SERVOMECHANISMS and REGULATING SYSTEM DESIGN

VOLUME I

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PREFACE

The material in this book has its origin in our experience and the experience of other General Electric Company engineers in designing control systems and regulators for the armed services and for industry. Although we have worked most intimately with position and steering controls, we have also profited greatly from our associates who have worked on voltage, speed, and other regulating systems. The association that we have had with the Servomechanisms Laboratory at the Massachusetts Institute of Technology, directed by Dr. Gordon S. Brown, and the M.I.T. Radiation Laboratory Servomechanism group under N. B. Nichols has also been beneficial in providing us with a broader understanding of the subject of servomechanism control systems.

For some years now the material contained in this and its companion volume, which is to follow, has been presented to General Electric engineers of the Aeronautic and Ordnance System Divisions and of the Technical Education Programs. This material has also been presented to graduate students of the Union College Extension Division. These students have had widely differing backgrounds of practical experience and previous training. The experience of teaching these students and of working with some of them on advanced design problems has guided us in developing the material in Volumes I and II.

This book, Volume I, is adapted to the needs of engineers and engineering students who have not had previous training or experience in the field of closed-loop control systems. Because the solution of linear differential equations during both transient and steady-state operation is important to an understanding of control system performance, this subject is presented briefly from the operational and Laplace transform points of view. Circuit theory and system stability are also presented because of their importance as background material. Chapters 7 through 9 describe closed-loop control system elements, operation, performance, and methods of analysis. The nomenclature and symbols recommended recently by the A.I.E. Subcommittee on Symbols for Feedback Control Systems are used throughout the book. Chapters 10 through 12 present the complex plane and attenuation-frequency methods of analyzing and synthesizing some simpler forms of servomechanisms and regulators. Chapter 13 extends the design procedure to the more complex multi-loop systems with multiple inputs frequently encountered in present-day practice. The comparison of steady-state and transient performance of servomechanisms in Chapter 14 presents information useful for system synthesis. To extend its usefulness to undergraduate and graduate students, we have given in this volume numerous problems illustrating and extending the material presented in the text.

Volume II, written at the same time as Volume I, is devoted to the needs of the practicing designer and of the advanced graduate student. It deals with such problems as the means for establishing design specifications as well as for reducing the effects of extraneous unwanted signal inputs. Factors influencing the selection of control elements for the power and stabilizing portions of control systems are discussed. The effects of non-linear operation as caused by saturation as well as deliberate gain changes are described. Amplifier design factors to minimize gain changes, drift, and changes in tubes and other components are presented. Chapters are also devoted to all a-c servomechanism design and to the very important subject of measurement techniques.

We wish to acknowledge the assistance we have received from our colleagues A. P. Adamson and P. Cushman. We wish also to acknowledge the assistence of our former associates, F. H. Andrix, J. L. Bower, Sidney Godet, and J. R. Moore. Mrs. Cecile Lester's calculations and Miss Anna Kosinski's typewriting have helped greatly in the preparation of this book. We are also grateful to many of the students and the staff of the Technical Education Divisions who have contributed to the make-up of this book. Dr. C. F. Green, Mr. W. B. Jordan, Mr. R. O. Dunham, and Mr. P. L. Alger deserve particular mention for their interest in this endeavor.

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1

The Automatic Control Problem

1.0 Introduction

Recent trends in the development of modern civilization have been in the direction of greater control. With the advent of the steam engine and the material improvements brought about by the industrial revolution, man has had available greater quantities of power for his use. To utilize this power effectively, he has been required to learn how to control and how to regulate it. The increased interdependence of individuals and groups of individuals upon one another during this period has made it necessary for a greater measure of regulation to be exerted in the field of social activity as well.

As part of the control process, whether it be of persons or of things, certain standards are established. The performance of the equipment or of the individual is compared to these standards, and according to the difference, appropriate action is taken to bring about a closer correspondence between the desired objectives and the actual performance. Examples of this type of regulating action are common in the operation of production or accounting groups in business as well as in the operation of mechanical, electrical, thermal, and other engineering equipment.[112]* In some of these regulated systems a long interval of time occurs before the performance being controlled is compared to the desired objectives. An annual manufacturing inventory or a quarterly financial statement illustrates this point. For the automatically controlled engineering equipment, the comparison may be made many times per second. The point of interest is that the need for good control is present in many phases of our existence. The problem is to determine the desired objectives and the best ways of producing these objectives. Although the material of this book is limited to a study of problems pertaining to the field of engineering applications of automatic control, there is the possibility that some of the philosophy of feedback control set forth here might be employed to advantage for other forms of control problems.

* Superior reference numbers refer to Bibliography at end of book.

Fortunately for engineers, the desired objectives for their equipment are fairly well defined and the means for measuring the performance of this equipment are relatively simple and accurate. For example, it is desired that the voltage and frequency of a power system be maintained within certain tolerances,[18, 22] that the speed or position of a motor shaft should obey certain prescribed conditions,[25] or that the motion of one instrument or shaft should faithfully duplicate the motion of another.[24, 104] All these are problems in which the nature of the control requirements and references for describing these requirements can be established fairly well in advance. The meters, oscillographs, or error-sensing elements for determining the quality of performance are generally available.[113, 114]

The advances of the technical arts have made available, in form suitable for control, amounts of power with high speeds of response such that man is unable to compete with machines for the power portion of the control. Furthermore, in the measuring or error-sensing portion of the control, instruments are in many cases more accurate, more rapid, more reliable, and cheaper than a human being who might be called upon to do a similar job manually. Thus man has found means for having some of his mental labor as well as much of his physical work done by machines. With these power and sensing means available, the field of automatic control is opened to relieve man of many of the monotonous and disagreeable aspects of many routine tasks and to make possible the accomplishment of labors that are humanly impossible. It is these features that make automatic control so attractive. Not only is it possible to reduce the amount of manual labor required but it may also be possible to achieve a higher degree of performance than would otherwise be possible.

In this chapter the general form of the feedback control problem is presented. The factors of stability and accuracy associated with such a control are indicated. Next the means that have been developed for solving these engineering problems are described. To emphasize the various phases of the actual solution to the design problem, the general steps in solving a typical control problem are outlined. A brief review is presented of the development of the regulator and servomechanism arts up to the present time, when they are being merged to form the more unified science of feedback control systems. An attempt is made to indicate the direction that will be taken by future developments in the field of automatic control.

1.1 Description of Feedback Control System

The terms servomechanism and regulator have been described and defined many times in the literature. As used in this book, they refer to a feedback control system in which the difference between the *reference input* and some function of the *controlled variable* is used to supply an *actuating error* signal to the control elements and the controlled system.[111] The amplified actuating error signal endeavors to reduce to zero the difference between reference input and the controlled variable. A supplemental source of power is available in such systems to provide amplification at one or more points in the feedback control system so that the possibility exists for self-excited oscillations or instability.

Figure 1.1–1 is a block diagram representation of a simplified feedback control system. In addition to the principal variables described

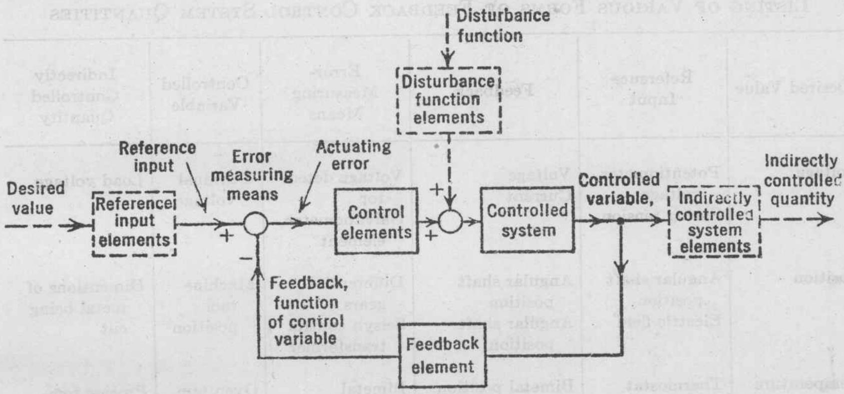

FIGURE 1.1–1. Block diagram of simplified feedback control system.

above and shown by the solid-line portions of the diagram, the *desired value*, the *indirectly controlled quantity*, and the *disturbance function* are shown. The desired value represents the value it is desired that the control system reproduce, and it differs from the actual reference input by the characteristics of the reference input elements. The indirectly controlled quantity represents the quantity that is the actual system output. It differs from the controlled variable by the characteristics of the indirectly controlled system elements. The disturbance function represents an unwanted input to the system that tends to cause the controlled variable to differ from the reference input. The disturbance function elements shown by dotted lines are intermediate between disturbance function and the controlled system itself. The dotted lines associated with the reference input and indirectly controlled system elements serve to indicate that these elements, when equal to unity, have a limited effect on the feedback control problem.

In many feedback control systems, the operation of all parts of the control system is continuous and automatic. By means of the continued use of the actuating error, such a feedback control system can be made accurate without requiring high accuracy or constant performance characteristics for all the control elements.

The principles of feedback control operation may be used effectively for any one of a number of different physical kinds of control problems whether they employ electrical, mechanical, thermal, or other forms of control elements. Table 1.1–1 indicates some common forms that these variables and elements take for a few types of feedback control systems.

TABLE 1.1–1

LISTING OF VARIOUS FORMS OF FEEDBACK CONTROL SYSTEM QUANTITIES

Desired Value	Reference Input	Feedback	Error-Measuring Means	Controlled Variable	Indirectly Controlled Quantity
Voltage	Potentiometer voltage Spring tension	Voltage Current	Voltage detector Galvanometer element	Terminal voltage	Load voltage
Position	Angular shaft position Electric field	Angular shaft position Angular shaft position	Differential gears Selsyn control transformer	Machine tool position	Dimensions of metal being cut
Temperature	Thermostat contact setting Thermoelectric voltage	Bimetal position Thermoelectric voltage	Bimetal thermostat Thermocouple	Oven temperature	Process temperature
Speed	Standard voltage Calibrated spring position	Voltage Governor position	Voltage detector Linkage position	Governor speed	Alternator speed

Requirements of Stability and Accuracy. The basic principle of feedback control or closed-loop operation tends to make for accurate performance since the control system endeavors continually to correct any error that exists. However, this corrective action can give rise to a dangerous condition of unstable operation when used with control elements having a large amount of amplification and significant delays in their time of response.[6] An unstable control system is one that is no longer effective in maintaining the controlled variable very nearly equal to the desired value. Instead, large sustained oscillations or erratic control of the controlled variable may take place, rendering the control useless.

If stable feedback control system performance is like that of a manually controlled system with a capable and well-trained operator, unstable feedback control system performance may be compared to that of the manually controlled system with an untrained and irresponsible operator. Rapid and destructive response of the system may result in which adequate control is impossible, and destructive action of the controlled variable may occur.

If in an effort to increase the accuracy of the control system, one increases the amplification of the control without taking adequate steps to insure stable operation, the advantages of the feedback control principle prove illusory. Furthermore, it is necessary to do more than have a system that is stable; one must have a system that has an adequate margin of stability and can recover rapidly and smoothly from the shocks of irregular inputs or of severe disturbances.

The requirements of stability and accuracy are mutually incompatible. The higher the desired accuracy, the smaller is the actuating error that can be tolerated for proper corrective action, and the sooner must full corrective action be initiated. Thus, to be accurate, a system requires high amplification. However, with high amplification more corrective action of the controlled variable can take place for a given error, and the time during which the corrective action is required is decreased.

Time delays in the various control elements and the controlled system that were not significant in a low gain system may become appreciable for the system with high amplification. After the corrective action is started and the need for correction has ceased, the inherent time delay of the system elements may prevent stopping the action of the control elements in time to prevent an overshoot by the controlled variable. The overshoot may be greater than that which initiated the control motion, and the process of continued corrective action, building up to violent oscillations, is thus started.

Following the above line of reasoning, one sees that the time delays present in the control elements cause the instability. By appropriate use of "anticipation" means to compensate for the inherent time "delays" in the control elements, it is possible to obtain a high gain system with satisfactory stability. Although improved accuracy and adequate stability can be obtained, they are achieved only at the expense of additional equipment or complexity.

Mathematical Basis for Stability. Essential and valuable as is the physical picture of stability, mathematical definitions provide more useful and exact means of describing system performance. The principal mathematical means for determining stability of linear control systems are the following:

1. Locating by analytical or graphical means the actual position on the complex plane of each of the roots of the characteristic equation of the system.
2. Applying Routh's stability criterion [2] to the coefficients of the system's characteristic equation.
3. Applying Nyquist's criterion [6] to a graphical plot of the open-loop response of the system as a function of frequency for a sinusoidal driving function.

The labor involved in locating the exact position of the roots of the characteristic equation or in calculating their values is such as to limit the use of this method. The Routh criterion involves the use of a brief, simple algebraic process and permits the ready determination of the system stability. However, the graphical data necessary for applying Nyquist criterion provide quantitative information on the accuracy of the system, the degree of system stability, as well as the system stability itself. Hence it is the Nyquist stability criterion in one or more of its modified forms that is used most extensively to determine system stability.

The Nyquist stability criterion places on a firm mathematical basis the well-known physical fact that, when the fed-back signal to a control element is equal in magnitude and in phase with the actuating signal producing it, instability will result. Thus the Nyquist criterion establishes the necessary conditions for stability in terms of the ratio between the sinusoidal actuating signal and the feedback signal. The ratio is expressed by an amplitude and a phase relationship as a function of frequency. This ratio can be determined with the feedback not connected to the error-measuring element; thus the system need not be a closed-loop one under the conditions during which the stability as a feedback control system is evaluated. As such, the analysis of the problem is reduced somewhat in complexity, although the results are valid for the more complicated feedback control condition of system operation.

Features of Feedback Control System Performance. The two principal advantages of feedback control over control without feedback are that lower tolerances and greater time delays can be permitted for the control elements. To appreciate some of the advantages of the feedback control system, a comparison will be made between the open-loop and closed-loop (feedback) control systems. Figures 1.1–2 and 1.1–3 are block diagrams showing how the controlled variable C is related to the reference input R for each of these two systems. The ratios of the controlled variable to the reference input for the open-loop and feedback control systems are, respectively,

$$\frac{C}{R} = G \tag{1.1-1}$$

and

$$\frac{C}{R} = \frac{G_1}{1 + G_1} \tag{1.1-2}$$

where $C/E = G_1$ and E is the actuating error.

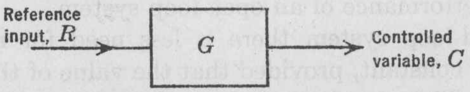

FIGURE 1.1–2. Open-loop control system.

The terms G and G_1 represent the transfer functions of the control elements. In addition to gain or constant terms, the transfer functions may contain time functional relationships having a wide range of values from 0 to ∞ under varying input conditions.

In contrast with the feedback control system in which the controlled variable is compared directly to the reference input to provide the error signal that actuates the control system elements, the open-loop control

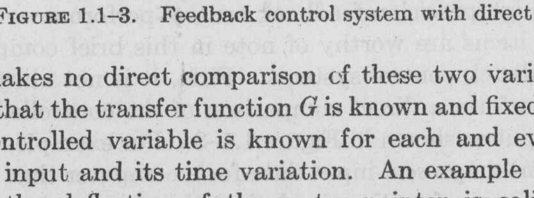

FIGURE 1.1–3. Feedback control system with direct feedback.

system makes no direct comparison of these two variables. Thus it is assumed that the transfer function G is known and fixed so that the value of the controlled variable is known for each and every value of the reference input and its time variation. An example of this is a meter element; the deflection of the meter pointer is calibrated against a standard input, for example, a voltage. Subsequently it is assumed that the meter response is identical with its calibration figure and that the same input voltage will produce the same pointer deflection. However, a change in the characteristics of the transfer function G of the meter element may cause the same input to produce a value of output different