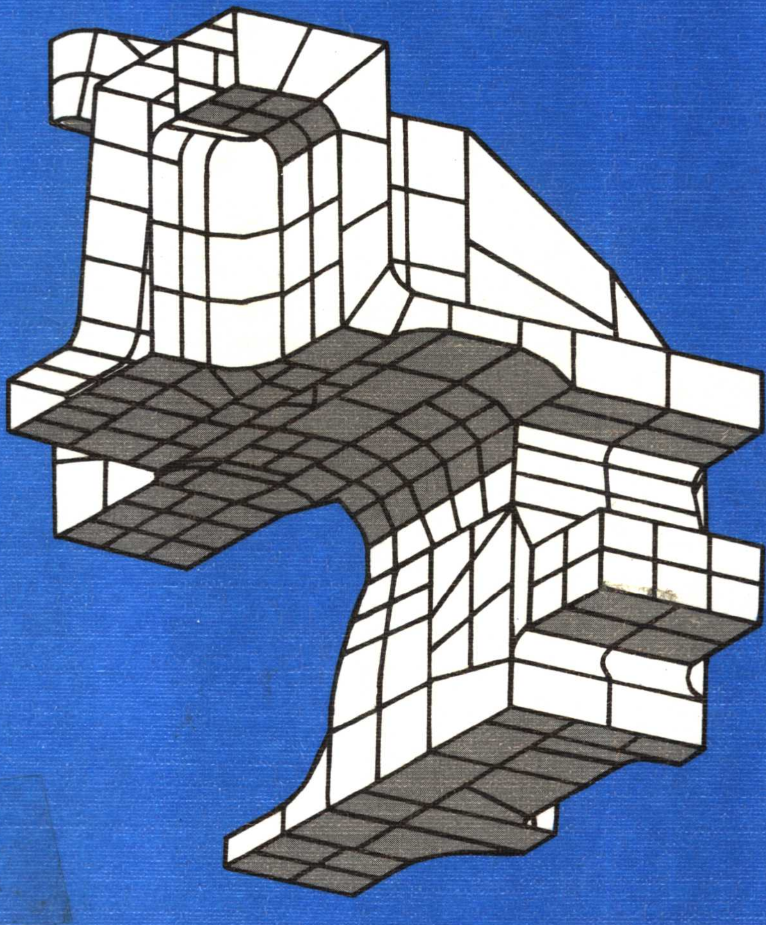
Boundary Elements
An Introductory Course

C.A. Brebbia and J. Dominguez



Computational Mechanics Publications
McGraw-Hill Book Company

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Computational Mechanics Publications
Southampton Boston

Co-published with

McGraw-Hill Book Company
New York St. Louis San Francisco Colorado Springs
Mexico Montreal Oklahoma City San Juan Toronto

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British Library Cataloguing in Publication Data

Brebbia, C.A. (Carlos Alberto)
Boundary Elements An Introductory Course
1. Mathematics. Boundary element methods
I. Title II. Dominguez, Jose
515. 3'53

ISBN 0-905451-76-7

Library of Congress Catalog Card Number 88-72042

ISBN 0-905451-76-7 Computational Mechanics Publications, Southampton
ISBN 0-931215-66-8 Computational Mechanics Publications, Boston
ISBN 0-07-007414-3 McGraw-Hill Book Company New York

Printed in Great Britain by The Bath Press, Avon

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Why Boundary Elements?

Engineers who have been exposed to finite elements may ask themselves why it is necessary to produce yet another computational technique. The answer is that finite elements have been proved to be inadequate or inefficient in many engineering applications and what is perhaps more important is in many cases cumbersome to use and hence difficult to implement in Computer Aided Engineering systems. Finite Element Analysis is still a comparatively slow process due to the need to define or redefine meshes in the piece or domain under study.

Boundary elements [1] have emerged as a powerful alternative to finite elements particularly in cases where better accuracy is required due to problems such as stress concentration or where the domain extends to infinity. The most important features of boundary elements however is that it only requires discretization of the surface rather than the volume. Hence boundary element codes are easier to use with existing solid modellers and mesh generators. This advantage is particularly important for designing as the process usually involves a series of modifications which are more difficult to carry out using finite elements. Meshes can easily be generated and design changes do not require a complete remeshing.

This point is illustrated in figure 1 by two views of a turbine blade section, one discretized using a finite element code and the other with boundary elements. Notice the presence of a series of cooling ducts in the blade whose size, position and number have to be reviewed during the design process. Such a variation creates difficulties for finite elements as some elements may easily become distorted or have bad dimension ratios. The boundary element mesh instead is easy to modify. Figure 1 describes a two dimensional application; these problems are of course compounded for finite elements when working in three dimensions.

Boundary element meshes, especially three dimensional ones can easily be linked to CAE systems as the structure is defined using only the boundary. The discretization process is even simpler when using discontinuous elements, which are not admissible in finite elements. The mesh shown in figure 2 represents the surface discretization of one eighth of a problem, i.e. a cylinder with a cylindrical perforation across. Notice that the use of elements which sometimes do not meet at corners and are consequently discontinuous in terms of their variables, facilitates the meshing. In addition there is no need to use elements on the planes of symmetry.

Figure 3 describes part of a bearing cap using discontinuous elements and taking full advantage of symmetry. These views can be easily rotated to check that elements on the surface are not missing. Notice that discontinuous elements allow for a simple mesh grading. The reason why these elements are possible in boundary elements is explained in some of the chapters in this book. From the user's point

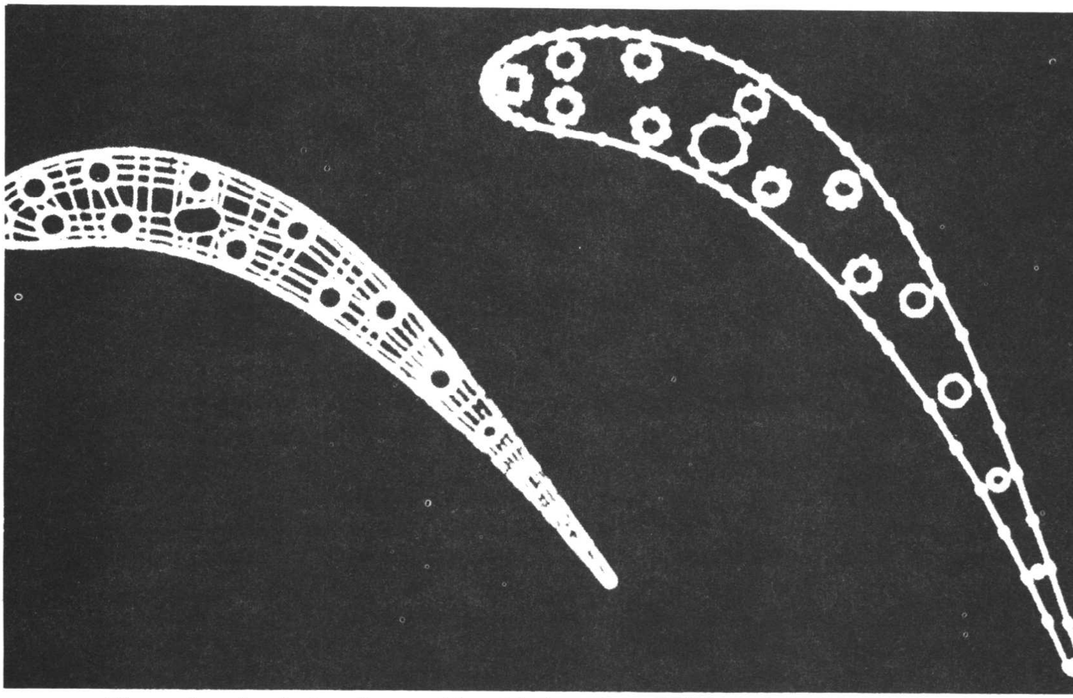


Figure 1. Analysis of a turbine blade using FEM and BEM. Notice that a variation in the configuration of cooling elements creates difficulties for the FE code (from a colour original)

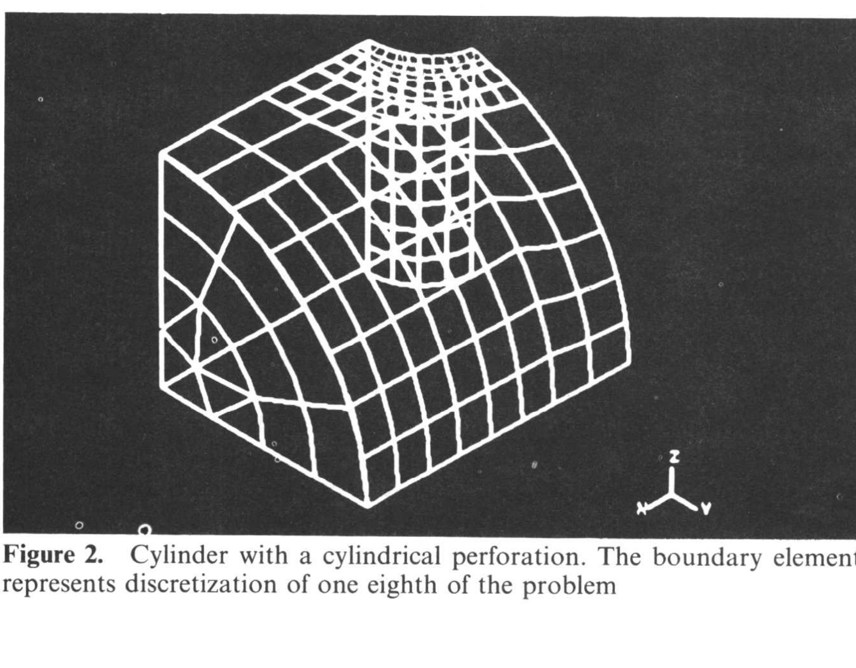


Figure 2. Cylinder with a cylindrical perforation. The boundary element mesh represents discretization of one eighth of the problem

of view they offer many advantages in terms of alterations of the meshes and general versatility. Figure 4 shows some displacements contours obtained for this cap bearing which are plotted using well known post processing systems.

More complex three dimensional structures such as part of the piston shown in figure 5 can be discretized relatively easily using a combination of continuous and discontinuous elements [2].

Boundary element codes can be used to analyse rapidly stresses or temperatures in different types of components. Figure 6 describes the mesh used to analyse part of a crankshaft. Results for the Von Mises stresses on the surface are also plotted. Figure 7 describes a section with a cylindrical hole at an angle which is analysed to determine its temperature distribution which is plotted in figure 8.

It is evident from these examples that boundary elements are an ideal tool for CAD mainly because it is easy to generate the data required to run a problem and carry out the modifications needed to achieve an optimum design. With computer costs declining while engineers' time becomes (or should become!) more expensive, the saving in engineers' time is of primary importance. (Also, engineers need relief from the dreary task of preparing finite element data.) More important still, any tool that can shorten the 'turn around' time taken by the analysis and design can bring forward the completion date of a project.

The future of BEM in engineering is promising and will continue to be so as long as the developers do not alienate the users by producing codes which are unreliable or cumbersome to use. Most of the advantages of BEM are related to

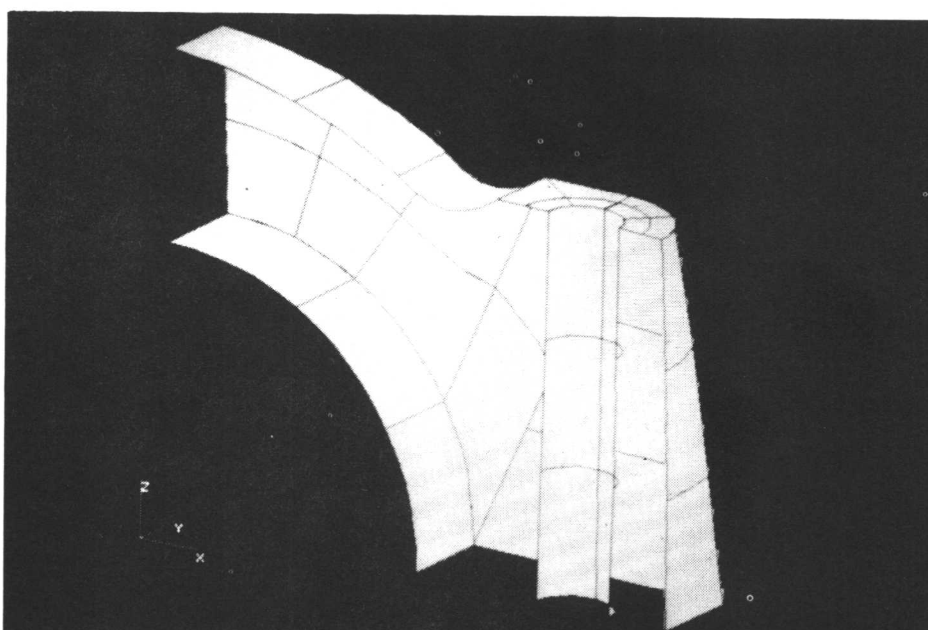


Figure 3. Discretization of the bearing cap into continuous and discontinuous elements (from a colour original)

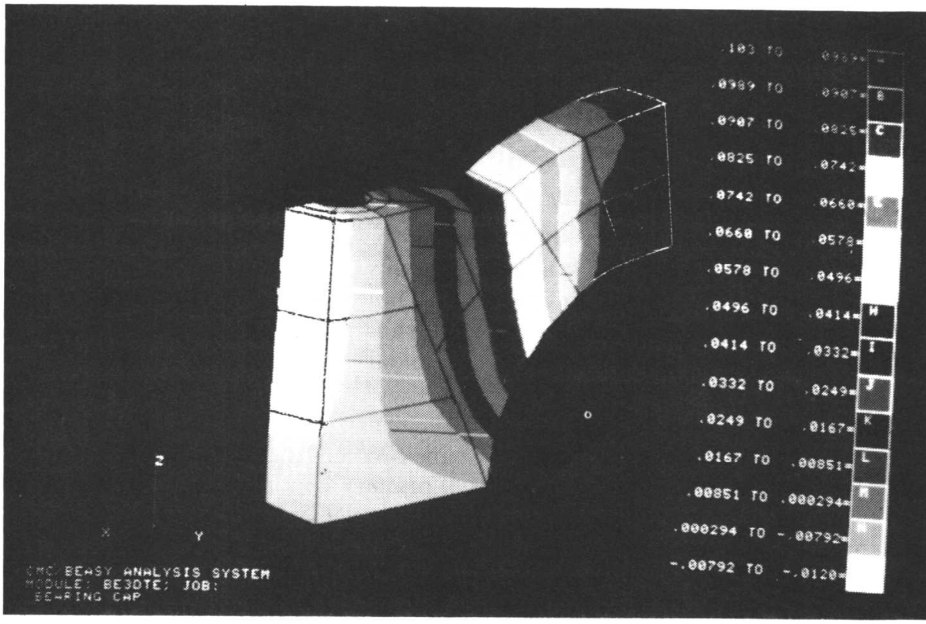


Figure 4. Contours for displacements in the cap bearing obtained using PATRAN for post processing (from a colour original)

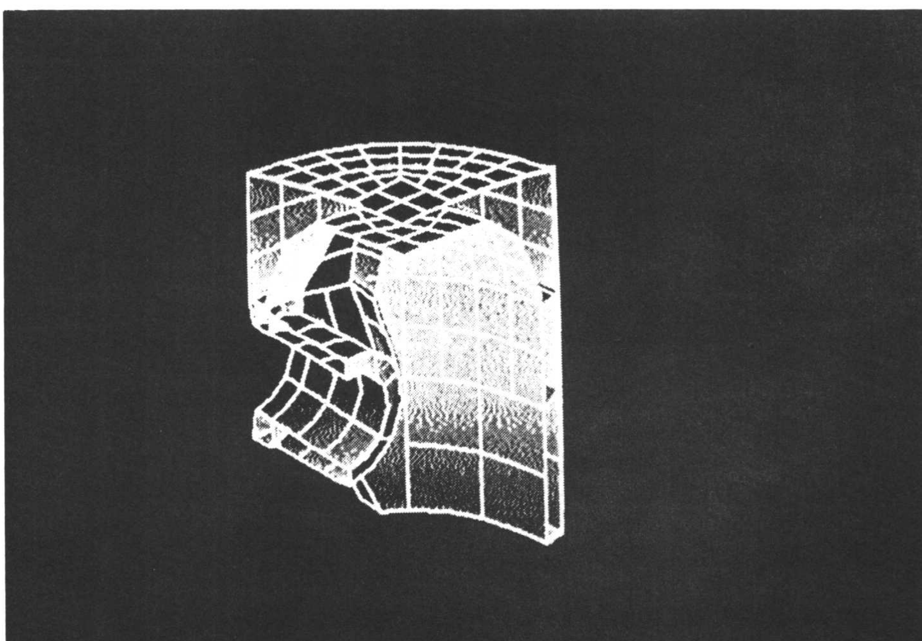


Figure 5. Part of a piston discretized using boundary elements (from a colour original)

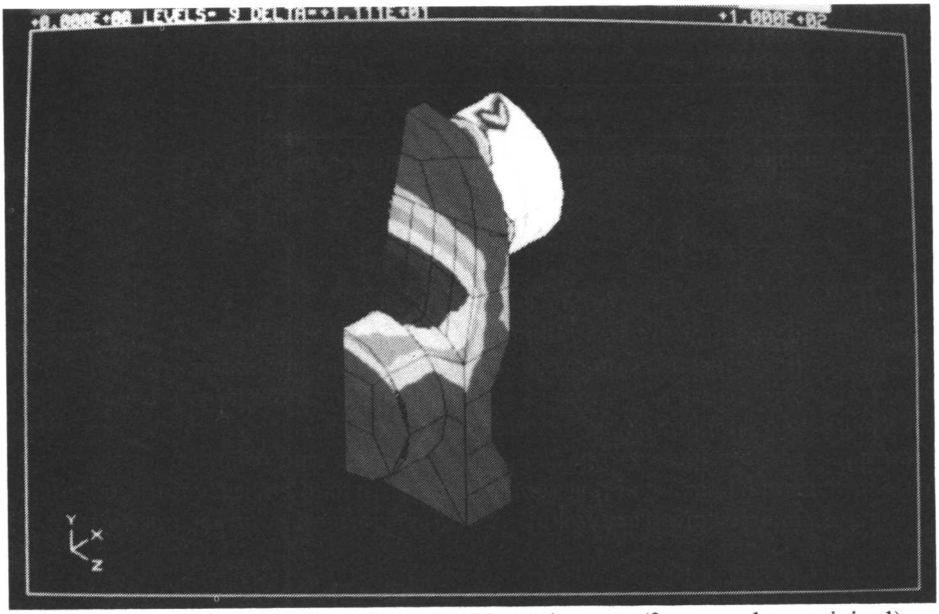


Figure 6. Part of a crankshaft discretized into elements (from a colour original)

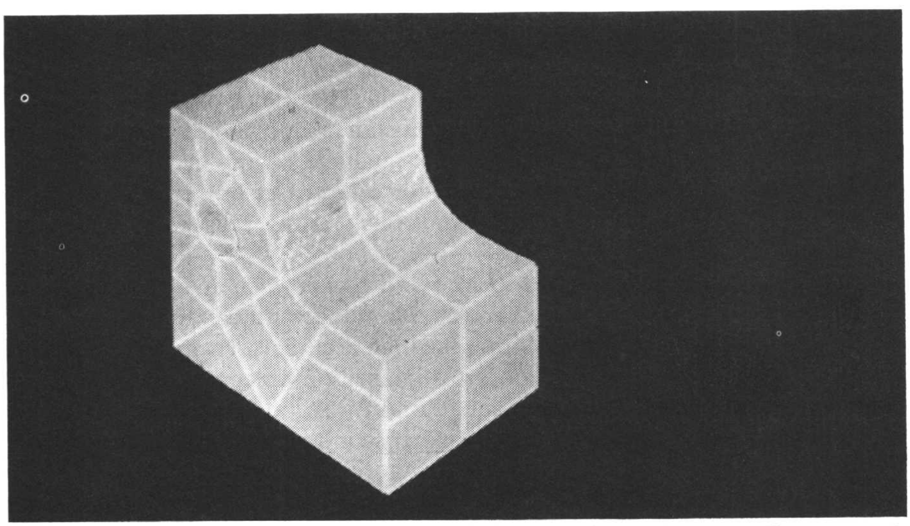


Figure 7. Discretization of a section with hole into elements (from a colour original)

its more complex mathematical foundations. This provides a high degree of versatility and accuracy in well-written codes but can have disastrous consequences in the case of poorly written BEM codes. The BEM is more susceptible to errors when the appropriate numerical techniques are not used and it is then important for developers to understand properly the theory of the method.

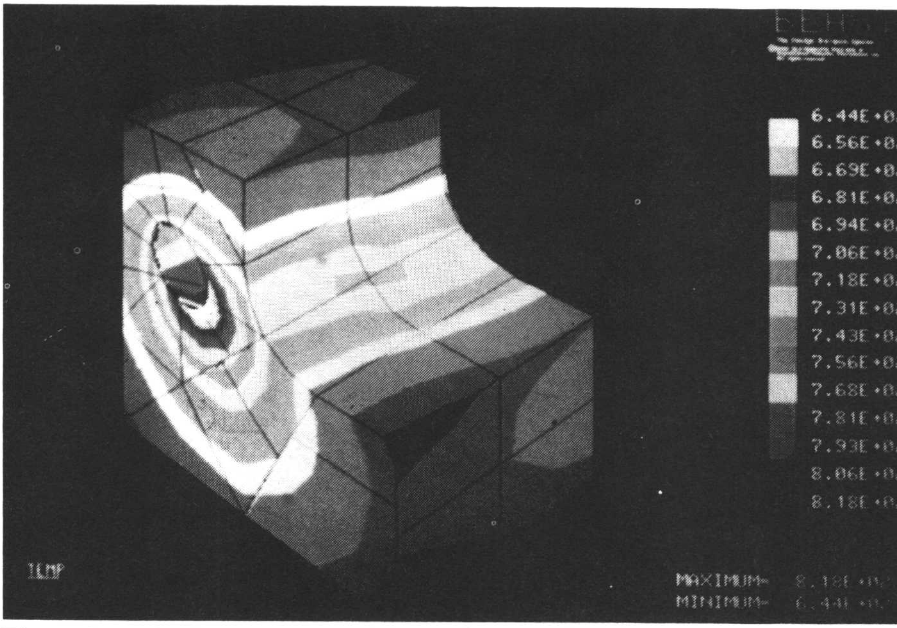
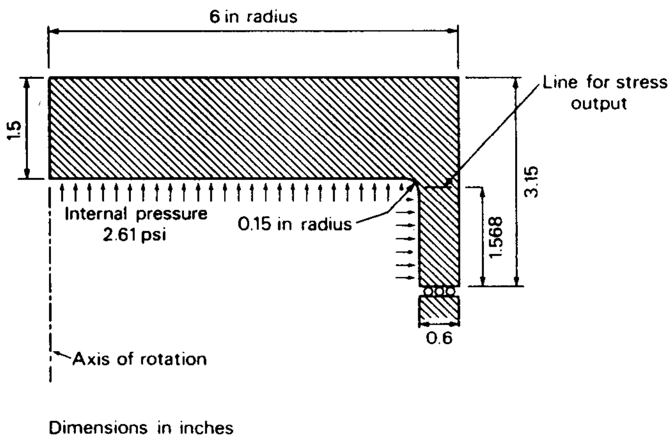


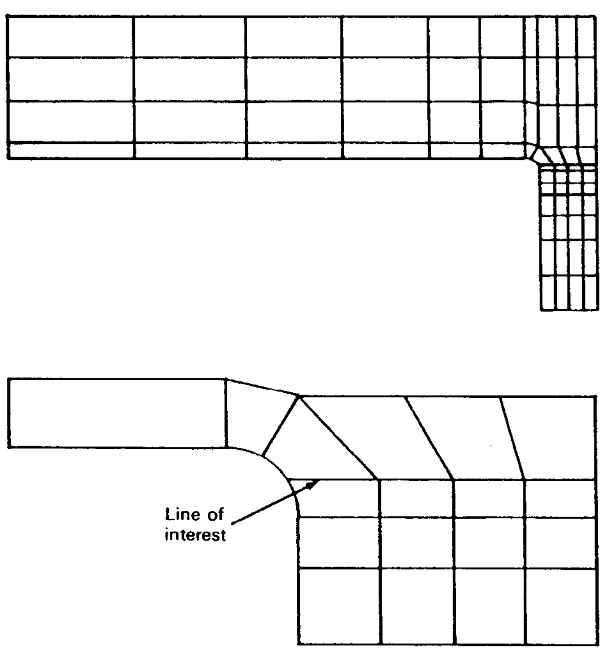
Figure 8. Distribution of temperatures over the surface (from a colour original)

Although better computational performance is important in BEM, particularly for three dimensional problems, improvements in CPU times should not come at the expense of precision and accuracy. For instance, applying coarse numerical integration techniques to BEM codes can result in large savings in computer codes and give reasonable results in many cases. For other cases however the solution may be of very poor accuracy or give non-convergent results. This makes such codes unreliable.

Another important advantage of BEM over FEM is when analysing problems with stress (or other type) concentrations. Many such studies have now been carried out and they tend to demonstrate the high accuracy of boundary elements for problems such as re-entry corners, stress intensity problems and even fracture mechanics applications. It is not our intention in this introduction to review all these studies but rather to point out the difference in results that can be obtained using one or the other numerical method. As an illustration one of the finite element solutions found along a line in the neighbourhood of a re-entry corner (figure 9) of a pressure vessel is shown in figure 10. The problem was also analysed using a photo-elastic model and boundary elements. Results for a finite element mesh consisting of approximately 500 degrees of freedom (69 elements) and using eight nodes elements are compared against BEM solutions obtained using only 20 elements. It is evident from the figures that while the finite element results show lack of equilibrium in the domain as well as on the boundary, reasonably accurate solutions were obtained using boundary elements. It was only when using a very refined finite element mesh that the FE results were in agreement with the boundary

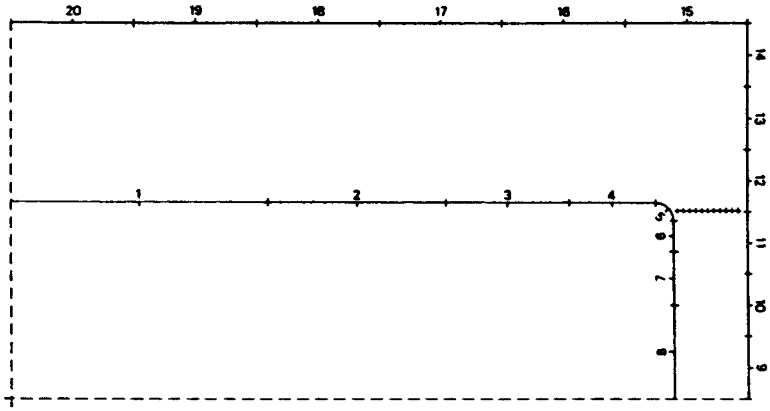


Geometry of the region under consideration



69 finite elements mesh
(with mid side nodes)
approximately 500 degrees of
freedom

Figure 9. Re-entry corner in pressure vessel



20 boundary elements mesh,
120 degrees of freedom

Figure 9 *continued*

elements and photo-elastic model solution. For a full discussion of these results the reader is directed to references [3] to [5].

The development of more powerful hardware, especially supercomputers, favours the use of BEM. These computers are better suited to deal with the fully populated matrices and the type of operations which are characteristics of boundary elements. Large problems can now be solved in a comparatively short time using machines such as the CRAY which produced the results for the bracket type component shown in figures 11 and 12.

Problems other than stress or temperature analysis can be solved using boundary elements. Typical applications include torsion, diffusion, seepage, fluid flow and electrostatics. Corrosion engineers have used the method to design better cathodic protection systems for offshore structures, ships and pipelines. Many of these structures are basically three dimensional and the region of interest extends to infinity. Consequently they could not be effectively analysed before the development of boundary elements. Early attempts to use finite differences or finite elements to solve these problems met with little success. For these cases the computer model has to represent the potential field around the structure, representing the shielding effect of the structural geometry and the effect of the different materials involved. Unlike a structural model the cathodic protection model is concerned with the seawater around the structure and the interface between the seawater and the structure. Hence the use of FEM to analyse the problem would require the subdivision of the seawater surrounding the structures which is a Herculean task.

The use of boundary element method represents the only practical solution for this problem. The advantage of the method is that only the structure needs to be defined as the BEM automatically takes care of the field – i.e. the seawater – extending to infinity. Figure 13 shows the first three dimensional BEM cathodic protection application which was the study of the tension leg platform (TLP) built by CONOCO in the Hutton Field in the North Sea. Figure 14 shows the

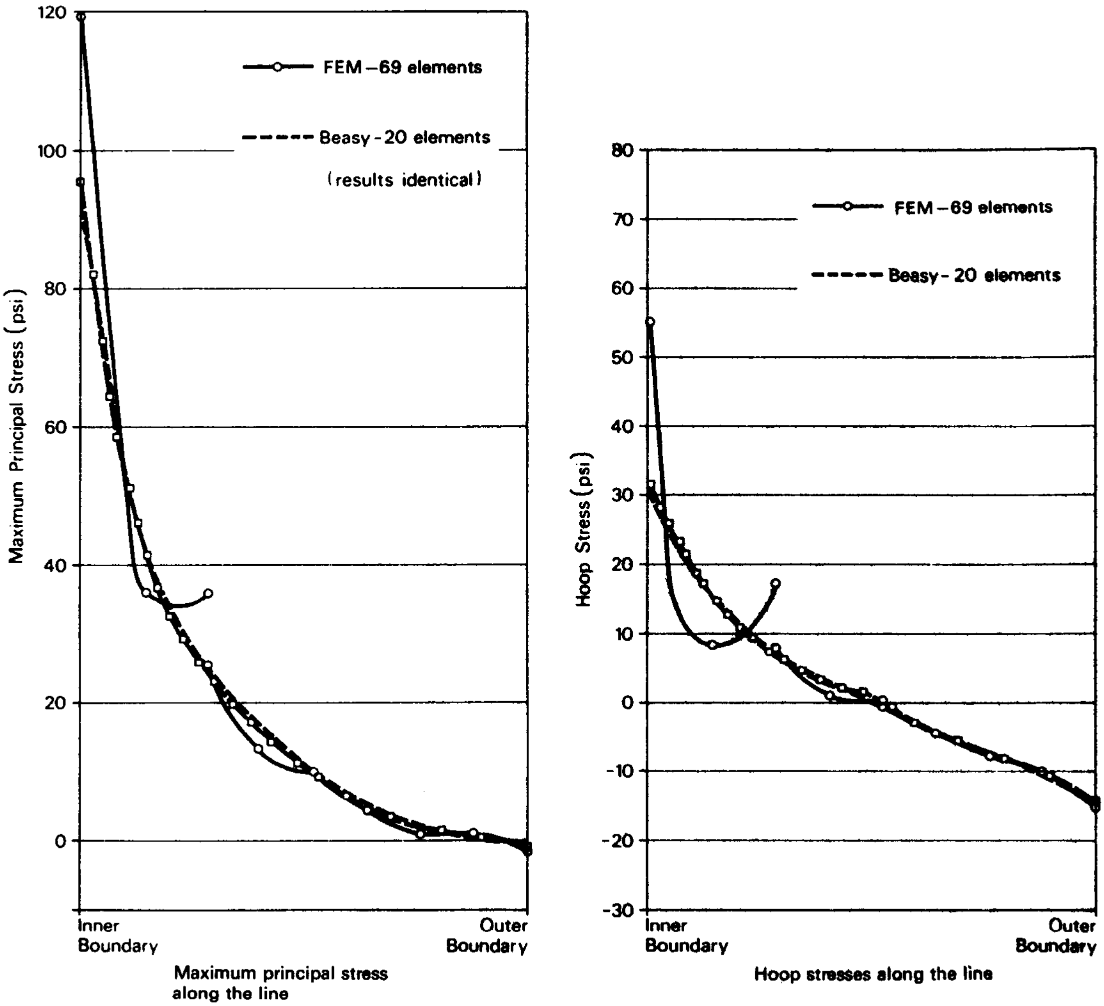


Figure 10. Comparison of FEM and BEM results along the line indicated in figure 9

discretization of a quarter of the structure into the boundary elements used in the analysis and figure 15 the results obtained for the potentials on the surface for a particular configuration of the improved code system used. Since then the boundary element method has become the key to the successful and practical analysis of cathodic protection systems and further work has been carried out in this regard particularly at the Computational Mechanics Institute, Southampton, UK. A system is now available which allows the corrosion engineer to evaluate design options, look at problem areas, interpret experimental observations, optimize the design and predict with accuracy and confidence the degree of protection and life expectancy of a cathodic protection system.

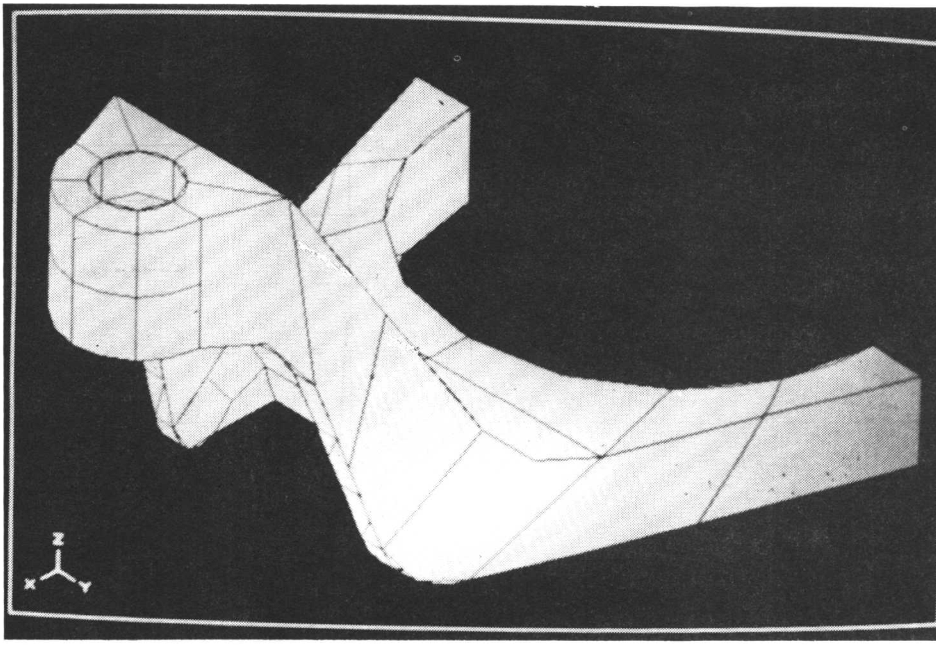


Figure 11. Boundary element mesh for the bracket (from a colour original)

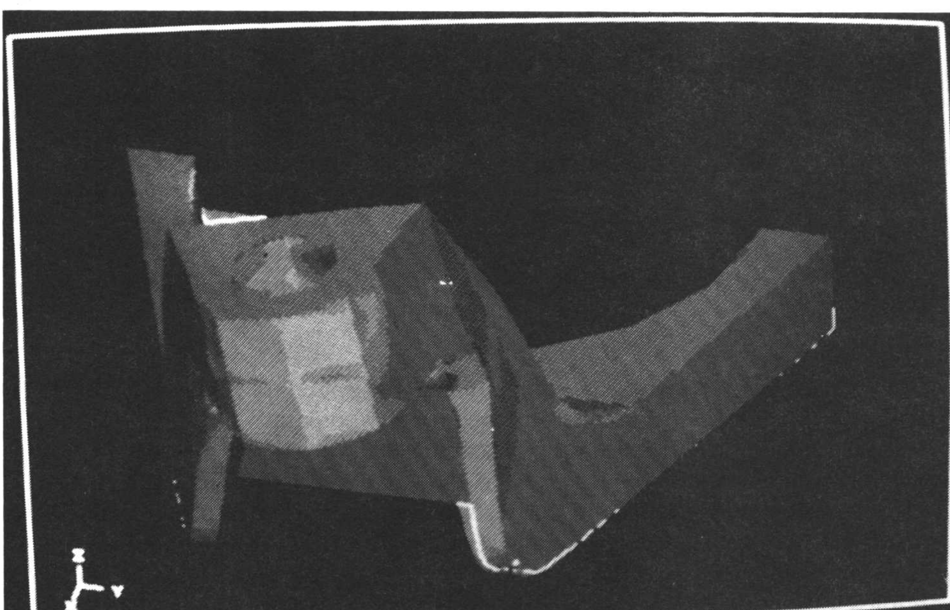


Figure 12. Defined shape for above. Results obtained in a CRAY XMP plotted with SUPERTAB (from a colour original)

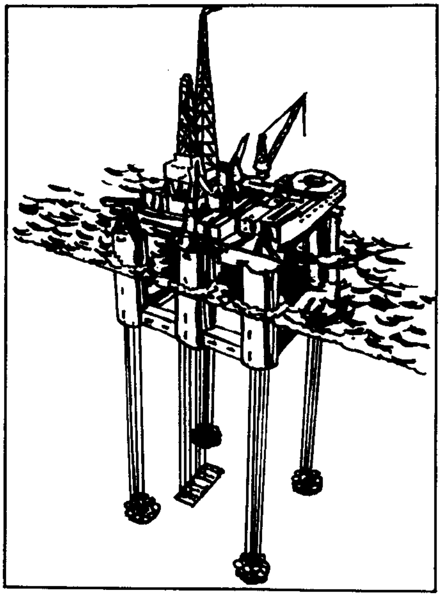


Figure 13. The CONOCO Hutton TLP (Tension Leg Platform)

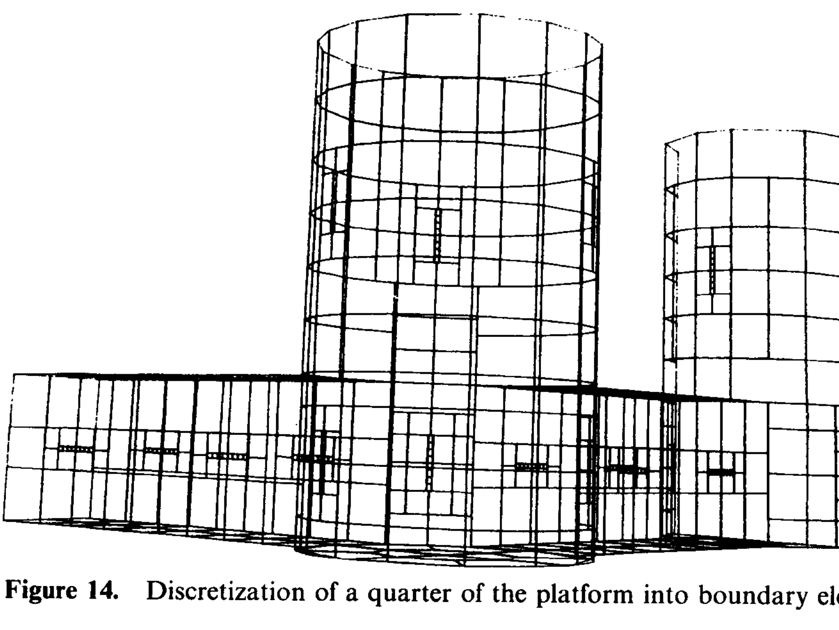


Figure 14. Discretization of a quarter of the platform into boundary elements

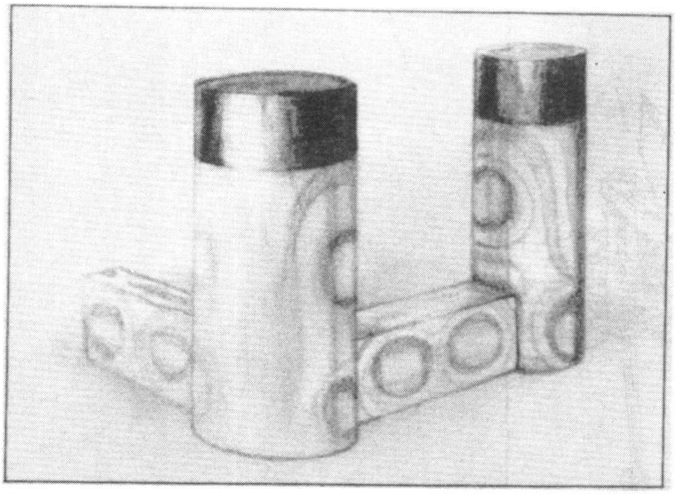


Figure 15. Model of the TLP showing contours of potentials

The advances made in cathodic protection modelling using boundary elements are just one of the applications of the technique for systems extending to infinity. The method is nowadays extensively used in other problems with infinite or semi-infinite domains such as those occurring in geomechanics, ocean engineering, foundations, aerodynamics, flow through porous media and many others.

This brief introduction has attempted to point out the advantages of BEM for a wide variety of engineering problems and the reason why the method should be taught on an undergraduate as well as a graduate level. University courses should include the fundamentals of the method and provide workshops on applications while short courses with hands-on applications will help to bring the method to the attention of practising engineers. This book has been written to provide a simple and up to date introduction to the method to help popularize the technique amongst engineers.

The future of BEM hinges on its acceptance by practising engineers, in particular as a design tool. Developers should aim to make the method more accessible to engineers by writing codes which are easy to use and by explaining the fundamentals of the method on the basis of engineering rather than mathematical concepts. This book has been written in a form that can be used as a textbook at undergraduate or graduate level and for the engineer in practice who wants to learn the fundamentals of the technique unaided. Of particular interest is the way in which the mathematics concepts are introduced and immediately applied in simple computer codes. These codes (4 for potential and 2 for elasticity) will facilitate the comprehension of BEM.

This book is based on the authors' many years experience as researchers and teachers of boundary elements. It is designed to teach in the most effective manner the fundamentals of the method rather than to attempt to demonstrate erudition on the subject. Many topics have been deliberately omitted to avoid confusing