Social Choice and Multicriterion Decision-Making

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Introduction

The current models used in operations research for the multicriterion ranking of a finite set of alternatives often lack firm (mathematical) foundations. This book intends to derive, from the lessons of social choice theory, possible foundations for multicriterion models effective for one type of decision frequently occurring in industry.

Consider a large number of alternatives and a large number of criteria, where "large" means greater than four and less than, say, five hundred. Suppose that each criterion ranks the alternatives according to its weak ordering, from the best to the worst one. Our decision problem consists of ranking the alternatives from the best to the worst according to a nontrivial weak ordering that is a legitimate synthesis of the criteria. We call this problem the industrial outranking problem.

Since the publication of Arrow's impossibility theorem [1951] and Black's work, summarized in his book [1958], much effort has been spent on the analysis and rationalization of committee decision-making. Noncontroversial progress in the theory of social choice and in committee decision techniques has come from the study of strategic voting (see, for instance, Gibbard [1973] and Satterthwaite [1975]) and from what has been called implementation (see, for instance, Fine and Fine [1974]). There has been less consensus in the field of multicriterion operations research. The accent has been on building a large collection of multicriterion decision-making recipes without being able to decide which of them were the best. Papers like "Douze méthodes d'analyse multicritère" (Twelve methods for multicriterion decision-making), by G. Bernard and M. L. Besson [1971], show great ingenuity in inventing new recipes. It is difficult, however, to determine which one makes more sense than the others.

A celebrated example of such a recipe is the Electre Method in its early development (cf. Susmann et al. [1967] or, for a short presentation, annex 1). For over ten years, at least in France, many discerning managers have thought that Electre was *the* way to deal with difficult multicriterion decision-making, even though this method, as far as we know, does not satisfy any system of consistent and appealing axioms.

We think that the professional decision maker should know that, especially in multicriterion analysis, he can be the victim of a series of personal biases that he should wish to avoid. In particular, he should hope for something better than just a recipe—rather, a true method that would at least appear to have solid foundations. Through dealing with **real** problems, it is clear that a recipe that allows for creativity, intuition, and adaptation to specific conditions will perform better than any elegant mathematical model. But if the recipe is inspired by safe principles, the decision maker should avoid the danger of using a method either so versatile that it will not prevent the influence of his personal biases or so artificially rigid that its application would appear unacceptable.

A few indisputable lessons arise nevertheless from the success of the Electre Method:

1. Decision makers liked the way Electre somehow describes an ideal behavior, extending to a large number of

criteria and alternatives a technique not without psychological value for small sets of data.

2. Decision makers enjoyed very much being able to understand the principles of the method.

3. With its large collection of arbitrary parameters, Electre is a very versatile method, which did not diminish the responsibility of the decision maker in the decision process.

These three points stress qualities that any effective method should possess.

The motivations that have inspired our work on the industrial outranking problem are simple:

• On one hand, ranking into a reasonable number of classes finite sets of "projects" already ordered according to a finite number of (most of the time) qualitative criteria became, for one of us, a daily burden! He felt that he had to offer more than a list of empirical recipes. In addition, he shared the assertion made by Eckenrode [1965] and pointed out by Johnsen [1968] that the more ordinal the data, the more consistent the result.

• On the other hand, the work on social choice was almost the only theoretical approach dealing with the foundations of multicriterion decision-making, in the domain where the decision consists in ordering a finite set of alternatives already ranked according to a finite set of criteria.

These two reasons explain our conscious limitation to the purely ordinal case and our research technique; it is by extending and translating present results in social choice theory that we found the main results presented in this book.

The first one is an extended list of conditions for efficacy of the majority method when applied to our case, not algebraic conditions but conditions with interpretations satisfactory for an industrial problem. The second result shows that these cases of applicability of the majority method form, statistically speaking, an asymptotically null set; in other words, these conditions will be met in practice only in very exceptional cases. The third result is a noncontradictory axiomatic system that makes sense for the industrial outranking problem (a system that includes a generalization of the Condorcet condition). And the fourth result is the identification of some of the practical methods satisfying this axiomatic system and the study of some of their properties.

All these concerns have induced the following outline of the book. Its first part is devoted to some of the simplest classical psychological considerations and results, which, further on, will help us to model an ordering process that allows for conflicting and incommensurable criteria. The fact that the more satisfactory axiomatic systems lead directly to unacceptable paradoxes will be treated in this first part. Because of its huge historical importance, the majority method, along with its limitations and its domain of efficacy, will comprise the entire second part. It will be shown how indeed it is theoretically of limited interest in solving our industrial decision maker's problem. The third and last part will concentrate on the new axiomatic system, which leads to practical techniques solving satisfactorily this same problem.

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Part I

1 The Problem

This chapter contains a miscellany that indeed could be omitted from our subject, being devoted to purely mathematical questions. But it seemed appropriate to outline the type of multicriterion ranking process that suggested the logical itinerary followed throughout the book. The problem is precisely one met everyday by one of us, as a consultant in industrial decision-making.

When hired to consult, it is very often about an ordering process in which two steps can be clearly identified:

1. an identification step, which consists of selecting relevant alternatives and selecting relevant criteria—both very approximate operations;

2. a processing step, which consists of selecting an aggregation method (selection is usually reduced to the mere acceptance of the first method proposed) and applying it to the data, and almost in any case a weighted majority method.

It is through the description of these steps that the needs of the decision scientist we hope to fulfill in this book will be made obvious.

1.1 The Identification Step

1.1.1 Selecting Relevant Alternatives Is an Approximate Operation

When a decision maker has to make one of the **decisions** we have in mind, he identifies a set of alternatives as being the supposedly competing ones. This set can seem very restricted (e.g., selection of one best candidate among a few or choice of a best strategy among a small number of proposals). More often, its limits are not so clearly fixed; one can have to take into account last minute candidates, or new combinations of strategies suggested by last minute information.

A small number of alternatives is the exception; a large but finite set of possibilities is the rule. This extensive set of alternatives is often represented by a continuum. Our methodology is to take the alternatives as identified by the decision maker (i.e., a large but finite set); but we attempt to derive methods that avoid the risk of instability in the results caused by the introduction or the deletion of a really noncompeting alternative. In other words, in those methods, only an outstanding new alternative would be able to disturb the ranking of the best elements.

1.1.2 Selecting Criteria Is Very Approximate As Well

Our decision maker also identifies a set of criteria that are supposed to be the pertinent ones for the decision.

We distinguish between two types of criteria:

a. *attributes,* which are rather well identified aspects of the alternatives, but are only correlated with the desired outcome of the alternatives;

b. *objectives,* which are directly connected with the desired

outcomes, but for which the estimation will be in general fuzzy.

Consider, as an example, the choice of advanced research projects for new production in a business firm. Let the desired outcome be a very profitable project. Correlated with this outcome, "having a fast return on investment" can be the first financial objective. The statement, "The head of the research lab thinks that the project is the most feasible from among the others," is only an attribute. But because its evaluation will be much more accurate than the evaluation of the financial objective, it will likely be more important.

In industrial multicriterion problems, we mainly encounter multiattribute problems. In this sense, the expected profit of an alternative can generally be considered as an attribute, when the desired outcome will, for instance, be to make safer the equilibrium of the productions of the firm. These attributes, easily obtained from the decision makers, are likely to be very large in number. For instance, Duncker [1903] already quotes attributes related to

- market standing,
- innovation level,
- productivity,
- physical and financial resources,
- profitability,
- manager performance and development,
- public responsibility.

In the M.A.R.S.A.N. method (Susmann et al. [1967]), as many as 49 potentially valid attributes were listed and actually taken into account.

In developing a recent model for the selection of diversification projects in an international business firm, a committee of the top decision makers in the firm suggested to one of us for their particular case not less than 25 strongly independent criteria.

1.2 The Aggregation Process

Underlying the criteria are ordinal preference structures: in other words, criteria that may look a priori to be numerical in nature can often be expressed as semiorders without any loss of information. Let us, for instance, consider the size of the investment required by a project. For many firms, it is not the numerical value of the investment that is important but the fact that the firm is used to dealing with this "precise" size of investment. From five to seven equivalence classes will in general be sufficient to describe the size of the investment precisely.

The name of aggregation is due to the fact that the desired result of the process is similarly a semiorder, with a reasonable number of equivalence classes. The frequency of this problem in industry is easy to understand. Let us consider a set of projects for a particular business firm. The real decision will be to realize some of these projects. During the course of their realization, some should rapidly prove unfeasible, some will need to be deleted, some, on the contrary, will suddenly become obsolete. The effective realization of an industrial project is highly unpredictable, and the best tool for a decision maker is a priority order on the projects, judged independently, in such a way that if a project has to be abandoned, or if the available budget for the projects is enlarged, the new projects to be realized will be the next ones along the "aggregated" order.

1.2.1 Then Comes the Choice of an Aggregation Method

What really happens inside a man's brain when he undertakes such a complex operation as in a multicriterion aggregation is still a conjecture. Even for very simple decisions indeed, we are far from being able to present realistic models of the phenomenon. As a proof of this assertion, we need only recall the experiments of Ungar [1973] on darkness avoidance in the rat. Conditioning rats to make the decision—from a learning process—to avoid darkness and prefer light leads to the synthesis, by the brain of the animal, of a special polypeptide, called scoptophobin, the presence of which in the brain is significantly correlated with darkness avoidance in nonconditioned rats. The building of such a substance is the result of an extremely complex mechanism, which we cannot even imagine. We are hence condemned to make a phenomenological model of what happens in the brain by only looking at very superficial, but indisputable, facts.

For instance, we know that a decision maker tries to make his decisions through knowledge and enlightening experiences. For an important decision, he would like to be able to process all the information his memory has stored that would be relevant to the subject. This, of course, cannot be done, and this impossibility can be explained especially by the structure of memory. For one hundred years (Ebbinghaus [1885]), it has been known that a human brain possesses a short-term memory that allows the storage, for a very limited time, of a very limited number of items available for treatment. Even if training this memory can prove effective in the increase of the performance, its capacity remains quite limited (as everyday experience confirms).

A human long-term memory, on the contrary, stores so many items in a lifetime that its huge capacity can be considered unlimited. However, if these data have to be retrieved for treatment, it will be through a linear chain of associations, and the process for retrieval will be relatively long and somewhat painful: try, for instance, to remember a birthday once forgotten! The same process is, when unconscious, considered by some authors as describing intuition, which can seem much more efficient. But intuition is well known for not being guaranteed, and one should feel it inappropriate to leave the responsibility of a dangerous decision to a process totally uncontrolled and subject to many errors and biases.

We have to admit that a regular brain is not built in order to make complex multicriterion decisions: the quantity of information is too large to allow a simultaneous treatment by the short-term memory, and there is no clearly dominant methodology allowing a progressive treatment. This is often typical of the urge for help expressed by many decision makers: they estimate that they are not able to consider concurrently all the projects and all the desirable criteria in order to make the decision as they would like.

They have often, however, the feeling, and sometimes enough intuition, to make the decision effectively from a small number of criteria applied to a small number of alternatives. Based on personal experience, we estimate four criteria and as many alternatives to be the maximum humanly tractable complexity.

The task of the scientist who wishes to help the decision maker should be relatively easy, as long as the latter will be able to explain clearly his way of processing a small amount of data: it can be conjectured that his inability to solve bigger problems comes in large proportion from the limitations of the short-term memory.

The decision maker is almost never able to suggest a set of noncontradictory axioms, which would be the set he actually uses; rather he expresses the axioms he would like to follow; as we shall see in the next chapter, the more appealing the hypotheses, the more infeasible appears the corresponding axiomatic system.

1.2.2 The Weighted Majority Deadlock

Strangely, one part of the recipe, which is rather wishful thinking, is popular among the decision makers, and deserves to be described. Very often, after the evaluation of the alternatives for all the criteria, the decision maker realizes that his problem is not made easier. A little disappointed, he expresses the feeling that the light will come from "weighting" the criteria. The decision maker implicitly then builds up the following dangerous model. He considers a committee in which the criterion i is associated with W_i members of identical behavior, W_i being the integer weight of the criterion i. He will then apply the majority method of decision.

We think that these committee members, in order to make the model realistic, should not behave like committee members. For instance, they should not act strategically (a technical criterion of not being shrewd). In spite of this, the decision models that are then used by the decision makers are identical to the procedures used in committee decision-making: they begin by trying the weighted majority method. For reasons that will be made clear, it almost never works. Then they have to use methods, such as Borda's very widespread method, that have been thought of only for the case of committee decisions and appear inappropriate. Through this process, they obtain for diverse reasons, results that, even if appropriate to committees, usually do not conveniently solve industrial multicriterion decision-making. As a consequence, faced with the lack of a convenient method, the decision maker prefers to wait until the last moment to make the ranking decision. This attitude allows him to benefit, more or less consciously, either from additional information that will legitimately influence the choice and will make it easier or from a state of emergency imposing one survival criterion over the others. This attitude, of course, does not solve the question, but we tend to believe that the "weighting" of the pros and cons yields some insight.

Weighting can be considered as a direct measure of the reinforcement of a learning process: a criterion that has often been correlated in the past with success or failure will naturally be granted a higher "weight." We know of no experimentation supporting the fact, but it fits with Ungar's experiments: a special polypeptide is present in the brain of rats submitted to a special training, and this polypeptide acts as a facilitator to the learning of this behavior when injected into untrained rats, as additional weights for the corresponding criteria.

One must add (which is confirmed by our experience) that the weights given by the decision maker are always subject to considerable fluctuations from one day to the next, even from one hour to the next! But the fact that they insist on assigning weights probably means that any serious method will have to cope with unstable weights. In other words, the method should not be too sensitive to a change in the weights. Practical experience shows (which was anyway very likely) that a very restricted set of permitted weights is often enough to satisfy the urge for precision expressed by the decision maker.

In conclusion, we have to recognize that, in industrial problems, *many criteria with inconstant weights and many alternatives in a domain with variable frontiers* will constitute the

data from which a decision has to be made. The decision maker would then need methods ranking the alternatives from the best to the worst, in order to choose the best ones inside a limited budget. Many books on industrial decision-making, which do not dare to fight on dangerous battlefields, insists on the analysis of the set of alternatives, and on their ranking along the criteria, without mention of any aggregation method. They claim that a good solution will emerge naturally from a good analysis.

On the contrary, we think that the decision maker does not need the intervention of a decision scientist in the cases where the solution emerges so easily, and the content of this book tends to show that the choice of an effective aggregation model has to be closely dependent on the context. By this we mean the nature of the data as well as the external context of the decision.

This book will help, we hope, the decision scientist to make clearer the domain of application of the just quoted weighted majority method, and will propose, in order to fight the weaknesses of this method, a set of methods with axiomatic justification taking into account the special outline of industrial choices.

2 The Paradoxes

There is very likely no unique method used by minds to make decisions. It is well known that individuals are generally not very logical, and that their decision behavior can be modified by the surrounding culture or by the acquisition of some special skill. In spite of this, it has to be admitted that, given a specific decision, a specific mind will use a specific method.

The decision process will then fail if the data are too heavy for the brain capacities of treatment, if the method is inadequate, or if the problem has no real solution (which can mean that the set of alternatives has been overrestricted).

We explained previously that the aim of models in operations research should be only to improve upon—as it becomes necessary—the natural, but then unsuccessful, method of a real decision maker. Very often, however, unable to explain what his brain does or should do, the decision maker, when interviewed about his multicriterion decision process, will answer by describing a set of axioms that he tries to follow. Of course, it would be ideal if the mathematician had only to listen to a set of axioms, and could derive from them a list of consistent corresponding methods. But the straightforward application of this process works much more poorly than one would like. In what follows, we shall even show how the two more natural ways to do it lead to a complete—or at least very substantial—failure.

Remember that our scope is limited, in terms of decision-making problems, to the cases where criteria and alternatives are finite in number. Each criterion consists of a linear ranking of the alternatives. We denote by X the set of alternatives, by $\Omega = (\theta_1, \ldots, \theta_N)$ the indexed set of criteria, by $E(X)$ a profile $\{\theta_1(X), \ldots, \theta_N(X)\}$. For any Y included in X, $E(Y)$ will denote the sequence $\theta_1(Y), \ldots, \theta_N(Y)$ of the restrictions of the θ_i to only the elements in Y.

2.1 Arrow's Axiomatic System

Since a full-scale discussion of Arrow's theorem is not within the scope of this book, we shall use a restricted but very simple form of the result: the criteria are taken to be total rankings of the alternatives. Additionally, the set of criteria and the set of alternatives are both finite and each contains more than two elements. We suppose that we need a method D able to give, for each profile, one satisfying total ranking of the alternatives deserving the name of multicriterion ranking. We shall call D a multicriterion decision function.

Axiom 2.1 (Unrestricted domain)
The criteria should be unrestricted and D should respect unanimity. Hence, the values that can be taken by the criteria and the decision function will be unrestricted.

Axiom 2.2 (Independence of irrelevant alternatives)
If D denotes the multicriterion decision function, X denotes the set of alternatives, and $\theta_1, \ldots, \theta_N$ the sequence of criteria, then $\forall Y \subset X$, $D[E(X)](Y) = D(E(Y))$.

In other words, the restriction of the multicriterion ranking to only the alternatives in Y is the result one obtains by applying the method to the restriction of the criteria to only the alternatives in Y.

This means that it is of no importance for the decision if you have forgotten in the application of the method some (poorly ranked) alternatives: we know from our first chapter that the complete set of alternatives is always very large and only a relatively small subset can be identified. It is thus essential that the result of the method on a small set of alternatives not vary if forgotten alternatives are taken into consideration.

Axiom 2.3 (Positive responsiveness)
Let us consider a pair of alternatives $\{x,y\}$ such that $D[E(X)]$ restricted to $\{x,y\}$ ranks x before y.

Let A be the set of the criteria that ranked x before y in E.

If E' is another profile where the criteria in A still rank x before y, then the restriction of $D[E'(X)]$ to $\{x,y\}$ should still rank x before y.

The axiom, written in this form, contains, of course, axiom 2.2, but in terms of interpretation, it is interesting to separate axiom 2.3, which only means, in a weak but precise form, that the more the criteria judge that some alternative should outrank some other, the more likely it is to be found in the value of the decision.

Let us now consider two alternatives, say, x, y, in this order and a set G_D^{xy} of criteria such that if the criteria in G_D^{xy}, for a particular profile E, unanimously rank x before y, then $(D(E))(\{x,y\}) = (x,y)$.

In other words, G_D^{xy} is a set of criteria such that, when applying D, their unanimity in ranking x before y assures the fact that x will be before y in the multicriterion decision ranking.

Any such set will be said to be decisive for $\{x,y\}$. Such sets always exist (e.g., the unanimity of criteria is a decisive set for any pair $\{x,y\}$ according to axiom 2.1). What is more, they can be uniquely characterized as follows:

Consider any profile $E(X)$ such that, in $D[E(X)]$, x is before y. The set of criteria that placed x before y in $E(X)$ is a decisive set for $\{x,y\}$. This is a direct application of axiom 2.3.

We can now proceed with the proof of Arrow's result.

Lemma 2.1
Any G_D^{xy} such that $|G_D^{xy}| \geq 2$ contains one smaller decisive set.

Proof Consider a profile E where the criteria in G_D^{xy} are separated into two nonempty sets G' and G''. Then consider a third alternative z and restrict the profile E to the set $\{x,y,z\}$. Denote the complement of a set S by $\mathbf{C}S$. Suppose G' ranks the alternatives z,x,y; G'' ranks the alternatives x,y,z; and, $\mathbf{C}G_D^{xy}$ ranks the alternatives y,z,x. In $D(E\{x,y,z\}))$, x will necessarily be before y, but z can occupy any of the three remaining ranks. However:

1. If $D(E(\{x,y,z\})) = (z,x,y)$, then, as the criteria in G' are the only ones that rank z before y, G' is G_D^{zy}.
2. If $D(E(\{x,y,z\})) = (x,y,z)$, then, as the criteria in G'' are the only ones that rank x before z, G'' is G_D^{xz}.
3. If $D(E(\{x,y,z\})) = (x,z,y)$, then, for the same reasons as in (1) and (2), G' is G_D^{zy} and G'' is G_D^{xz}.

Lemma 2.2
Any G_D^{xy} is G_D^{wz} for any (w,z).

Proof Consider E_1, in which all the criteria in G (which is G_D^{xy}) rank w before x before y and those in $\mathbf{C}G$ rank y before

w before x. $D(E_1)$ places x before y, as G ranked x before y, and w before x by unanimity (axiom 2.1).

Then $D(E_1) = (w,x,y)$, which contains w before y. As only the criteria in G ranked w before y, G is G_D^{wy}.

Consider E_2 in which all the criteria in G (which is now G_D^{wy}) rank w before y before z when those in $\mathbf{C}G$ rank y before z before w. For similar reasons, G is G_D^{wz}. The conclusion now, of course, follows immediately.

From lemma 2.1 applied recursively, there is a decisive set of one criterion, and from lemma 2.2, it is decisive on any pair of alternatives, hence on any ranking. In other words, the decision function is identical to the ranking of this unique criterion.

Theorem 2.1
A multicriterion decision function satisfying axioms 2.1, 2.2, and 2.3 must coincide with exactly one of the criteria.

You wanted to make a real, wise, *multi*criterion decision, and the simplest and most natural axioms drive you toward a *mono*criterion one!

In the political sciences, Arrow's axiomatic system is considered to be necessary for a democracy (in which voters are the criteria), and the paradox is that it leads to dictatorship.

In our context, this result suggests that one particular criterion should overcome the influence of the others. Maybe, if the set of criteria has the good luck to contain one that efficiently combines the others, from among the possible "dictatorships" at least one will be acceptable.

What does all of this mean in practice? Let us look at what probably works and what probably does not in a real, then in an ideal, decision maker's brain.

As we discussed earlier, a human decision maker probably uses different methods at different stages. Some elimination steps are probably undertaken in order to verify that all characteristics of the alternatives meet certain minimal requirements. The remaining ones will be admissible candidates.

The mind will then proceed to rank the candidates by means of more or less conscious methods. For biological reasons, we suppose that no method involving sophisticated global calculations can be used by the brain. As a consequence, a "dictatorship" of one utility criterion will occur in a brain only if this utility is available with almost no calculation.

We have in effect supposed earlier that in the type of decision problems which we solve here, the time needed to construct a valid numerical model for the trade-off ratios would in any case be considerable enough to ensure their obsolescence before use.

If "weights" have been so popular in the multicriterion literature, it is because something like weighting probably occurs often. This "something" can be described either as the approximate computation of a linear function or, in less numerical issues, as the aggregation of pairwise comparisons, obtained from the weighting of a not very constant battery of criteria. In fact, as soon as the brain cannot easily find a unique criterion that clearly synthesizes the others, it will decide to separate the analysis along different criteria (related to different past situations which can introduce many irrelevant items) and will begin to falter.

Why should it be so fuzzy?

1. Because the brain is not able to concentrate its attention on a large number of items that are themselves ranked by a large number of criteria.

2. Because the brain is probably not able to rank with precision all possible alternatives along all possible criteria.

3. Because during the ranking of a long series of alternatives, the set of relevant criteria can be substantially altered: for instance, if the first fifty alternatives have all called for a certain set of criteria and a new criterion becomes pertinent from the fifty-first alternative onward, the brain might well stick to the criteria that have been "efficient" for fifty! And this only because of functional fixity (Duncker [1903]).

It is thus on a very uncertain background that the brain has to build a method of decision. Remember that it cannot do much, because its most sophisticated capabilities of treatment are limited to a local, very small area of memory. This can explain the success of certain "simple" minds that stick to one idea, to one criterion, and can, by this means, always decide in the same coherent direction. This is, of course, rarely the best solution, and it is in fact the multitude of counterexamples that drove operational researchers away from, for example, a maximized profit that would not be balanced by the risk of generating strikes!

2.2 May's Axiomatic System

As a counterpoint to dictatorship, public opinion has, since the Marquis de Condorcet [1785], considered the simple majority rule as a panacea. In this method alternative a will be ranked before alternative b in the decision if and only if a majority of criteria ranks a before b.

Although this method was universally adopted, it was not until 1952 that it became characterized by an axiomatic system making sense from the decision point of view. The detailed discussion of this axiomatic system will be found in May [1952]. Let us recall it here briefly.

The set of possible decisions is limited to two alternatives, x and y. The individuals can vote $+1$ (which means x

before y), -1, or 0 (which means indifference). The decision can be $+1$, -1, or 0.

The proper axioms are

1. The decision rule is well and everywhere defined.

2. The decision rule is symmetric (i.e., neutral with respect to the individuals as well as the alternatives). This condition can be expressed by two subconditions: (a) the decision depends only on the number of votes for $+1$, -1, and 0; and (b) if $f(d_1, \ldots, d_N)$ denotes the value of the decision when the voter i votes d_i, then $f(-d_1, \ldots, -d_N) = -f(d_1, \ldots, d_N)$.

3. The decision has the property of positive responsiveness: if $f(d_1, \ldots, d_N) \geqslant 0$, for all i, $d'_i \geqslant d_i$, and for at least one i_0, $d'_{i_0} > d_{i_0}$, then $f(d'_1, \ldots, d'_N) = 1$.

The fundamental consequence of these axioms is the following.

Theorem 2.2
The decision rule that satisfies May's axiomatic system is unique: it is the simple majority decision rule.

Proof Let us define a profile as an indexed set of individual ballots corresponding to a simple issue. Consider any profile $(d_1, \ldots, d_N)$ where the number of those who voted $+1$ equals the number of those who voted -1:

$$f(d_1, \ldots, d_N) = f(-d_1, \ldots, -d_N) = -f(d_1, \ldots, d_N) = 0.$$

Any additional ballot in favor of $+1$ would shift (cf. axiom 2.3) the decision f to the state $+1$.

Considering the symmetric profile, it is clear that any additional vote in favor of -1 would push the decision to the state -1, and thus the theorem is proved.

So, up to this point, the majority method has the nice outlook we would like for our aggregation method. This

could by itself be a sufficient reason to explain the huge success of the majority method. This success can be equally credited to the excellent opportunities for strategies that the method offers to the shrewd.

2.3 Strategic Majority Voting

The previous part shows that if a vote is taken once and if only two alternatives are compared, the majority method works in a satisfactory way. Grave problems occur for a succession of votes and for more than two alternatives.

2.3.1 The Cake

Let us consider, for instance, a cake to be shared among 100 people. Suppose one of them has some power and political training. His first aim will be to grant undue favors to, say, 50 people against the other 49. With their "cooperation" thus ensured he will form the "Party of the 51." These 51 are going to use majority voting to decide legally to throw 49 people in jail and redistribute their 49 shares among the 51 members of the party.

This shrewd person, if not overcome by a competitor, will again make alliances to form a new Party of the 26, etc., until only two people remain as potential cake-eaters! The majority method has concentrated power and goods into the hands of a few.

2.3.2 The Drunkard, the Miser, and the Health Freak

The majority method can work much more discreetly to the same effect, if the members of an assembly use strategic amendment techniques.

The theorems that can be obtained say roughly that if your assembly is divided enough, if there are enough alter-