Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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Category Theory

Applications to Algebra, Logic and Topology
Proceedings, Gummersbach, 1981

Edited by K.H. Kamps, D. Pumplün, and W. Tholen

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During the last stages of the preparation of this volume the editors learnt of the tragic death of our colleague Graciela Salicrup. Her personality and her work will always be remembered by all of us.

PREFACE

The International Conference on Category Theory - Applications to Algebra, Logic and Topology - was held in Gummersbach, July 6-10, 1981; it was attended by 93 mathematicians from 19 different countries.

Financial support for this conference was provided by a grant of the Deutsche Forschungsgemeinschaft (grant no. 4851/140/80) and by additional means of the Minister für Wissenschaft und Forschung des Landes Nordrhein-Westfalen. The organizers would like to express their sincere thanks for this financial assistance, without which this conference would not have been possible.

The conference had been divided into three sections: General category theory, category theory and logic, and applications of category theory to analysis, topology and computer science. It was very much appreciated by the organizers that John Gray agreed to be chairman of this conference and special thanks are due to him for his essential contribution to its success. The organizers are also very grateful to Horst Herrlich for his help as chairman for the section on applications of category theory to analysis, topology and computer science.

The organizers would like to express their thanks to the Rektor of the Fernuniversität, Prof. Dr. Dr. h.c. O. Peters, for his opening of the conference and for the welcome he extended to the participants on behalf of the Fernuniversität. During the conference and during its preparation essential and effective help was given by the administration of the Fernuniversität, and this help has been gratefully acknowledged by the organizers. Especially Mr. Blümel from the university administration should be mentioned for his engagement for this conference.

Thanks are due to the Fachbereich Mathematik und Informatik of the Fernuniversität for supporting this conference in every respect. Many colleagues advised and assisted us during the conference and its preparation. We would like especially to thank the secretaries Mrs. I. Müller and Mrs. K. Topp for their most efficient work.

Last, but by no means least, we would like to express our thanks to Dr. G. Greve, Dr. W. Sydow, Dr. D. Brümmer, Dr. B. Hoffmann and Dr. T. Müller, all members of the Fachbereich Mathematik und Informatik of the Fernuniversität for their engagement. It is due to their efforts that there were no organizational difficulties during the conference and they did their best to make the participants of the conference feel at ease.

This volume of Springer Lecture Notes constitutes the proceedings of this conference. We would like to thank the editors of the Springer Lecture Notes in Mathematics for accepting the proceedings for this series. All contributions to this volume have been refereed and our sincere thanks go to all the referees for their work.

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CONTENTS

A Note on the Homology of Regular Nearness Spaces

H. L. Bentley

Abstract: It is shown that the homology and cohomology groups of a regular nearness space can be defined by means of a variation on the Čech method, which uses nerves of uniform covers: the variation involves associating with each uniform cover, not the nerve, but a complex, called the vein, defined by means of nearness.

In a recent paper, the author showed that the Čech homology and cohomology groups (= Vietoris homology and Alexander cohomology groups) of merotopic and nearness spaces satisfy, in a variant form, all the axioms of Eilenberg-Steenrod. For definitions of these groups and for historical information, the reader is referred to that paper [1]. We are interested here in regular nearness spaces (for the definition, see Herrlich [5]) and in the possibility of using what is, formally, a different definition of the homology and cohomology groups, but a definition which we prove gives rise to the usual Čech groups.

By a pair (X, Y) of nearness spaces we mean a nearness space X together with a nearness subspace Y of X. A uniform cover of (X, Y) is a pair $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ where $\mathcal{A}_1$ is a uniform cover of X, $\mathcal{A}_2 \subset \mathcal{A}_1$, and $\mathcal{A}_2 \cup \{X - Y\}$ is a uniform cover of Y.

The <u>nerve</u> $K(\mathcal{A})$ of a uniform cover $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ of (X, Y) is a pair of simplicial complexes $K(\mathcal{A}) = (K_1(\mathcal{A}), K_2(\mathcal{A}))$. The vertices of $K_1(\mathcal{A})$ are the elements of $\mathcal{A}_1$; a simplex of $K_1(\mathcal{A})$ is a finite subset $\mathcal{G}$ of $\mathcal{A}_1$ such that $\cap \mathcal{G} \neq \phi$. The vertices of $K_2(\mathcal{A})$ are the elements of $\mathcal{A}_2$; a simplex of $K_2(\mathcal{A})$ is a finite subset $\mathcal{G}$ of $\mathcal{A}_2$ such that $Y \cap \cap \mathcal{G} \neq \phi$.

Recall that a collection $\mathcal{G}$ of subsets of a nearness space X is said to be <u>near</u> in X if for each uniform cover $\mathcal{C}$ of X there exists $C \in \mathcal{C}$ such that for all $G \in \mathcal{G}$, $C \cap G \neq \phi$. Recall also that if Y is a nearness subspace of X then a collection $\mathcal{G}$ of subsets of Y is near in Y if and only if $\mathcal{G}$ is near in X.

Now we are ready to make our main definition; it is a variation on the definition of the nerve.

The <u>vein</u> $J(\mathcal{A})$ of a uniform cover $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ of (X, Y) is a pair of simplicial complexes $J(\mathcal{A}) = (J_1(\mathcal{A}), J_2(\mathcal{A}))$. The vertices of $J_1(\mathcal{A})$ are the elements of $\mathcal{A}_1$; a simplex of $J_1(\mathcal{A})$ is a finite subset $\mathcal{G}$ of $\mathcal{A}_1$ such that $\mathcal{G}$ is near in X. The vertices of $J_2(\mathcal{A})$ are the elements of $\mathcal{A}_2$; a simplex of $J_2(\mathcal{A})$ is a finite subset $\mathcal{G}$ of $\mathcal{A}_2$ such that $\mathcal{G} \wedge \{Y\}$ is near in Y.

If $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ and $\mathcal{B} = (\mathcal{B}_1, \mathcal{B}_2)$ are uniform covers of the pair (X, Y) of nearness spaces then we say that $\mathcal{A}$ is a <u>refinement</u> of $\mathcal{B}$ if $\mathcal{A}_1$ is a refinement of $\mathcal{B}_1$ and $\mathcal{A}_2$ is a refinement of $\mathcal{B}_2$. Under this relation of refinement, the set of all uniform covers of a pair of nearness spaces becomes a directed set.

Thus, there is a spectrum of complexes

$$K(\mathcal{A}) \longrightarrow K(\mathcal{B})$$

and of complexes

$$J(\mathcal{A}) \longrightarrow J(\mathcal{B})$$

for $\mathcal{A}$ a refinement of $\mathcal{B}$. From these spectra there arise two spectra of homology groups and two of cohomology groups.

From now on, let G be a fixed abelian group. G will be the coefficient group of our homology and cohomology theories but explicit denotation of G will be suppressed.

The direct spectrum of cohomology groups

$$\alpha_{\mathcal{B}}^{\mathcal{A}} : H^n(K(\mathcal{B})) \longrightarrow H^n(K(\mathcal{A}))$$

has for its limit group the <u>n-dimensional Čech cohomology group</u> of (X, Y) which we will denote by $\check{H}^n(X, Y)$. The inverse spectrum of homology groups

$$\beta_{\mathcal{B}}^{\mathcal{A}} : H_n(K(\mathcal{A})) \longrightarrow H_n(K(\mathcal{B}))$$

has for its limit group the <u>n-dimensional Čech homology group</u> of (X, Y) which we will denote by $\check{H}_n(X, Y)$.

The direct spectrum of cohomology groups

$$\gamma_{\mathcal{B}}^{\mathcal{A}} : H^n(J(\mathcal{B})) \longrightarrow H^n(J(\mathcal{A}))$$

has for its limit group the <u>n-dimensional vascular cohomology group</u> of (X, Y) which we will denote by $\dot{H}^n(X, Y)$. The inverse spectrum of homology groups