.

It is common for process dynamics to be included as the introductory portion of a process control textbook, however, there are a number of limitations to this approach. Since the emphasis of most of the textbooks is on process control, there is too little space to give adequate depth to modeling, analysis, and simulation of dynamic systems. The focus tends to be on transfer function-based models that are used for control system design. The prime motivation for my textbook is then to provide a more comprehensive treatment of process dynamics, including modeling, analysis, and simulation. This textbook evolved from notes developed for a course on dynamic systems that I have been teaching at Rensselaer since 1991. We have been fortunate to have a two-semester sequence in dynamics and control, allowing more depth to the coverage of each topic.

Topics covered here that are not covered in a traditional text include nonlinear dynamics and the use of MATLAB for numerical analysis and simulation. Also, a significant portion of the text consists of review and learning modules. Each learning module provides model development, steady-state solutions, nonlinear dynamic results, linearization, state space and transfer function analysis and simulation. The motivation for this approach is to allow the student to “tie-together” all of the concepts, rather than treating them independently (and not understanding the connections between the different methods).

An important feature of this text is the use of MATLAB software. A set of m-files used in many of the examples and in the learning modules is available via the world wide web at the following locations:

http://www/rpi.edu/~bequeb/Process_Dynamics
<http://www.mathworks.com/education/thirdparty.html>

Additional learning modules will also be available at the RPI location.

A few acknowledgments are in order. A special thanks to Professor Jim Turpin at the University of Arkansas, who taught me the introductory course in process dynamics and control. His love of teaching should be an inspiration to us all. Many thanks to one of my graduate students, Lou Russo, who not only made a number of suggestions to improve the text, but also sparked an interest in many of the undergraduates that have taken the course. The task of developing a solutions manual has been carried out by Venkatesh Natarajan, Brian Aufderheide, Ramesh Rao, Vinay Prasad, and Kevin Schott.

Preliminary drafts of many chapters were developed over cappuccinos at the Daily Grind in Albany and Troy. Bass Ale served at the El Dorado in Troy promoted discussions about teaching (and other somewhat unrelated topics) with my graduate students; the effect of the many Buffalo wings is still unclear. Final revisions to the textbook were done under the influence of cappuccinos at Cafe Avanti in Chicago (while there is a lot of effort in developing interactive classroom environments at Rensselaer, my ideal study environment looks much like a coffee shop).

Teaching and learning should be dynamic processes. I would appreciate any comments and suggestions that you have on this textbook. I will use the WWW site to provide updated examples, additional problems with solutions, and suggestions for teaching and studying process dynamics.

B. Wayne Bequette

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