Nonrelativistic Quantum Mechanics

Anton Z. Capri

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To Kim, Karin, Irene and Skaidrite

for giving me time and love.

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Editor's Foreward

Everyone concerned with the teaching of physics at the advanced undergraduate or graduate level is aware of the continuing need for a modernization and reorganization of the basic course material. Despite the existence today of many good textbooks in these areas, there is always an appreciable time-lag in the incorporation of new viewpoints and techniques which result from the most recent developments in physics research. Typically these changes in concepts and material take place first in the personal lecture notes of some of those who teach graduate courses. Eventually, printed notes may appear, and some fraction of such notes evolve into textbooks or monographs. But much of this fresh material remains available only to a very limited audience, to the detriment of all. This series aims at filling this gap in the literature of physics by presenting occasional volumes with a contemporary approach to the classical topics of physics at the advanced undergraduate and graduate level. Clarity and soundness of treatment will, we hope, mark these volumes, as well as the freshness of the approach.

Another area in which the series hopes to make a contribution is by presenting useful supplementing material of well-defined scope. This

may take the form of a survey of relevant mathematical principles, or a collection of reprints of basic papers in a field. Here the aim is to provide the instructor with added flexibility through the use of supplements at relatively low cost.

The scope of both the lecture notes and supplements is somewhat different from the "Frontiers in Physics" series. In spite of wide variations from institution to institution as to what comprises the basic graduate course program, there is a widely accepted group of "bread and butter" courses that deal with the classic topics in physics. These include: Mathematical methods of physics, electromagnetic theory, advanced dynamics, quantum mechanics, statistical mechanics, and frequently nuclear physics and/or solid-state physics. It is chiefly these areas that will be covered by the present series. The listing is perhaps best described as including all advanced undergraduate and graduate courses which are at a level below seminar courses dealing entirely with current research topics.

Finally, because the series represents a continuing experiment on the part of the editors and the publisher, suggestions from interested readers as to format, contributors, and contributions will be most welcome.

DAVID PINES

Preface

Most textbooks start as a set of lecture notes which expand and undergo numerous revisions over the years. The present version is no exception and is the culmination of many revisions of such lecture notes. I have taught quantum mechanics at several different levels and thus attempted to make this book of sufficiently broad scope for different courses. The book includes material for undergraduate courses as well as a one-semester graduate course. As a guide for possible division of the material in this book I have included a few pages in the following section entitled "How to use this Book".

It is a long way from lecture notes to a text-book and on this journey I have been greatly helped by many people. My first debt is to my students who forced me to clarify and expand my lecture notes. Without their prodding this book would never have been started.

Dr. M. Razavy has also been most generous with his time and ideas. Not only did he test this book in his quantum mechanics classes, but he willingly proof-read all the present version. He also helped with many pertinent references and comments; his criticism of chapter 21 was particularly useful. Without his constant encouragement and support my enthusiasm would have flagged many times and this book might never have been written.

Preface

During the early stages, Dr. W. Brouwer also helped considerably with his detailed criticisms of the first fourteen chapters. I thank him for his generous contribution of time and ideas.

The first version of this book was expertly typed by Mrs. M. Yiu. Had she not been here to read my scrawl, I am sure this book would never been written. I thank her for her patience and tolerance.

The diagrams are due to the heroic efforts of Mrs. J. Hube. Her attention to detail turned some rather skimpy sketches into actual diagrams. I am most grateful for her skills.

Finally, I acknowledge with many thanks the expertly accomplished job of preparing the final manuscript. This was ably handled by Christine Fischer and Laura Heiland. Their friendly attitude did much to make a painful job easier.

I would also like to thank all of my colleagues who, in one way or another, influenced me during our coffee-room discussions. Their contributions, although less tangible, are nevertheless very real.

It is my hope that in writing this book I may have helped a few students to discover the beauty of quantum mechanics.

Anton Z. Capri

Edmonton, Alberta
April, 1985

How to Use This Book

This book is designed for use in any of the following programs:

A) A one-semester introductory course for undergraduates;

B) A two-semester introductory course for undergraduates;

C) A one-semester graduate courses.

To indicate which material is suitable for inclusion in any of the above programs I have added the appropriate letter or letters "A,B,C" after the title in the Table of Contents. Thus, for example, all section headings for Chapter 1, except 1.10 and 1.11, are followed by the letters "AB" to indicate that this material is suitable for both a one-semester introductory course as well as a two-semester course. Sections 1.10 and 1.11 are labelled only B and should be omitted in a one-semester undergraduate course.

The material to be included will also vary from class to class. Thus it is not feasible, nor should it be attempted, to include all the sections marked with a "B" in a two-semester undergraduate course. For example, I have found that I could include either the Rayleight-Ritz variational approach or a somewhat watered-down version of the WKB approximation in a two-semester undergraduate course, but not both.

How to Use This Book

Similarly in a one-semester graduate course a selection from the material marked "C" is necessary. This will, of course, depend on the special interests of the students. The point is that not all the material in any given course division (A,B or C) can be covered in the time normally allotted for such a course. The interests of classes vary from year to year and so I have included extra topics from which a selection can be made.

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Chapter 1

The Breakdown of Classical Mechanics

During the nineteenth century many of the great advances in physics of the eighteenth century were consolidated and extended. In addition, the theory of electromagnetism was completed by J.C. Maxwell. Except for a few unexplained effects or anomalies there seemed little more in terms of fundamental physics to be done by the beginning of the twentieth century. Yet it is precisely in the year 1900 that quantum theory starts with Planck's formula for blackbody radiation.

Soon there were a host of experimental results, both new ones and earlier ones that again attracted attention. All of these pointed to flaws in the physics of the nineteenth century. In almost all cases these anomalies resulted when Newtonian mechanics and electromagnetism were simultaneously involved. In trying to elucidate these various experimental facts a new theory of physics, quantum theory, was born.

In the next few sections we briefly examine several of these experiments and discuss them with some modern hindsight. First we consider blackbody radiation from the prequantum or classical point of view. We then turn to a consideration of the stability of the classical Rutherford atom. Although Rutherford had experimentally demonstrated the planetlike structure of the electrons in atoms, his model caused a lot of theoretical problems.