

MICROMECHANICAL STRESSES IN  
MONOLITHIC CERAMICS AND  
CERAMIC COMPOSITE MATERIALS

LI, ZHUANG

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MICROMECHANICAL STRESSES IN MONOLITHIC CERAMICS  
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by

ZHUANG LI

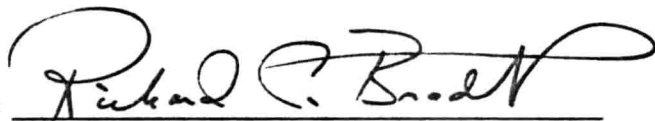
A dissertation submitted in partial fulfillment  
of the requirements for the degree of

Doctor of Philosophy

University of Washington

1988

Approved by



(Chairperson of Supervisory Committee)

Program Authorized  
to Offer Degree Materials Science and Engineering

Date

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University of Washington

Abstract

MICROMECHANICAL STRESSES IN MONOLITHIC CERAMICS  
AND CERAMIC COMPOSITE MATERIALS

by Zhuang Li

Chairperson of  
the Supervisory Committee:

Professor Richard C. Bradt  
Department of Materials  
Science and Engineering

Thermoelastic properties of the cubic and hexagonal polytypes of SiC have been determined. The thermal expansion,  $\alpha$ , is highly temperature dependent, but only slightly dependent on the crystallographic orientation, while the elastic properties are highly dependent on crystal orientation, but not very temperature dependent. However, the thermoelastic stress index,  $\alpha E$ , is highly dependent both on temperature and the crystallographic orientation.

By developing a modification of the Eshelby method and applying it in matrix form, the micromechanical stresses and strain energy density within polycrystalline ceramics and ceramic matrix composites have been determined. For cubic polycrystalline ceramics the micromechanical stress concentration and the strain energy density are related to the elastic anisotropy of individual large grains within a fine grain size microstructure. Those large grains are



ideal sites for fracture initiation, directly influencing the strength of cubic polycrystalline ceramics.

Micromechanical stresses and strain energy densities of non-cubic polycrystalline ceramics are not only dependent on the elastic anisotropy, but also on the thermal expansion anisotropy. Even without external loading, internal stresses are generated during temperature changes and may result in spontaneous microcracking. Besides the thermoelastic anisotropy, the geometry of the crystals or grains is also important and influences the micromechanical stresses within polycrystalline ceramics.

Micromechanical stresses in SiC reinforced ceramic matrix composites are often in the GPa range and may be expected to influence the strength, fracture toughness and the R-curve behavior. Four microstructural factors affect these composite micromechanical stresses: (i) the thermal expansion mismatch,  $\Delta\alpha$ , which primarily influences the magnitude and the distribution of the internal stresses, (ii) the elastic modulus difference,  $\Delta E$ , which strongly affects the distribution of any externally applied loads, (iii) the geometric shape of the reinforcing phase,  $(L/d)$ , which can either amplify or reduce the thermal and elastic effects on the internal stresses, and (iv) the volume fraction of the reinforcing phase,  $V_f$ , which directly affects the magnitudes of the stresses. To design an optimal composite, those four factors must be considered.

## ACKNOWLEDGMENTS

The author wishes to express his gratitude and sincere appreciation to Professor Richard C. Bradt for his guidance and constant encouragement throughout the course of this study. Special thanks also go to Professor M. Taya for his valuable advice and helpful discussions. In addition he would like to thank Professors W. D. Scott and R. G. Stang for serving on the reading committee and providing their useful comments.

Appreciation is also expressed to his fellow graduate students, including Asish Ghosh, Masaki Kaji, Henry Larmie, Sue Laurich-McIntyre, Hong Li and Lana Wong for their friendly support and enjoyable discussions throughout the study.

Finally the author would like to express his sincere gratitude to his parents and his sister for their lovely support and most importantly, to his wife, Grace, for her boundless love and deep understanding during the completion of this study.

The financial support for this study by NASA Grant No. NAGW-199, is also gratefully acknowledged.

**To:**

**My Parents and My Wife**

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