Truly Nonlinear Oscillations

Harmonic Balance, Parameter Expansions, Iteration, and Averaging Methods

Ronald E Mickens

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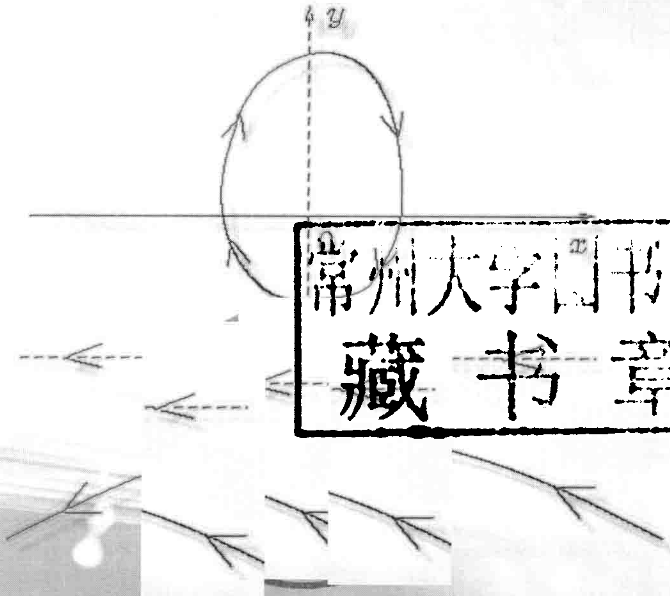

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NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data
Mickens, Ronald E., 1943–
Truly nonlinear oscillations : harmonic balance, parameter expansions, iteration, and averaging methods / by Ronald E. Mickens.
p. cm.
Includes bibliographical references and index.
ISBN-13: 978-981-4291-65-1 (hardcover : alk. paper)
ISBN-10: 981-4291-65-X (hardcover : alk. paper)
1. Approximation theory. 2. Nonlinear oscillations. I. Title.

QA221.M53 2010
511'.4--dc22

2009038794

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Printed in Singapore by B & JO Enterprise

Preface

This small volume introduces several important methods for calculating approximations to the periodic solutions of "truly nonlinear" (TNL) oscillator differential equations. This class of equations take the form

$$\ddot{x} + g(x) = \epsilon F(x, \dot{x}),$$

where $g(x)$ has no linear approximation at $x = 0$. During the past several decades a broad range of calculational procedures for solving such differential equations have been created by an internationally based group of researchers. These techniques appear under headings such as

- averaging
- combined and linearization
- harmonic balance
- homotopy perturbation
- iteration
- parameter expansion
- variational iteration methods.

Further, these methodologies have not only been applied to TNL oscillators, but also to strongly nonlinear oscillations where a parameter may take on large values. Most of these techniques have undergone Darwinian type evolution and, as a consequence, a large number of papers are published each year on specializations of a particular method. While we have been thorough in our personal examination of the research literature, only those papers having an immediate connection to the topic under discussion are cited because of the magnitude of existing publications and because an interested user of this volume can easily locate the relevant materials from various websites.

We have written this book for the individual who wishes to learn, understand, and apply available techniques for analyzing and solving problems involving TNL oscillations. It is assumed that the reader of this volume has a background preparation that includes knowledge of perturbation methods for the standard oscillatory systems modeled by the equation

$$\ddot{x} + x = \epsilon F(x, \dot{x}).$$

In particular, this includes an understanding of concepts such as secular terms, limit-cycles, uniformly-valid approximations, and the elements of Fourier series.

The basic style and presentation of the material in this book is heuristic rather than rigorous. The references at the end of each chapter, along with an examination of relevant websites, will allow the reader to fully comprehend what is currently known about a particular technique. However, the reader should also realize that the creation and development of most of the methods discussed in this book do not derive from rigorous mathematical derivations. This task is a future project for those who have the interests and necessary background to carry out these procedures. However, these efforts are clearly not relevant for our present needs.

The book consists of seven chapters and several appendices. Chapter 1 offers an overview of the book. In particular, it presents a definition of TNL equations, introduces the concept of odd-parity systems, and calculates the exact solutions to four TNL oscillatory systems.

Chapter 2 provides a brief discussion of several procedures for a priori determining whether a given TNL differential equation has periodic and/or oscillatory solutions. The next four chapters present introductions to most of the significant procedures for calculating analytical approximations to the solutions of TNL differential equations. These chapters discuss, respectively, harmonic balance, parameter expansion, iteration, and averaging methods. Each chapter gives not only the basic methodology for each technique, but also provides a range of worked examples illustrating their use.

The last chapter considers six TNL oscillator equations and compares results obtained by all the methods that are applicable to each. It ends with general comments on TNL oscillators and provides a short listing of unresolved research problems.

We also include a number of appendices covering topics relevant to understanding the general issues covered in this book. The topics discussed range from certain mathematical relations to basic results on linear second-order differential equations having constant coefficients. Brief presentations are given on Fourier series, the Lindstedt-Poincaré perturbation method, and the standard first-order method of averaging. A final appendix, "Discrete Models of Two TNL Oscillators," illustrates the complexities that may arise when one attempts to construct discretizations to calculate numerical solutions.

I thank my many colleagues around the world for the interest in my work, their generalization of these results and their own original "creations" on the subject of TNL oscillations. As always, I am truly grateful to Ms. Annette Rohrs for her technical services in seeing that my handwritten pages were transformed into the present format. Both she and my wife, Maria Mickens, provided valuable editorial assistance and the needed encouragement to successfully complete this project. Finally, I wish to acknowledge Dr. Shirley Williams-Kirksey, Dean of the School of Arts and Sciences, for providing Professional Development Funds to assist in the completion of this project. Without this support the writing effort would not have been done on time.

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Atlanta, GA
August 2009

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Chapter 1

Background and General Comments

This chapter introduces the basic, but fundamental concepts relating to the class of oscillators we call "truly nonlinear." The two phrases "truly nonlinear oscillators" and "truly nonlinear differential equations" are used interchangeable. In Sections 1.1 and 1.2, respectively, we define truly nonlinear (TNL) functions and TNL oscillators. Section 1.3 presents general comments regarding time reversal invariant systems and odd parity oscillators. Section 1.4 discusses the important topic of the elimination of dimensional quantities in the physical nonlinear differential equations through the use of scaling parameters. The existence of and exact solutions to four TNL oscillators are given in Section 1.5; this is followed by a brief overview of four methods that can be used to construct analytic approximations to the periodic solutions for TNL oscillator differential equations. We conclude the chapter with a set of possible criteria that may be used to judge the value of a calculational method for generating approximate solutions.

1.1 Truly Nonlinear Functions

A TNL function is defined with respect to its properties in a neighborhood at a given point. For our purposes, we select $x = 0$. Thus, for a function $f(x)$, we make the following definition:

Definition 1.1. $f(x)$ is a TNL function, at $x = 0$, if $f(x)$ has no linear approximation in any neighborhood of $x = 0$.

The following are several explicit examples of TNL functions

$$f_1(x) = x^3, \quad f_2(x) = x^{1/3}, \quad f_3(x) = x + x^{1/3}. \tag{1.1.1}$$