FUZZY SETS AND APPLICATIONS:

Selected Papers by L.A. Zadeh

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PREFACE

At the beginning of 1985 the four of us decided that the 20th anniversary of the publication of the first paper on fuzzy sets would be a good time to undertake the project that resulted in this collection of papers. Our original conception was to be a personal tribute to Professor Zadeh who had in each case been instrumental in encouraging our efforts in the area of fuzzy sets. However, we soon realized that a selection of Professor Zadeh's papers would be of interest to a wide audience, and so by degrees our small project evolved into this volume.

The order of our names in the list of editors was determined in a way that we hope will appeal to all those who have had the fortune to be entertained by Professor Zadeh at any one of his favorite Chinese restaurants in Berkeley. During one of our dinner meetings to discuss the form and content of the book we decided that we should take the messages from those ubiquitous cookies and use them to define a lexical ordering based on the first word of the fortune. So:

"He loves you as much as he can, but he cannot love you very much."

"To open a book brings profit."

"You love sports, horses and gambling but not to excess."

"You will triumph over your enemy."

We leave the reader to decide whether these have any special significance!

All efforts of this kind require contributions from others. Accordingly, we thank Maria Taylor of Wiley for her support in putting together this volume. We also wish to give a special thank you to Margaret Tong, who achieved the nearly impossible by retrieving the paper reprints from the Zadeh archive. Then finally, of course, we must thank Professor Zadeh himself for providing us with the opportunity and the encouragement that he alone could give.

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CONTENTS

INTRODUCTION

Selecting from Professor Zadeh's extensive list of publications proved to be a challenging task. In the end we were limited more by the format of the book than by our ability to select important papers. The final choice was guided by our desire both to provide some historical perspective on his work and to assemble a collection that could be used as a reference by those working in the field.

The works are unedited, and furthermore, since we believe each is a classic in its own way, we have refrained from adding any significant, interpretative comments of our own. In particular, we have not tried to perform any scholarly analysis of the impact of these papers on the work of others, nor have we made any attempt to disentangle the many themes that run through them. We leave this to those who in the future may wish to publish "The Collected Works."

We start with a recent interview with Professor Zadeh that appeared in the Communications of the ACM [1]. This illustrates his basic philosophy, and so gives us a perspective from which to interpret this significant body of scientific work. Following that are eighteen papers that are representative of his enormous intellectual output in the years 1965 to 1986. Although we have grouped them into a number of presentational categories, they form a consistent and remarkably comprehensive approach to the problem of reasoning and problem solving under uncertainty.

FORMAL FOUNDATIONS

We have chosen seven papers in this category and present them chronologically. They contain formal developments of issues in fuzzy sets and possibility theory, and provide the framework within which to interpret the more application-oriented papers. As we might expect, they also tend to be the earlier publications.

We start, naturally, with the very first published paper on fuzzy sets [2] in which Professor Zadeh introduces the seminal idea and defines inclusion, union, intersection, complement, relation and convexity. Although all the material is familiar by now, it is especially remarkable for its recognition that a large part of human reasoning is concerned with problems "...in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the

presence of random variables." This is the central theme in all of Professor Zadeh's work and we see it in many forms in the papers in this collection.

Our second selection contains the first development of the idea of fuzzy events and their associated probability measures [3]. In this paper Professor Zadeh is again concerned with extensions of conventional mathematical ideas to what he considers to be everyday experience. In this case it is the interpretation of probabilistic statements about events such as "It is a warm day." The paper defines fuzzy event, the probability of a fuzzy event, independence, conditional probability and entropy.

The third selection is the highly influential paper coauthored with Richard Bellman that appeared in 1970 [4]. In it Professors Bellman and Zadeh develop a decision theory based on fuzzy goals and constraints. This paper provides the first explication of goals, constraints and decisions in environments that are fuzzy rather than random. With these concepts defined, the authors show how they can be applied to multistage decision problems in which the system under control is either deterministic or stochastic.

The fourth selection is concerned with the concept of fuzzy binary relations [5]. Here we see Professor Zadeh defining such important concepts as similarity relations and fuzzy orderings and also developing some valuable mathematical techniques for the resolution of fuzzy relations and the description of the transitivity properties of similarity relations.

The next paper is included because in many ways it is a summary of the results and ideas developed in 1973 [6]. This paper from *IEEE Transactions on Systems, Man, and Cybernetics* also has the distinction of being amongst the most often cited of Professor Zadeh's early works. Indeed for many of us it was our first introduction to the concepts in fuzzy sets theory. The paper contains a statement of the "Principle of Incompatibility" and presents an approach to the description of the behavior of complex systems using linguistic variables and fuzzy algorithms.

The sixth paper is also concerned with mechanisms for the description and definition of complex and imprecise concepts [7]. In it Professor Zadeh extends his ideas on fuzzy algorithms, focussing especially on those that can be structured as branching questionnaires.

The final paper in this section is another landmark. In it Professor Zadeh introduces for the first time the concept of a possibility distribution [8]. Published in 1978, it represents his next major step in the formal development of a theory of uncertain reasoning. Here we see clearly stated the premise that "... the imprecision that is intrinsic in natural languages is, in the main, possibilistic rather than probabilistic in nature." The mathematics of possibility distributions is thus intended as a unifying principle around which to address many questions of natural language interpretation. According to Professor Zadeh this is particularly important in the context of trying to develop machines "... which can simulate the remarkable human ability to attain imprecisely defined goals in a fuzzy environment."

APPROXIMATE REASONING

With this next category the focus of the papers changes. The overall vision remains constant, but Professor Zadeh is now primarily concerned with modelling the human ability to reach conclusions when the information available is imprecise, incomplete and not totally reliable. For this Professor Zadeh uses the especially apt phrase "approximate reasoning." The six papers included in this category are still formal—and especially logical—approaches to approximate reasoning, but we begin to see how the ideas may be transformed into computer-based reasoning systems. The work thus parallels, and in many cases anticipates, the efforts of workers in artificial intelligence.

The first three papers are to be read together. In them Professor Zadeh defines the concept of linguistic variables, performs a complete analysis of their properties and shows how they are used in approximate reasoning [9, 10, 11]. Among the many important topics discussed are the extension principle, fuzzy sets of type 2, marginal and conditional restrictions, separability and non-interaction, and the compositional rule of inference.

The next paper in this category is in the nature of a reprise for the work in approximate reasoning, while at the same time setting the scene for new developments [12]. In summarizing what he now describes as a theory of approximate reasoning, Professor Zadeh draws together the results from several earlier papers showing how the ideas of fuzzy logic, linguistic variables and possibility distributions all work together to provide a system for reasoning from imprecise premises to imprecise conclusions.

The fifth paper is an exposition on the role of fuzzy logic in expert systems [13]. Drawing on the attempts by various workers in artificial intelligence to construct expert systems that manipulate imprecise information, Professor Zadeh shows how the rules of inference in fuzzy logic can be used to model many of the problems that arise.

The last paper contains the most recent developments in the use of syllogisms in approximate reasoning systems [14]. In this work Professor Zadeh identifies the notion of a fuzzy syllogism as being a central unifying concept around which to formalize our ideas on commonsense reasoning. In addition, the computational framework which the syllogisms provide can be used to support a more satisfactory scheme for combining evidence than that used in the current generation of expert systems.

MEANING REPRESENTATION

The third and final category contains five papers that address directly the problem of meaning representation in natural language, with the primary effort now being directed towards a mathematically oriented interpretation of commonsense reasoning. In Professor Zadeh's conception of this problem, the key is to provide an

interpretation of natural language through the representational mechanisms of fuzzy sets and possibility theory. These papers overlap with those in the preceding categories, but a distinguishing characteristic is their focus on the direct translation of various propositional forms of natural language, rather than on the representational mechanisms themselves.

The first paper is an early attempt to use fuzzy sets to capture the meaning of hedges in natural language [15]. In this paper Professor Zadeh shows how a linguistic hedge such as "very," "more or less," "much," "essentially," or "slightly," may be viewed as an operator which acts on the fuzzy set representing the meaning of its operand. More complex hedges whose effect is strongly context-dependent require, in Professor Zadeh's view, the use of a fuzzy-algorithmic mode of characterization.

The second selection is the landmark paper on PRUF [16] which provides many of the basic elements in Professor Zadeh's approach to meaning representation. PRUF departs from the conventional approaches to meaning representation in several important ways. First, it assumes that imprecision in language is possibilistic rather than probabilistic. Second, the underlying logic is fuzzy rather than two-valued. Finally, it allows quantifiers to be linguistic. The paper contains formal definitions of four translation rules: modification, composition, quantification and qualification.

The third selection contains a discussion of what Professor Zadeh considers to be the crucial role of fuzzy quantifiers in meaning representation [17]. In this paper, fuzzy quantifiers are treated as fuzzy numbers that can be manipulated through the use of fuzzy arithmetic and fuzzy logic. The computational approach can be viewed as a derivative of fuzzy logic and test-score semantics and makes extensive use of the concept of the cardinality of a fuzzy set.

The next selection contains the beginnings of a theory of commonsense knowledge [18] that is based on the idea that this form of knowledge can be viewed as a collection of dispositions, that is, prepositions with implied fuzzy quantifiers. So for example, a typical dispositional statement that we may consider as being part of a commonsense knowledge base is "Snow is white." Professor Zadeh shows that if we assume the intended meaning of this is "Usually snow is white" then it has a much simple interpretation in terms of his test-score semantics than that provided by truth-conditional semantics.

Finally, we include a recent paper that shows how the test-score semantics can provide the basis of a computational approach to the representation of meaning [19]. Here Professor Zadeh illustrates the use of test-score semantics in the representation of the meaning of propositions, predicates, dispositions, and commands. He places much emphasis on the idea of a canonical form which "... may be viewed as a possibilistic analog of an assignment statement" and notes that semantic networks are actually specia' cases of this concept.

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The Selected Papers

Coping with the Imprecision of the Real World: an Interview with Lotfi A. Zadeh

The tools researches use to probe certain AI problems, says this Berkeley professor, are sometimes too precise to deal with the "fuzziness" of the real world.

Q. Professor Zadeh, in this interview today, you agreed to talk mainly about the limits of traditional logic in dealing with many of the problems in the field of artificial intelligence (AI) and your approach toward helping to overcome those difficulties. Before getting into those issues, though, could you first give our readers a brief overview of what you see as the major areas for computer applications in the years ahead?

ZADEH. In the years ahead, there will be three major areas of computer applications. One, in the traditional vein, is the use of computers for purposes of numerical analysis. Numerical analysis will be very important in a number of fields—particularly in scientific computations and simulation of large-scale systems.

For such purposes, there will be a need for larger and larger computers. This is especially true for applications in meteorology, in nuclear physics, in modeling of large-scale economic systems, in the solution of partial differential equations, and in the simulation of complex phenomena like turbulence, fluid flow, etc.

Area number two will be concerned with masses of data—large databases. This is the sort of thing that is playing and will be playing an important role in banking, insurance, records processing, information retrieval, etc. What will be important in these areas is not so much number-crunching capabilities as the capability to store massive amounts of data and to access whatever data are needed rapidly and at a reasonably low cost.

Furthermore, in these areas, computer networking, of course, will be playing an essential role. For you will