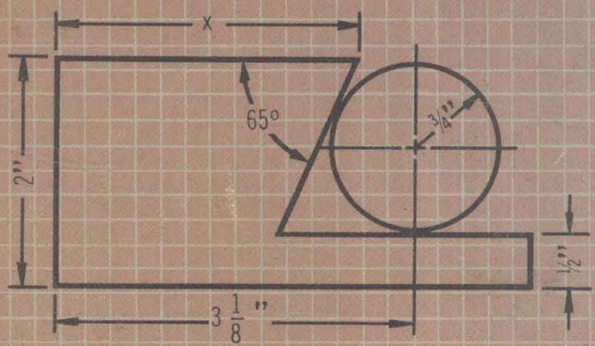
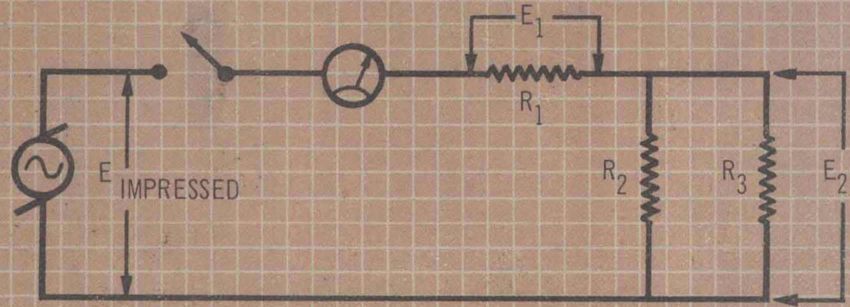
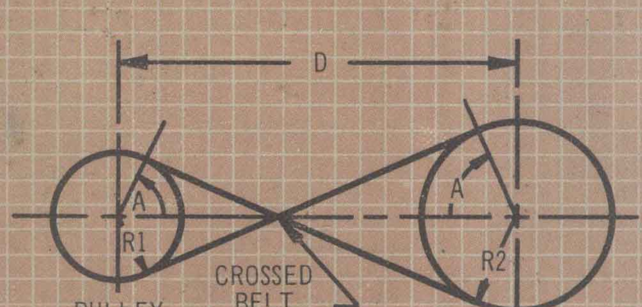
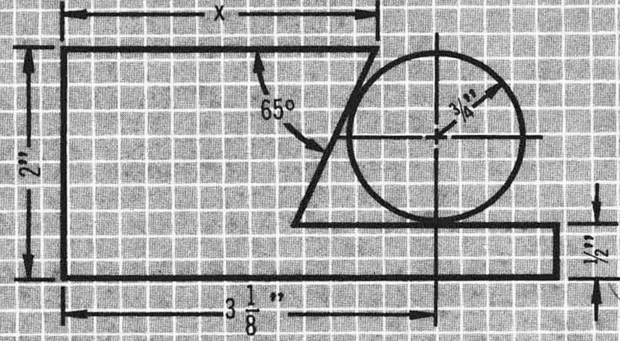


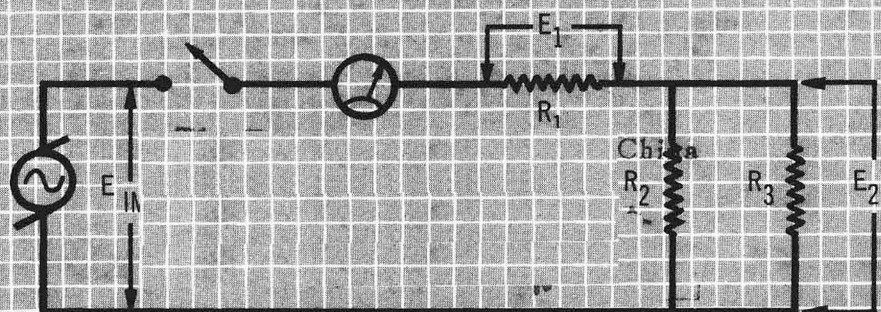


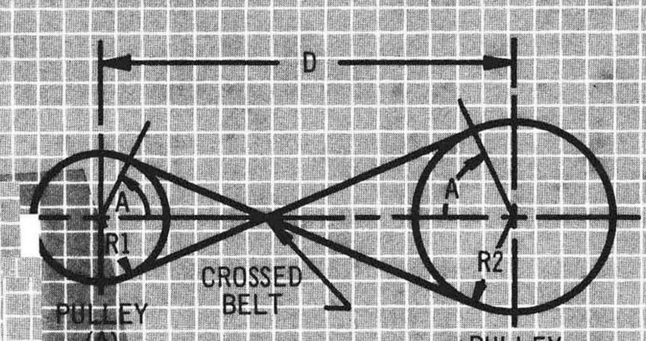


TECHNICAL TRIGONOMETRY

Price and Stillwell







TECHNICAL TRIGONOMETRY

Price and Stillwell

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Preface

The burgeoning growth of instructional programs of a technical nature has brought with it instructional problems of suitable curriculum materials. For those students who are pursuing the two-year technical program, it has been found that instructional material designed for four-year college engineering programs is not suitable; nor, on the other hand, is conventional academic or vocational material. Instead, those who are teaching such students have found that while great emphasis must be placed on fundamental skills and concepts, there must also be emphasis on the application of these fundamentals. Thus, both theory and practical application are joined and the terminal objectives of technical training are satisfied.

As a result of much experience in teaching two-year technical students, and after much experimentation, trial and error, this new approach to the presentation of trigonometry has evolved.

TECHNICAL TRIGONOMETRY is intended to cover the specific portions of the subject of trigonometry which pertain directly to practical applications. Thus the content is directed toward solving problems normally encountered in trade and industrial applications. It should be pointed out that in this respect, and in the manner in which the content is cataloged, this text differs from other, more conventional trigonometry texts. It is this terminal approach which allows the principal concepts of the subject to be explored from the practical view.

It should be recognized that the study of trigonometry will require a working knowledge of algebra and geometry. As an introduction to the presentation of the trigonometry content in this text, a brief review of algebra and geometry is given. This review section merely points up some of the more pertinent content in algebra and geometry that is applicable to the study of trigonometry. It is not intended as sufficient coverage of such subjects.

A good foundation in mathematics cannot be laid by mere mastery of technique. The uses to which this text will be put will be varied, and it would be impossible to illustrate all such uses by specific examples. Nevertheless, the more immediate applications of trigonometry to problems such as are encountered in electrical and electronic work, tool design, hydraulics and pneumatics, are dealt with in specific instructional units.

Besides the step-by-step development of the necessary principles required for an understanding of basic techniques, and in addition to the varied applications mentioned above, the text provides drill material for mastery of basic skills. Thus, the presentation follows a logical sequence: (1) presentation of the principle, (2) drill for mastery of the basic skill, and (3) application of this skill to practical problems.

September 15, 1963
Albany, New York

William G. Dickson,
Editor

To the Instructor

From the vocational approach to the teaching of trigonometry, the authors feel some explanation as to the past history of industrial mathematics is in order. After many years of searching for a textbook that would present the vocational aspects of the subject, little success was realized. The level of presentation of the contents in many of the texts reviewed was above the level of the practical concept with little or no practical application. Almost all trigonometry textbooks are reasonably standard in that the theory is established. Therefore, that which remains to be done in preparing a vocational-type trigonometry textbook is to govern the presentation of the already standardized material.

The unusual format of this text is the result of considerable teaching and experimentation with classes at the technical level. The subject matter is presented in the sequence which has been found desirable not only for the instructor, but for the student. Since each local situation varies, however, adaptations to meet local requirements are easily made because of the careful subject matter breakdown. The instructor should feel at liberty to amplify the given material, or to omit those areas not applicable to his specific needs.

Since the student enrolled in a technical program is mainly concerned with learning a vocation, he should not be burdened with extremely difficult derivations: in this presentation, only the rules of basic concepts are derived, and these derivations are explained in detail. He should, however, be advised of the subject matter directly related to his field of interest.

Dennis H. Price

ACKNOWLEDGMENTS

I wish to take this opportunity to express my gratitude to all those persons who have so graciously contributed toward the completed manuscript. My many thanks to Mr. C. J. Laible and Mr. F. M. Moore, associates at Avco Corporation, Electronics Division. Mr. Laible and Mr. Moore reviewed the original manuscript and offered comments and suggestions too numerous to mention. And to my associates at Avco Corporation, Electronics Division, Cincinnati, Ohio, I express my sincere thanks for their many comments. In particular, Mr. J. H. Mason, Mr. C. E. Weber, Mr. R. H. Knese, and Mr. R. W. Desserich deserve much credit for their personal contributions.

Mr. Jack Cahall, Dean of the Evening College of the Ohio Mechanics Institute, Ohio College of Applied Science, permitted us the opportunity of applying this work in direct classroom practice. I, therefore, thank Mr. Cahall for his consideration and understanding. The late Mr. John Johnson, Director of Department of Mathematics, also reviewed the original manuscript and was extremely helpful in his guidance.

I want to thank Mrs. Harry Stillwell, Sr. for assisting in the preparation of the final manuscript. I would like to point out that my wife, Elaine, was the motivating force behind the entire program. She reviewed, edited, and helped type the final manuscript. This work is, therefore, dedicated to her.

H. R. Stillwell

INTRODUCTION

A SUMMARY OF MATHEMATICS HISTORY

Early mathematicians experienced many difficulties arising from a lack of paper, pencils, chalk, erasers, standardized notation, and an adequate number system. The abacus, a simple but effective computer consisting of rows of beads suspended on strings in a wood frame, solved many of the early problems. Not until Hindu symbols came into use did the processes of arithmetic become simple enough for them to be within the grasp of the nonprofessional. But it was only after the adoption of our present system of numerical notation and standard symbols, that mathematics became a truly universal language. In our study of mathematics we are indeed fortunate that we have the necessary tools with which to bring about solutions to problems which only a few short years ago were impossible to solve. In view of this, Table I and Table II are presented and give the common symbols used in much of today's mathematics.

SYMBOL	IDENTITY	SYMBOL	IDENTITY
X OR ·	MULTIPLIED BY	$>>$	IS MUCH GREATER THAN
÷ OR :	DIVIDED BY	$<<$	IS MUCH LESS THAN
$+$	POSITIVE, PLUS OR ADD	$\geq$	GREATER THAN OR EQUAL TO
$-$	NEGATIVE, MINUS OR SUBTRACT	$\leq$	LESS THAN OR EQUAL TO
$\pm$	POSITIVE OR NEGATIVE	$\therefore$	THEREFORE
$\mp$	NEGATIVE OR POSITIVE	$\angle$	ANGLE
= OR : :	EQUALS	Δ	INCREMENT OR DECREMENT
$\equiv$	IDENTITY		
$\cong$	IS APPROXIMATELY EQUAL TO OR IS CONGRUENT TO	$\perp$	PERPENDICULAR TO
$\neq$	DOES NOT EQUAL	$\parallel$	PARALLEL TO
$>$	IS GREATER THAN	$\lvert n \rvert$	ABSOLUTE VALUE OF n.

UPPER CASE	LOWER CASE	IDENTITY	UPPER CASE	LOWER CASE	IDENTITY
A	α	ALPHA	N	ν	NU
B	β	BETA	Ξ	ξ	XI
Γ	γ	GAMMA	O	o	OMICRON
Δ	δ	DELTA	Π	π	PI
E	ϵ	EPSILON	P	ρ	RHO
Z	ζ	ZETA	Σ	σ	SIGMA
H	η	ETA	T	τ	TAU
Θ	θ	THETE	Υ	υ	UPSILON
I	ι	IOTA	Φ	φ	PHI
K	κ	KAPPA	X	χ	CHI
Λ	λ	LAMBDA	Ψ	ψ	PSI
M	μ	MU	Ω	ω	OMEGA

The following review material is presented for the student's information. This material is basic and should be referred to often. Although there are no problems submitted with these basics, the student should apply the principles with little or no difficulty.

ALGEBRA

A. Ratio and Proportion

(1) If $a:b = c:d$ or if a is to b as c is to d, or, as it is more commonly written,

$$\frac{a}{b} = \frac{c}{d}$$

then $ad = bc$, and, in addition to the proportion already stated, we can write these other two proportions,

$$\frac{a}{c} = \frac{b}{d} \text{ and } \frac{b}{a} = \frac{d}{c}$$

(2) If the two middle terms are the same, $\frac{a}{b} = \frac{b}{d}$, then $ad = b^2$ and b is called a mean proportional between a and b.

B. Monomials

A monomial is a single term together with its preceding plus (+) or minus (-) sign indicating whether the monomial is positive or negative.

$$(+5,\ -7x^2y^3,\ 8ab^2c)$$

C. Binomials

A binomial is an algebraic expression of two terms.

$$(9 - 3y);\ (4x + 3y);\ (3a^2b + 4c^3d^2)$$

D. Trinomials

A trinomial is an algebraic expression of three terms.

$$(5x - 7xy + 9y)$$

E. Polynomials

A polynomial is an algebraic expression of two or more terms. Thus, binomials and trinomials may also be defined as polynomials.

F. Exponent

An exponent is a small figure or letter written to the right and slightly above a quantity to indicate how many times the quantity is to be used as a factor.

(7^3) means $7\cdot7\cdot7$; $(a + b)^3$ means $(a + b)(a + b)(a + b)$

G. Power

A power of a number is the result obtained by multiplying that number by itself a definite number of times.

8 is the third power of 2; $2^3 = 2 \cdot 2 \cdot 2 = 8$ $\qquad 2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$

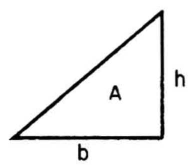
In the study of trigonometry there will be many opportunities to use formulas and theorems which have been developed in earlier courses. A few of these are outlined below for reference.

GEOMETRY

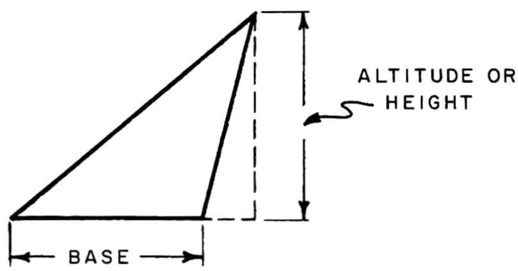
A. Triangle

(1) The area of a right triangle is equal to one-half the product of the base and height.

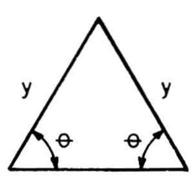
$$A = \frac{1}{2} bh$$



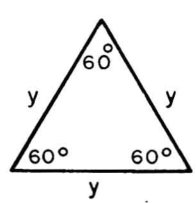
Any one of the sides of a triangle may be considered the base. Note, too, that an altitude may fall outside the triangle.



(2) Two sides, and hence two angles, of an isosceles triangle are equal.



(3) An equilateral triangle has three equal sides, hence each angle contains 60°.



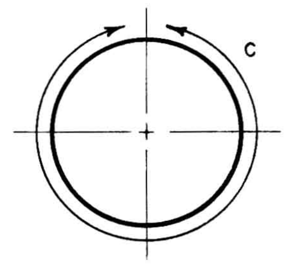
4. The sum of the three angles of any plane triangle must equal 180°.

$$\angle A + \angle B + \angle C = 180°$$

B. Circle

(1) The circumference of a circle is equal to 2π times the radius, or π times the diameter. (π is equal to 3.14159)

$$C = \pi d, \text{ or } C = 2\pi r$$

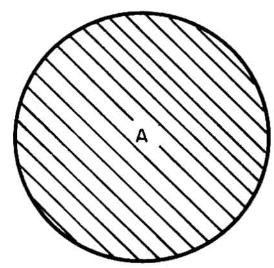


(2) The area of a circle is equal to π times the square of the radius.

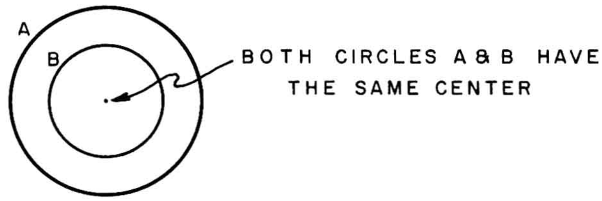
$$A = \pi r^2$$

Also, since the radius of a circle is $\frac{1}{2}$ the diameter D or $r = \frac{D}{2}$, the area can be expressed as:

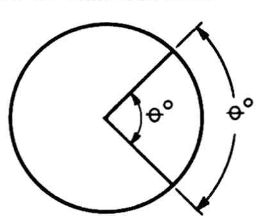
$$A = \pi \left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$$



(3) Two or more circles are said to be concentric when they have the same center.



(4) Any angle measured at the center of a circle intercepts an arc of the same angle on the circumference of the circle.



C. Function Notation

One of the most important fundamental mathematical concepts is the function notation. A simple explanation of function in terms of our everyday surroundings will serve to simplify the concept. One function of our police department is traffic control; another function is protection of our homes and property.

If we were to express this relationship in mathematical terms we could let x stand for the police department, y stand for traffic control, and z for protection.

Therefore, a function of x is y and another function of x is z. In mathematical symbols we can write $y = f(x)$, $z = f(x)$, or $y,z = f(x)$. The notation of $f(x)$ does not mean f multiplied by x, but that f is simply an operational symbol or operator.

If we were to let x stand for another agency of our government, the functions y and z would stand for entirely different things, such as tax collection or waste disposal. Therefore, the interpretation of y and z depend upon the selection of x. Since y and z depend upon x, we define x as the independent variable and y and z or both as the dependent variable or variables.

The same reasoning as above applies to mathematical formulas. For example,

$$y = x, \text{ if } f(x) = 3 \quad \text{then } y = f(3) \text{ or } y = 3$$

$$y - 3 = x, \text{ if } f(x) = 4 \quad \text{then } y - 3 = 4 \text{ or } y = 7$$

$$y = x - 2q, \text{ if } f(x) = (q^2 + 1) \quad \text{then } y = q^2 - 2q + 1 = (q - 1)(q - 1) = (q - 1)^2$$

$$y = 6^x, \text{ if } f(x) = 2 \quad \text{then } y^2 = 36$$

We can now more precisely define a function as the medium between two sets of numbers, such that when one number from the first set is given, the number from the second set can be determined. Therefore, y is called a function of x if, whenever x is known, y can be found.

Examples:
1. $y = 6x + 3$
2. $y = x^2 + 3x^3 - 4x + 4$
3. $y = 6^x$
4. $y = \dfrac{x^3 + 4x^2 - 5}{x - 5}$

As shown above, in each case, if x is known, y can be computed. This then allows us to call x the independent variable and y, the dependent variable. Should we wish to speak of an unknown function, or functions in general, the accepted symbol is:

$$y = f(x)$$

and reads y is equal to f of x, or y equals the function of x. Further, it does not imply that x is multiplied by f but merely an operator that produces from each quantity for x, a quantity for y.

Examples: $y = 6x + 3$ from above and $x = 3$ $\quad f(x) = 6 \cdot 3 + 3 = 21$

Contents

APPENDIX

INDEX

Unit I

RADIAN MEASURE

There are two basic methods for determining the dimensions of a circle, namely, physical or actual measurement and computation or mathematical solution.

1. ____

CIRCUMFERENCE OF A CIRCLE

We know that the distance around the perimeter of a circle is the circumference, and that this dimension can be computed as well as actually measured.

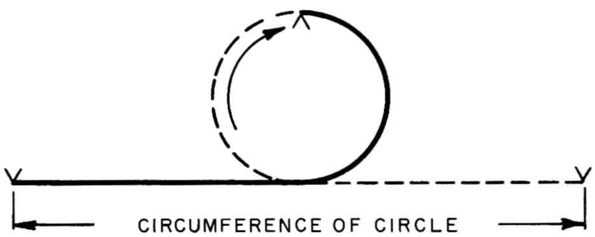


FIG. I—I

PHYSICAL MEASUREMENT

The physical measurement method is very simple for circles or wheels of small size or those not too heavy to handle; however, in those cases where we are dealing with circles or wheels which do not lend themselves to physical measurement, we must compute this quantity. The formula used is:

$$\text{Circumference} = 2\pi\text{Radius} = 360^\circ$$

Notice that a circle is actually a generated figure. This basic fact is needed for the balance of this Unit. Above we state that: $C = 2\pi R = 360^\circ$. The question now arises: What does 360 degrees actually mean?

As an aid in answering this question, let us generate a circle. We can do this by a simple process using a thumbtack, a piece of string and a pencil. Tying the string between the thumbtack and the pencil, push the thumbtack into a flat board, and pull the string tight while holding the pencil vertical to the board.

(R) STRING

(O) THUMB TACK

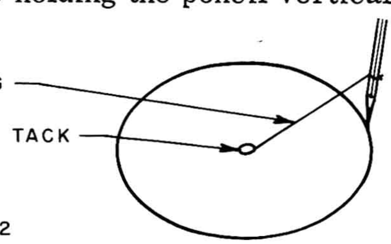


FIG. I—2

Now rotate the pencil all the way around the thumbtack and back to the original starting point, scribing a neat circle. Let us call the string "R" or "Radius" and the thumbtack "O" or the "Origin".

It is now feasible to consider the measurement of the angle. If we rotate the pencil again, but this time through only a short distance, mark the finish point, and connect this point with the center of the circle, we have generated an "<u>arc</u>".

A preferred method of describing this is to visualize a rigid form of radius such as a pole or stick, fastened so that it may be rotated about the origin, free to stop at any desired point.

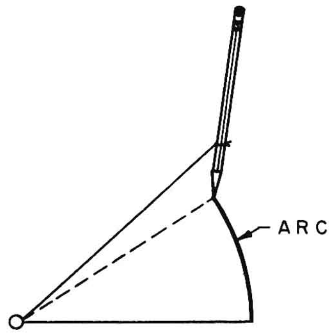


FIG. 1—3

> The increments of rotation of the radius from a starting point are most commonly referred to as DEGREES. (1)

Any segment of the circumference expressed in degrees may be divided into smaller segments by ordinary division. A degree is 1/360 of the circumference of a circle or an angle at the center of a circle that intercepts 1/360 of the circumference.

$$C = 2\pi R = 360^\circ$$

1 Degree (°) is 1/360 of a circumference
1 Minute (′) is 1/60 of a degree or $60' = 1^\circ$
1 Second (″) is 1/60 of a minute or $60'' = 1'$

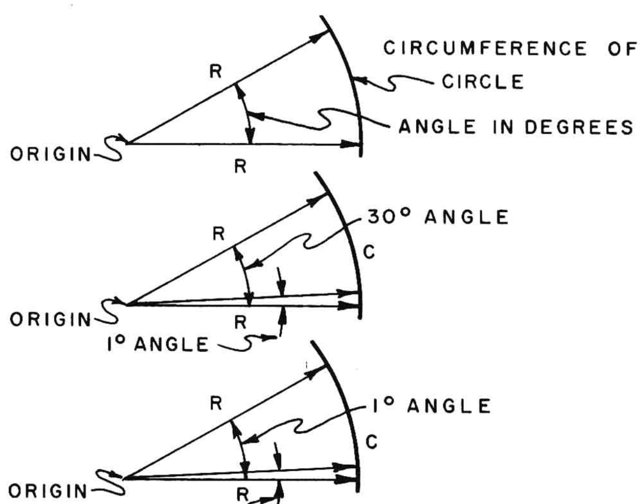


FIG. 1—4

(1) There is no mathematical reason for the use of the degree, minute, or second. It is conjecture that the Babylonians used 360 as the base of their number system, which, in turn, was based on their year having 360 days.