Applied

Probability FOR

Engineers And Onional

Scientists

EPHRAIM SUHIR

# Applied Probability Engineers and Scientists

### **Ephraim Suhir**

Bell Laboratories, Lucent Technologies, Physical Sciences and Engineering Research Division

#### Library of Congress Cataloging-in-Publication Data

Suhir, Ephraim.

Applied probability for engineers and scientists / Ephraim Suhir.

p. cm.

Includes bibliographical references (p. 529-533) and index.

ISBN 0-07-061860-7

1. Probabilities. I. Title.

OA273.S773 1997

519.2'02462--dc21

96-48238 CIP

### McGraw-Hill



#### A Division of The McGraw-Hill Companies

Copyright © 1997 by The Bell Laboratories, Lucent Technologies, Inc. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a data base or retrieval system, without the prior written permission of the publisher.

1 2 3 4 5 6 7 8 9 Doc/Doc 9 0 2 1 0 9 8 7

ISBN 0-07-061860-7

The sponsoring editor for this book was Harold B. Crawford, the editing supervisor was Patricia V. Amoroso, and the production supervisors were Donald F. Schmidt and Suzanne W. B. Rapcavage. It was set in Century Schoolbook by Santype International.

Printed and bound by R. R. Donnelley & Sons Company.

Information contained in this work has been obtained by The McGraw-Hill Companies, Inc., ("McGraw-Hill") from sources believed to be reliable. However, neither McGraw-Hill nor its authors guarantee the accuracy or completeness of any information published herein and neither McGraw-Hill nor its authors shall be responsible for any errors, omissions, or damages arising out of use of this information. This work is published with the understanding that McGraw-Hill and its authors are supplying information but are not attempting to render engineering or other professional services. If such services are required, the assistance of an appropriate professional should be sought.

McGraw-Hill books are available at special quantity discounts to use as premiums and sales promotions, or for use in corporate training programs. For more information, please write to the Director of Special Sales, McGraw-Hill, 11 West 19th Street, New York, NY 10011. Or contact your local bookstore.

# Applied Probability for Engineers and Scientists

Chance governs all

John Milton, Paradise Lost

There are things in this world, far more important than the most splendid discoveries—it is the methods by which they were made Gottfried Leibnitz

此为试读,需要完整PDF请访问: www.ertongbook.com

To my family, Raisa, Elena, and Eugene Suhir, to my mother Betty, and to the memory of my father, Leonid Suhir

## **Preface**

"Education is man's going forward from cocksure ignorance to thoughtful uncertainty."

DON CLARK "SCRAPBOOK"

"The only real voyage to discovery consists not in seeking new landscapes, but in having new eyes." MARCEL PROUST

"All the general theories stem from examination of specific problems." RICHARD COURANT

This book provides an introduction to probabilistic methods used in engineering and some other areas of applied science. Its main objective is to be suitable for young engineers and engineering students of different majors who, with some background in calculus, ordinary and partial differential equations, and general engineering disciplines, would like to gain an initial insight into the substance and use of probabilistic methods. The book describes the major concepts, methods, and approaches of the applied theory of probability and provides numerous examples from various areas of engineering: mechanical, structural, civil, earthquake, industrial, reliability, materials, ocean, aerospace, electronic, telecommunications, etc. Many applied physics problems and everyday life situations are also addressed. It is these examples (God is in details!) which is the book's forte.

Although the overall structure of the book is shaped with a particular audience in mind, it is the author's belief that it will prove of interest to a rather broad audience of engineers and applied scientists who, in one way or another, are encountered with, or are interested in, uncertainties in engineering analyses: practicing and research engineers, technical managers, college professors and students, instructors of various continuing education programs, etc. The book is also well-suited as a guide for self-study. I have tried to provide a combination of a clear, reasonably detailed, palatable, and, to an extent possible, entertaining introductory manual, and a useful reference source on the basics of the Applied Probability. I hope that some chapters will be useful also for experimental physicists, biologists, chemists, and even economists and market analysts. The bibliography

#### xiv Preface

at the end of the book, though not meant to be exhaustive, is rather extensive and will be useful for further study.

Ephraim Suhir IEEE Fellow, ASME Fellow Lucent Technologies, Bell Laboratories Physical Science and Engineering Research Division Murray Hill, New Jersey.

## **List of Basic Symbols**

$\boldsymbol{A}$
a, b, c
C(n, m)
D, $d$
$D_x = D(X) = \text{var}(X) = \langle (x - \langle x \rangle)^2 \rangle$
$\sqrt{D_x} = \sigma_x$
<b>V</b>
$oldsymbol{E}$
$E(X) = \langle x \rangle = m_x$
F(t)
F(x)
F(x, y)
f(t)
$f(x) = \frac{dF(x)}{dx}$
f(x, y)
G
$G(s), G_x(s)$
ď
g H
h . ————————————————————————————————————
$i = \sqrt{-1}$

 $\boldsymbol{J}$ 

#### area

constants

combination of n elements taken m at a time

diameter

variance of the random variable X standard deviation of the random variable X

modulus of elasticity, characteristic function

mean (expected) value of the random variable  $\boldsymbol{X}$ 

time-to-failure distribution function cumulative distribution function of the random variable X

joint cumulative distribution function of the random variables X and Y probability density function for the time-to-failure

probability density function of the random variable X

joint density of the random variables X and Y

shear modulus (or elasticity)

generating function, spectrum of the random process X(t)

acceleration due to gravity

hypothesis height

"imaginary" unit

Jacobian

V	completion function (coverience) of	
$K_{xy}$	correlation function (covariance) of the random variables $X$ and $Y$	
$K_x( au)$	(auto) correlation function of the stationary random process $X(t)$	
M(t)	moment generating function	
P(A)	probability of the event A	
P(A/B)	conditional probability of event A given that event B has occurred	
$\bar{P}(A) = 1 - P(A)$	complement of the probability of the event A	
P(n, m)	permutation of $n$ elements taken $m$ at a time	
$R(t) = P(t \ge T)$	reliability function	
r	damping coefficient, correlation coefficient	
$S_x(\omega)$	spectrum of the random process $X(t)$	
T	time-to-failure (random variable)	
t	time	
$\langle t  angle$	mean time-to-failure (MTTF)	
$\langle t_r \rangle$	mean time-between-failures (MTBF)	
$v_x = \sqrt{D_x}/\langle x \rangle$	coefficient of variation of the random variable $\boldsymbol{X}$	
X, Y, Z	random variables	
x, y, z	actual values (realizations) of the corresponding random variables	
α	producer's (seller's) risk (probability that a good lot or item will be rejected)	
β	consumer's (buyer's) risk (probability that a bad lot or item will be accepted)	
$\delta(t)$	Dirac delta function	
λ	failure rate	
σ	stress, standard deviation	
$\langle \sigma  angle$	mean (expected) value of stress	
$\sigma_f$	fatigue limit	
$\Phi(x)$	Laplace function	
$\Phi(i\omega)$	complex frequency characteristic	
$\phi(t)$	characteristic function	
Ω	sample space	
ω	elementary event, frequency	
Ø	empty set	

# Applied Probability for Engineers and Scientists

此为试读,需要完整PDF请访问: www.ertongbook.com

### Introduction

"We see that the theory of probability is at heart only common sense reduced to calculation; it makes us appreciate with exactitude what reasonable minds feel by a sort of instincts, often without being able to account for it.... The most important questions of life are, for the most part, really only problems of probability."

PIERRE SIMON, MARQUIS DE LAPLACE

"Coincidences, in general, are great stumbling blocks in the way of that class of thinkers who have been educated to know nothing of the theory of probabilities; that theory to which the most glorious objects of human research are indebted for the most glorious of illustration."

EDGAR ALLAN POE,
The Murders in the Rue Morgue

"If you bet on a horse, that's gambling. If you bet you can make three spades, that's entertainment. If you bet the structure will survive for a hundred years, that's engineering. See the difference?"

UNKNOWN ENGINEER

Engineering products must have a worthwhile lifetime and operate during this time and under stated conditions successfully to match the user's expectations. To achieve this, the engineer must understand the ways in which the useful life-in-service of the product can be evaluated and to incorporate this understanding in the design. His tasks are to analyze, design, test, manufacture, operate, and maintain the product, system, or structure at all the stages of its creation and use, from the moment of the conception and substantiation of the initial idea to the moment of writing off and wrapping the product, so

that it does not fail during the service period. When solving these problems, the engineer inevitably encounters variability in the employed materials, loads, manufacturing processes, testing techniques, and applications.

The traditional engineering approaches, when dealing with these problems, are referred to as *deterministic*, i.e., do not pay sufficient attention to the variability of the parameters and criteria used. Such approaches are acceptable and can be justified in many cases, when the deviations ("fluctuations") from the mean values are small, when the design parameters are known or can be predicted with reasonable accuracy, and when the processes and procedures that the engineer deals with are "stochastically stable," i.e., when "small causes" result in "small effects."

There are, however, numerous situations in which the "fluctuations" from the anticipated (mean) values are significant and in which the variability, change, and uncertainty play a vital role. In such situations the product will most likely fail if these uncertainties are ignored. Therefore understanding the role and significance of the "laws of chance," and the causes and effects of variability in materials properties, dimensions, tolerances, bearing clearances, loading conditions, stresses and strength, applications and environments, and a multitude of other design parameters, is critical for the creation and successful operation of a viable and reliable product or structure. In numerous practically important cases, the random nature and various uncertainties in the design characteristics and parameters can be described on the basis of the methods of the theory of probability.

Probabilistic methods proceed from the fact that various uncertainties is an inevitable and essential feature of the nature of an engineering system or design and provide ways of dealing with quantities whose values cannot be predicted with absolute certainty. Unlike deterministic methods, probabilistic approaches address more general and more complicated situations, in which the behavior of the given characteristic or parameter cannot be determined with certainty in each particular experiment or a situation, but, for products manufactured in large quantities and for experiments, which are repeated many times in identical conditions, follow certain, "statistical," relationships. These manifest themselves as trends in a large number of random events.

Probabilistic models reflect the actual physics of phenomena and the inevitable variability of the characteristics of an engineering system much better than the deterministic theories. Probabilistic models enable one to establish the scope and the limits of the application of deterministic theories, and provide a solid basis for a substantiated and goal-oriented accumulation, and effective use of empirical data. It would not be an exaggeration to say that all the fundamental theories and approaches of modern physics and engineering are probabilistic, and contain the corresponding deterministic models as first approximations. Realizing the fact that the probability of failure of an engineering product is never zero, the probabilistic methods enable one to quantitatively assess the degree of uncertainty in various factors, which determine the performance of the product, and to design on this basis a product with a low probability of failure. These methods, underlying modern methods of forecasting and decision making, allow one to extend the accumulated experience on new products and designs which differ from the existing ones by type, dimensions, materials, operating conditions, etc.

Probabilistic models are able to account for a substantially larger number of different practically important factors than the deterministic methods. It should be emphasized that the use of the probabilistic methods and approaches is due not so much to the fact that the available information is insufficient for a deterministic analysis, but, first of all, to the fact that the variability and uncertainty are inherent in the very nature of many physical phenomena, materials properties, engineering designs, and application conditions.

At the same time, it should be pointed out that probabilistic methods should not be viewed as a sort of panacea that cures all engineering troubles, and definitely should not be expected to perform miracles. Such methods have their limitations. For instance, they cannot be applied in situations where the conditions of an experiment or a trial are not reproducable or when the events are very rare. Was it possible to predict the outcome of the O. J. Simpson trials? Or was it possible to forecast the explosions in the World Trade Center or in the government building in Oklahoma City? Quite often a serious obstacle for applying probabilistic methods is also the difficulty of obtaining the necessary input information. In such cases the design has to be based on the worst case, i.e., in effect, on a "deterministic" situation, rather than on the application of probabilistic methods, or could be based on a combination of probabilistic and deterministic approaches. However, when the application of probabilistic methods is possible, justified, and is supported by reliable input information, these methods provide a powerful, effective, and wellsubstantiated means for engineering analyses and designs.

As it is clear from the book's title, it provides an introduction to probabilistic methods used in engineering and applied science. The book is written by an engineer and for engineers, and is not intended to be mathematically rigorous. The book is, first of all, a collection of examples (problems) and solutions. It is a "problem book," and

although it provides numerous sets of mathematical tools for attacking problems, presenting well-structural theoretical developments of various mathematical techniques is not among the book's objectives. Such tools are presented, in many cases, in the process of solving a particular problem. Although such a style might strike some pedantic readers as being unreasonable and lacking organization, it has been taken deliberately to keep the discussions descriptive, lively, friendly and interesting. The author realizes also that his judgements on what the reader does and does not know, as far as various mathematical techniques are concerned, are to a great extent subjective. although are based on the author's engineering and teaching experience. I assumed, for instance, that the reader is comfortable with multivariable calculus, partial differential equations and even with some special functions (such as Dirac delta-function or elliptic functions and integrals), but is not familiar with the theory of spectra or the theory of Markovian processes. Finally, the author realizes that some important topics (e.g. martingales) are not addressed in the book at all and that the presentation of others (e.g. queueing) can be made more interesting and consistant.

The author acknowledges, with thanks, the support of Dr. J. W. Mitchell and useful comments made by Dr. A. Weiss and Dr. L. Shepherd. He would also like to thank Mrs. C. Martin for her skill, patience, and care in typing the manuscript.

# **Contents**

Preface xili
List of Basic Symbols xv
Introduction xix

1. Random Events	1
Frequency and Probability	1
Axioms of the Probability Theory	11
Algebra of Events	15
Procedures for Counting the Sample Points	23
"Geometric" Probabilities	34
Bayes Formula	39
2. Discrete Random Variables	47
Random Variables	47
Frequently Encountered Discrete Distributions	52
2.2.1 Bernoulli trials and the binomial distribution	52
2.2.2 Poisson distribution: flow of events	60
2.2.3 Geometric distribution	69
2.2.4 Hypergeometric distribution	70
Generating Functions	72
3. Continuous Random Variables	79
Characteristics of Continuous Random Variables	79
Bayes Formula for Continuous Random Variables	82
Frequently Encountered Continuous Distributions	84
3.3.1 Uniform distribution	84
3.3.2 Exponential distribution	85
3.3.3 Normal (Gaussian) distribution	88
3.3.4 Distributions associated with the normal law	93
3.3.5 Rayleigh distribution	95
	Axioms of the Probability Theory Algebra of Events Procedures for Counting the Sample Points "Geometric" Probabilities Bayes Formula  2. Discrete Random Variables Random Variables Frequently Encountered Discrete Distributions 2.2.1 Bernoulli trials and the binomial distribution 2.2.2 Poisson distribution: flow of events 2.2.3 Geometric distribution 2.2.4 Hypergeometric distribution Generating Functions  3. Continuous Random Variables Characteristics of Continuous Random Variables Bayes Formula for Continuous Random Variables Frequently Encountered Continuous Distributions 3.3.1 Uniform distribution 3.3.2 Exponential distribution 3.3.3 Normal (Gaussian) distribution 3.3.4 Distributions associated with the normal law

#### viii Contents

	3.3.6 Rice distribution	98
	3.3.7 Welbull distribution	99
	3.3.8 χ²-distribution. Gamma distribution. Erlang	distribution 101
	3.3.9 χ-distribution	103
	3.3.10 Cauchy distribution	103
	3.3.11 Beta distribution	104
	3.3.12 Student distribution	105
Chapter	4. Systems of Random Variables	107
4.1	System of Two Random Variables	107
4.2	System of Many Random Variables	113
4.3	Complex Random Variables	114
Chapter	5. Functions of Random Variables	117
5.1	Characteristics of Functions of Random Variables	117
5.2	Distributions of Functions of Random Variables. Co	nvolution
	of Distributions	121
5.3	Limit Theorems of the Probability Theory	144
Chapter	6. Entropy and Information	149
6.1	Uncertainty of an Experiment	149
6.2	Entropy	152
6.3	Information	156
6.4	Communication systems	160
Chapter	r 7. Random Processes: Correlation Theory	167
7.1	Random Functions and Processes	167
7.2	Characteristics of Random Processes	169
7.3	Stationary Random Processes	175
	Crossing the Given Level	184
7.5	•	
7.5	Dynamic Systems	186
Chapter	r 8. Random Processes: Spectral Theory	195
•	•	195
8.1	Theory of Spectra 8.1.1 Fourier series	195
	8.1.2 Fourier series 8.1.2 Fourier Integral	197
	8.1.3 Fourier transforms: Spectrum of a function	198
	8.1.4 Parseval's formula	200
	8.1.5 Solution of ordinary differential equations up	
	Fourier integral	201
	8.1.6 The complex frequency characteristic as a s	spectrum of the
	impulse response of a dynamic system	204
	8.1.7 The duration of a process and the width of it	is spectrum 206
8.2		208
0.2	"White Noise" Wiener's Process Nerrow Rand Pr	rocesses 211