

# Data Segmentation and Model Selection for Computer Vision

*A Statistical Approach*



Alireza Bab-Hadiashar  
David Suter

*Editors*

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# Data Segmentation and Model Selection for Computer Vision

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*To our students, our teachers,  
and our families*

# Preface

The primary focus of this book is on techniques for *segmentation of visual data*. By “visual data,” we mean data derived from a single image or from a sequence of images. By “segmentation” we mean breaking the visual data into meaningful parts or segments. However, in general, we do not mean “any old data”: but data fundamental to the operation of robotic devices such as the range to and motion of objects in a scene.

Having said that, much of what is covered in this book is far more general: The above merely describes our driving interests.

The central emphasis of this book is that segmentation involves model-fitting. We believe this to be true either implicitly (as a conscious or subconscious guiding principle of those who develop various approaches) or explicitly.

What makes model-fitting in computer vision especially hard? There are a number of factors involved in answering this question. The amount of data involved is very large. The number of segments and types (models) are not known in advance (and can sometimes rapidly change over time). The sensors we have involve the introduction of noise. Usually, we require fast (“real-time” or near real-time) computation of solutions independent of any human intervention/supervision. Chapter 1 summarizes many of the attempts of computer vision researchers to solve the problem of segmentation in these difficult circumstances.

However, despite the perhaps unique nature of the difficulties, the fundamental fact that segmentation boils down to model-fitting leads one to naturally turn to statistical science for guidance. Statisticians have long studied the problem of fitting models to noisy observational data. Similarly, they have studied methods of model validation and selection (from alternative models), albeit, usually in the context of situations where a trained statistician is “in the loop” of data collection-analysis-collection.

Chapters 2 and 3 summarize some of the principles and recent work of statisticians on these important problems. It is our contention that much of the more recent work (if not some of the earlier work as well!) should provide guiding principles for methods in computer vision seg-

mentation and model-fitting, particularly with respect to finding robust methods.

However, as argued in Chapter 4, the transfer of knowledge from statistical science to computer vision is no mere application of principles and techniques. This chapter argues that the model-fitting of computer vision problems is a much more geometrical concept than the related concept of regression in statistics. The chapter goes on to define geometrical fitting, and, consequently, geometrical versions of model selection criteria.

Since the work summarized in Chapter 4 is very recent and demands a fair degree of familiarity with statistical concepts, it is not surprising that the work has yet to have its major impact on computer vision researchers. However, Chapters 5 and 6 show that the underlying principles can be applied to real world computer vision problems to yield new methods with promising results. These chapters concentrate on range (Chapter 5) and motion (both chapters) segmentation. In relation to the latter, the two approaches, although sharing the same principles of using robust statistical algorithms, are complementary, in that chapter 5 considers segmentation of optical flow (dense and instantaneous image plane motion), whereas chapter 6 studies the segmentation of motion correspondences (sparse and discrete image plane motion). Chapter 6 also summarizes work that has gone a long way toward developing and testing methods of model selection in that context.

This book is an expression of collaboration between researchers from two usually very distinct communities: statistical science and computer vision researchers. We trust that this book will serve both communities well. In particular, statisticians interested in application of robust fitting and model selection will find that this book provides them with the general ideas and historical background of image and scene segmentation (Chapter 1 and References). Chapter 4 will provide such an audience with thought provoking developments in a geometrical fitting theory. In addition, Chapters 5 and 6, together with the cited references, will provide them with practical tips and picture of the current state of the art in scene segmentation and model selection using robust statistics. Likewise, image analysis researchers will find that Chapter 4 provides an inspiration for the adaptation of statistical concepts for their work in image analysis, which generally involves some form of segmentation and some form of model-fitting (if not selection as well). Those unfamiliar with the recent work in robust statistics for model selection and model-fitting will find that Chapters 2 and 3 provide a snapshot of such recent work.

We would like to express our sincere thanks to those that made this volume possible: the contributors of the chapters, each an expert in their field and each generous with their time and expertise. Ms. K. McKenzie is thanked for a very thorough copyedit of this manuscript. Lastly, we thank the staff at Springer: In particular, Dr. William R. Sanders whose



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