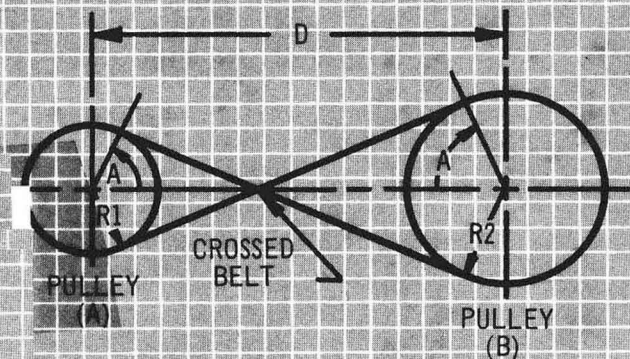
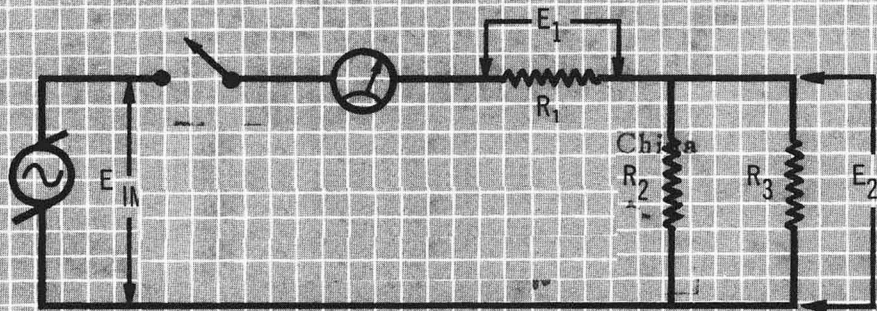
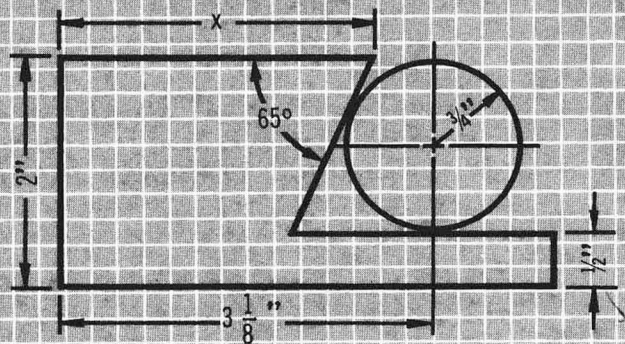


TECHNICAL TRIGONOMETRY

Price and Stillwell



TECHNICAL TRIGONOMETRY

Price and Stillwell

Copyright 1963
DELMAR PUBLISHERS, INC.

No part of this book may be reproduced in any form without written permission from the publishers. However, permission is granted to reviewers who may quote brief passages for a printed review.

Library of Congress Catalog Card Number:

62-21584

PRINTED IN THE UNITED STATES OF AMERICA

Preface

The burgeoning growth of instructional programs of a technical nature has brought with it instructional problems of suitable curriculum materials. For those students who are pursuing the two-year technical program, it has been found that instructional material designed for four-year college engineering programs is not suitable; nor, on the other hand, is conventional academic or vocational material. Instead, those who are teaching such students have found that while great emphasis must be placed on fundamental skills and concepts, there must also be emphasis on the application of these fundamentals. Thus, both theory and practical application are joined and the terminal objectives of technical training are satisfied.

As a result of much experience in teaching two-year technical students, and after much experimentation, trial and error, this new approach to the presentation of trigonometry has evolved.

TECHNICAL TRIGONOMETRY is intended to cover the specific portions of the subject of trigonometry which pertain directly to practical applications. Thus the content is directed toward solving problems normally encountered in trade and industrial applications. It should be pointed out that in this respect, and in the manner in which the content is cataloged, this text differs from other, more conventional trigonometry texts. It is this terminal approach which allows the principal concepts of the subject to be explored from the practical view.

It should be recognized that the study of trigonometry will require a working knowledge of algebra and geometry. As an introduction to the presentation of the trigonometry content in this text, a brief review of algebra and geometry is given. This review section merely points up some of the more pertinent content in algebra and geometry that is applicable to the study of trigonometry. It is not intended as sufficient coverage of such subjects.

A good foundation in mathematics cannot be laid by mere mastery of technique. The uses to which this text will be put will be varied, and it would be impossible to illustrate all such uses by specific examples. Nevertheless, the more immediate applications of trigonometry to problems such as are encountered in electrical and electronic work, tool design, hydraulics and pneumatics, are dealt with in specific instructional units.

Besides the step-by-step development of the necessary principles required for an understanding of basic techniques, and in addition to the varied applications mentioned above, the text provides drill material for mastery of basic skills. Thus, the presentation follows a logical sequence: (1) presentation of the principle, (2) drill for mastery of the basic skill, and (3) application of this skill to practical problems.

September 15, 1963
Albany, New York

William G. Dickson,
Editor

To the Instructor

From the vocational approach to the teaching of trigonometry, the authors feel some explanation as to the past history of industrial mathematics is in order. After many years of searching for a textbook that would present the vocational aspects of the subject, little success was realized. The level of presentation of the contents in many of the texts reviewed was above the level of the practical concept with little or no practical application. Almost all trigonometry textbooks are reasonably standard in that the theory is established. Therefore, that which remains to be done in preparing a vocational-type trigonometry textbook is to govern the presentation of the already standardized material.

The unusual format of this text is the result of considerable teaching and experimentation with classes at the technical level. The subject matter is presented in the sequence which has been found desirable not only for the instructor, but for the student. Since each local situation varies, however, adaptations to meet local requirements are easily made because of the careful subject matter breakdown. The instructor should feel at liberty to amplify the given material, or to omit those areas not applicable to his specific needs.

Since the student enrolled in a technical program is mainly concerned with learning a vocation, he should not be burdened with extremely difficult derivations: in this presentation, only the rules of basic concepts are derived, and these derivations are explained in detail. He should, however, be advised of the subject matter directly related to his field of interest.

Dennis H. Price

ACKNOWLEDGMENTS

I wish to take this opportunity to express my gratitude to all those persons who have so graciously contributed toward the completed manuscript. My many thanks to Mr. C. J. Laible and Mr. F. M. Moore, associates at Avco Corporation, Electronics Division. Mr. Laible and Mr. Moore reviewed the original manuscript and offered comments and suggestions too numerous to mention. And to my associates at Avco Corporation, Electronics Division, Cincinnati, Ohio, I express my sincere thanks for their many comments. In particular, Mr. J. H. Mason, Mr. C. E. Weber, Mr. R. H. Knese, and Mr. R. W. Desserich deserve much credit for their personal contributions.

Mr. Jack Cahall, Dean of the Evening College of the Ohio Mechanics Institute, Ohio College of Applied Science, permitted us the opportunity of applying this work in direct classroom practice. I, therefore, thank Mr. Cahall for his consideration and understanding. The late Mr. John Johnson, Director of Department of Mathematics, also reviewed the original manuscript and was extremely helpful in his guidance.

I want to thank Mrs. Harry Stillwell, Sr. for assisting in the preparation of the final manuscript. I would like to point out that my wife, Elaine, was the motivating force behind the entire program. She reviewed, edited, and helped type the final manuscript. This work is, therefore, dedicated to her.

H. R. Stillwell

INTRODUCTION

A SUMMARY OF MATHEMATICS HISTORY

Early mathematicians experienced many difficulties arising from a lack of paper, pencils, chalk, erasers, standardized notation, and an adequate number system. The abacus, a simple but effective computer consisting of rows of beads suspended on strings in a wood frame, solved many of the early problems. Not until Hindu symbols came into use did the processes of arithmetic become simple enough for them to be within the grasp of the nonprofessional. But it was only after the adoption of our present system of numerical notation and standard symbols, that mathematics became a truly universal language. In our study of mathematics we are indeed fortunate that we have the necessary tools with which to bring about solutions to problems which only a few short years ago were impossible to solve. In view of this, Table I and Table II are presented and give the common symbols used in much of today's mathematics.

SYMBOL	IDENTITY	SYMBOL	IDENTITY
\times OR \cdot	MULTIPLIED BY	\gg	IS MUCH GREATER THAN
\div OR $:$	DIVIDED BY	\ll	IS MUCH LESS THAN
$+$	POSITIVE, PLUS OR ADD	\geq	GREATER THAN OR EQUAL TO
$-$	NEGATIVE, MINUS OR SUBTRACT	\leq	LESS THAN OR EQUAL TO
\pm	POSITIVE OR NEGATIVE	\therefore	THEREFORE
\mp	NEGATIVE OR POSITIVE	\angle	ANGLE
$=$ OR $::$	EQUALS	Δ	INCREMENT OR DECREMENT
\equiv	IDENTITY	\perp	PERPENDICULAR TO
\approx	IS APPROXIMATELY EQUAL TO OR IS CONGRUENT TO	\parallel	PARALLEL TO
\neq	DOES NOT EQUAL	$ n $	ABSOLUTE VALUE OF n.
$>$	IS GREATER THAN		

UPPER CASE	LOWER CASE	IDENTITY	UPPER CASE	LOWER CASE	IDENTITY
A	α	ALPHA	N	ν	NU
B	β	BETA	Ξ	ξ	XI
Γ	γ	GAMMA	O	o	OMICRON
Δ	δ	DELTA	Π	π	PI
E	ϵ	EPSILON	P	ρ	RHO
Z	ζ	ZETA	Σ	σ	SIGMA
H	η	ETA	T	τ	TAU
Θ	θ	THETE	Υ	υ	UPSILON
I	ι	IOTA	Φ	φ	PHI
K	κ	KAPPA	X	χ	CHI
Λ	λ	LAMBDA	Ψ	ψ	PSI
M	μ	MU	Ω	ω	OMEGA

The following review material is presented for the student's information. This material is basic and should be referred to often. Although there are no problems submitted with these basics, the student should apply the principles with little or no difficulty.

ALGEBRA

A. Ratio and Proportion

(1) If $a:b = c:d$ or if a is to b as c is to d, or, as it is more commonly written,

$$\frac{a}{b} = \frac{c}{d}$$

then $ad = bc$, and, in addition to the proportion already stated, we can write these other two proportions,

$$\frac{a}{c} = \frac{b}{d} \text{ and } \frac{b}{a} = \frac{d}{c}$$

(2) If the two middle terms are the same, $\frac{a}{b} = \frac{b}{d}$, then $ad = b^2$ and b is called a mean proportional between a and b.

B. Monomials

A monomial is a single term together with its preceding plus (+) or minus (-) sign indicating whether the monomial is positive or negative.

$$(+5, -7x^2y^3, 8ab^2c)$$

C. Binomials

A binomial is an algebraic expression of two terms.

$$(9 - 3y); (4x + 3y); (3a^2b + 4c^3d^2)$$

D. Trinomials

A trinomial is an algebraic expression of three terms.

$$(5x - 7xy + 9y)$$

E. Polynomials

A polynomial is an algebraic expression of two or more terms. Thus, binomials and trinomials may also be defined as polynomials.

F. Exponent

An exponent is a small figure or letter written to the right and slightly above a quantity to indicate how many times the quantity is to be used as a factor.

$$(7^3) \text{ means } 7 \cdot 7 \cdot 7; (a + b)^3 \text{ means } (a + b)(a + b)(a + b)$$

G. Power

A power of a number is the result obtained by multiplying that number by itself a definite number of times.

8 is the third power of 2; $2^3 = 2 \cdot 2 \cdot 2 = 8$ $2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$

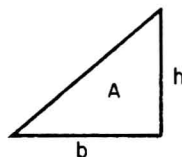
In the study of trigonometry there will be many opportunities to use formulas and theorems which have been developed in earlier courses. A few of these are outlined below for reference.

GEOMETRY

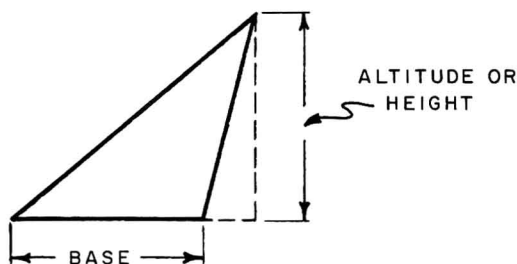
A. Triangle

- (1) The area of a right triangle is equal to one-half the product of the base and height.

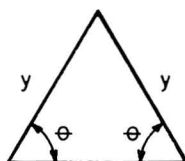
$$A = \frac{1}{2} bh$$



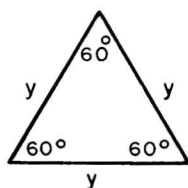
Any one of the sides of a triangle may be considered the base. Note, too, that an altitude may fall outside the triangle.



- (2) Two sides, and hence two angles, of an isosceles triangle are equal.



- (3) An equilateral triangle has three equal sides, hence each angle contains 60° .



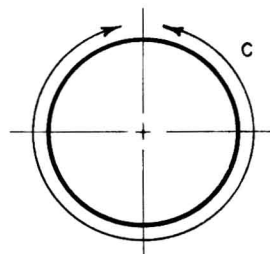
4. The sum of the three angles of any plane triangle must equal 180° .

$$\angle A + \angle B + \angle C = 180^\circ$$

B. Circle

- (1) The circumference of a circle is equal to 2π times the radius, or π times the diameter. (π is equal to 3.14159)

$$C = \pi d, \text{ or } C = 2\pi r$$

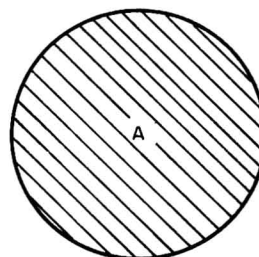


- (2) The area of a circle is equal to π times the square of the radius.

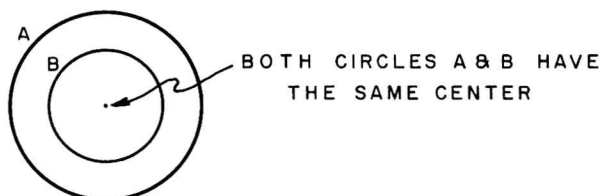
$$A = \pi r^2$$

Also, since the radius of a circle is $\frac{1}{2}$ the diameter D or $r = \frac{D}{2}$, the area can be expressed as:

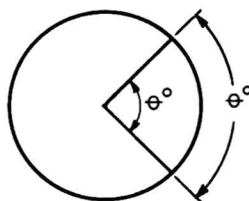
$$A = \pi \left(\frac{D}{2} \right)^2 = \frac{\pi D^2}{4}$$



- (3) Two or more circles are said to be concentric when they have the same center.



- (4) Any angle measured at the center of a circle intercepts an arc of the same angle on the circumference of the circle.



C. Function Notation

One of the most important fundamental mathematical concepts is the function notation. A simple explanation of function in terms of our everyday surroundings will serve to simplify the concept. One function of our police department is traffic control; another function is protection of our homes and property.

If we were to express this relationship in mathematical terms we could let x stand for the police department, y stand for traffic control, and z for protection.

Therefore, a function of x is y and another function of x is z . In mathematical symbols we can write $y = f(x)$, $z = f(x)$, or $y, z = f(x)$. The notation of $f(x)$ does not mean f multiplied by x , but that f is simply an operational symbol or operator.

If we were to let x stand for another agency of our government, the functions y and z would stand for entirely different things, such as tax collection or waste disposal. Therefore, the interpretation of y and z depend upon the selection of x . Since y and z depend upon x , we define x as the independent variable and y and z or both as the dependent variable or variables.

The same reasoning as above applies to mathematical formulas. For example,

$$y = x, \text{ if } f(x) = 3 \quad \text{then } y = f(3) \text{ or } y = 3$$

$$y - 3 = x, \text{ if } f(x) = 4 \quad \text{then } y - 3 = 4 \text{ or } y = 7$$

$$y = x - 2q, \text{ if } f(x) = (q^2 + 1) \quad \text{then } y = q^2 - 2q + 1 \\ = (q - 1)(q - 1) = (q - 1)^2$$

$$y = 6^x, \text{ if } f(x) = 2 \quad \text{then } y^2 = 36$$

We can now more precisely define a function as the medium between two sets of numbers, such that when one number from the first set is given, the number from the second set can be determined. Therefore, y is called a function of x if, whenever x is known, y can be found.

Examples: 1. $y = 6x + 3$

2. $y = x^2 + 3x^3 - 4x + 4$

3. $y = 6^x$

4. $y = \frac{x^3 + 4x^2 - 5}{x - 5}$

As shown above, in each case, if x is known, y can be computed. This then allows us to call x the independent variable and y , the dependent variable. Should we wish to speak of an unknown function, or functions in general, the accepted symbol is:

$$y = f(x)$$

and reads y is equal to f of x , or y equals the function of x . Further, it does not imply that x is multiplied by f but merely an operator that produces from each quantity for x , a quantity for y .

Examples: $y = 6x + 3$ from above and $x = 3$

$f(x) = 6 \cdot 3 + 3 = 21$

Contents

Preface	iii
To the Instructor	iv
Acknowledgments	iv
Table of Contents	v-ix

Introduction - Algebra and Geometry Review	xi-xv
--	-------

Unit I Radian Measure 1

1. Circumference of a Circle	1
2. The Radian	3
2.1 Skill Problems	4
3. Arc Length	4
4. Linear and Angular Velocity	5
4.1 Applications	6

Unit II Rectangular Coordinate System 9

1. The Graph and its Use	9
2. Plotting a Point on a Plane	9
2.1 Skill Problems	10
3. Algebraic Signs and Polarity	10
4. Trigonometric Angle	12
5. Definition of Trigonometric Functions	14
5.1 Applications	19
5.2 Applications	20
6. Tables of Trigonometric Functions	20
6.1 Applications	23
7. Related Angles	23
7.1 Skill Problems	23

Unit III Acute Angles 24

1. Acute Angles, Parts of Right Triangles	24
1.1 Skill Problems	24
2. Cofunctions	25
2.1 Skill Problems	25
3. Trigonometric Functions of 30° and 60° Angles	25
4. Trigonometric Functions of 45° Angles	26
5. Powers of Trigonometric Functions	26
5.1 Skill Problems	26
6. Reference Aid for Most Often Used Trigonometric Functions	27
6.1 Skill Problems	28

Unit IV	INTERPOLATION	29
1.	Approximations and Significant Figures	29
2.	Scientific Notation	29
2.1	Skill Problems	31
3.	Interpolation	31
3.1	Applications	33
Unit V	SOLUTION OF RIGHT TRIANGLES	34
1.	Right Triangle Nomenclature	34
2.	The Acute Angles in a Right Triangle	34
3.	Solution Defined	35
3.1	Skill Problems	37
3.2	Applications	37
4.	Area of a Triangle	40
4.1	Applications	41
Unit VI	ANGULAR DISPLACEMENT	43
1.	Definition	43
2.	Angle of Elevation and Depression	43
3.	Angle of Bearing	44
3.1	Applications	45
Unit VII	TRIGONOMETRIC IDENTITIES	50
1.	Algebraic Identities	50
1.1	Skill Problems	51
2.	Eight Fundamental Relations	51
3.	Proofs	52
4.	Variations of Basic Equations	54
4.1	Skill Problems	55
Unit VIII	FUNCTIONS OF TWO ANGLES	56
1.	Sine and Cosine Functions of Two Angles	56
2.	Proofs of Rules	56
2.1	Skill Problems	59
3.	Tangent Functions of Two Angles	60
3.1	Applications	62
4.	Double Angle Formulas	63
5.	Half Angle Formulas	64
5.1	Applications	66
6.	Product to Sum Formulas: Sum to Product Formulas	67
6.1	Skill Problems	68

Unit IX	OBLIQUE TRIANGLES	69
---------	-------------------	----

1. A Distance Formula Taken From Analytic Geometry	.	69
1.1 Skill Problems	71
2. Law of Cosines (SSS, SAS)	71
2.1 Applications	73
3. Law of Sines (SSA, ASA)	74
3.1 Applications	76
4. The Ambiguous Case (SSA)	76
5. Law of Tangents (SAS)	77
6. Half Angle Formulas (SSS)	78
7. Applications	78

Unit X	VECTORS	81
--------	---------	----

1. Vector Addition	82
1.1 Skill Problems	82
2. Vector Subtraction	83
2.1 Skill Problems	83
3. Parallelogram Method	84
3.1 Skill Problems	85
4. Triangle Method	86
4.1 Skill Problems	87
5. X and Y Components	87
6. Applications	90

Unit XI	LOGARITHMS	96
---------	------------	----

1. Characteristics	97
2. Scientific Notation	98
3. Characteristics of Numbers Less Than One	98
4. Mantissa	98
4.1 Skill Problems	100
5. Antilogarithms	100
6. Multiplication	100
6.1 Skill Problems	101
7. Multiplication and Interpolation	101
7.1 Applications	102
8. Division	103
9. Multiplication and Division	103
9.1 Applications	104
10. Powers and Roots	105
10.1 Applications	106

Unit XII	SLIDE RULE OPERATION	107
1. Areas and Volumes		107
1.1 Skill Problems		108
2. Logarithm and Cube Scales		108
2.1 Skill Problems		109
3. Sine and Tangent Scales		110
3.1 Skill Problems		113
Unit XIII	PLOTTING TRIGONOMETRIC GRAPHS	115
1. Plotting Trigonometric Graphs		115
2. Plotting $Y = \sin \theta$		115
3. Plotting $Y = \cos \theta$		116
4. Plotting $Y = \tan \theta$		117
5. Plotting $Y = \cot \theta$		117
6. Plotting $Y = \sec \theta$		118
7. Plotting $Y = \csc \theta$		118
8. Applications		118
9. Periodic Functions		119
9.1 Cycle of Periodic Curve		119
9.2 Definition of Period		119
9.3 Skill Problems		119
Unit XIV	INVERSE TRIGONOMETRIC FUNCTIONS	120
1. Definition of Inverse Functions		120
1.1 Skill Problems		120
2. Inverse Trigonometric Functions		120
3. Graphs of Inverse Trigonometric Functions		122
4. Principal Values of Inverse Functions		123
5. Applications		123
Unit XV	COMPLEX NUMBERS	124
1. Definition of Complex Numbers		124
1.1 Skill Problems		125
2. Cartesian Form		125
3. Applications		126
Unit XVI	ALTERNATING CURRENT CIRCUIT ANALYSIS	128
1. Alternating Current Circuit Analysis		128
1.1 Applications		129
2. The A-C Sine Wave		131
3. Inductance		132
3.1 Applications		132
4. Capacitance		133
4.1 Applications		133

Unit XVII	GENERAL PHYSICS	134
-----------	-----------------	-----

1. General Physics Defined	134
2. Statics	134
2.1 Vector Components of Force	136
2.2 Skill Problems	136
3. Dynamics	137
3.1 Skill Problems	138
4. Light Properties	138
4.1 Skill Problems	140
5. Heat Properties	140
5.1 Skill Problems	141

Unit XVIII	HYDROSTATICS	142
------------	--------------	-----

1. Hydrostatics Problem Analysis	142
1.1 Skill Problems	142
2. Bernoulli's Theorem	143
3. Applications	144

Unit XIX	SURVEYING	146
----------	-----------	-----

1. Surveying Defined	146
2. Applications	146

APPENDIX

Table A	Natural Values of Trigonometric Functions	. 150-153
Table B	Logarithms of Trigonometric Functions	. . 154-158
Table C	Common Logarithms of Numbers	. . 159-160
Table D	Logarithms of Constants	. . . 161
Table E	Radian Measure, 0° to 180° , Radius = 1	. . 162
Table F	Square Roots and Squares	. . . 163-166
Table G	Common Logarithms of Functions of Angles in Mils	. . . 167-170

INDEX

Unit I

RADIAN MEASURE

There are two basic methods for determining the dimensions of a circle, namely, physical or actual measurement and computation or mathematical solution.

1. _____

CIRCUMFERENCE OF A CIRCLE

We know that the distance around the perimeter of a circle is the circumference, and that this dimension can be computed as well as actually measured.



FIG. I-1

PHYSICAL MEASUREMENT

The physical measurement method is very simple for circles or wheels of small size or those not too heavy to handle; however, in those cases where we are dealing with circles or wheels which do not lend themselves to physical measurement, we must compute this quantity. The formula used is:

$$\text{Circumference} = 2\pi\text{Radius} = 360^\circ$$

Notice that a circle is actually a generated figure. This basic fact is needed for the balance of this Unit. Above we state that: $C = 2\pi R = 360^\circ$. The question now arises: What does 360 degrees actually mean?

As an aid in answering this question, let us generate a circle. We can do this by a simple process using a thumbtack, a piece of string and a pencil. Tying the string between the thumbtack and the pencil, push the thumbtack into a flat board, and pull the string tight while holding the pencil vertical to the board.

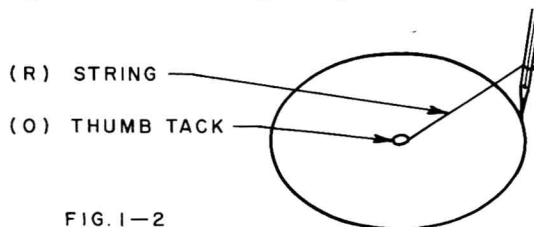


FIG. I-2

Now rotate the pencil all the way around the thumbtack and back to the original starting point, scribing a neat circle. Let us call the string "R" or "Radius" and the thumbtack "O" or the "Origin".

It is now feasible to consider the measurement of the angle. If we rotate the pencil again, but this time through only a short distance, mark the finish point, and connect this point with the center of the circle, we have generated an "arc".

A preferred method of describing this is to visualize a rigid form of radius such as a pole or stick, fastened so that it may be rotated about the origin, free to stop at any desired point.

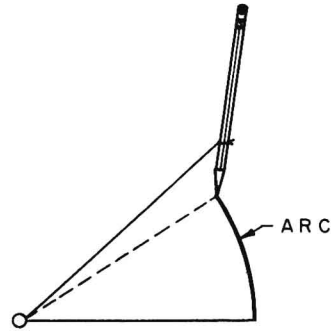


FIG. 1-3

The increments of rotation of the radius from a starting point are most commonly referred to as DEGREES. (1)

Any segment of the circumference expressed in degrees may be divided into smaller segments by ordinary division. A degree is $1/360$ of the circumference of a circle or an angle at the center of a circle that intercepts $1/360$ of the circumference.

$$C = 2\pi R = 360^\circ$$

1 Degree ($^\circ$) is $1/360$ of a circumference

1 Minute ($'$) is $1/60$ of a degree or $60' = 1^\circ$

1 Second ($''$) is $1/60$ of a minute or $60'' = 1'$

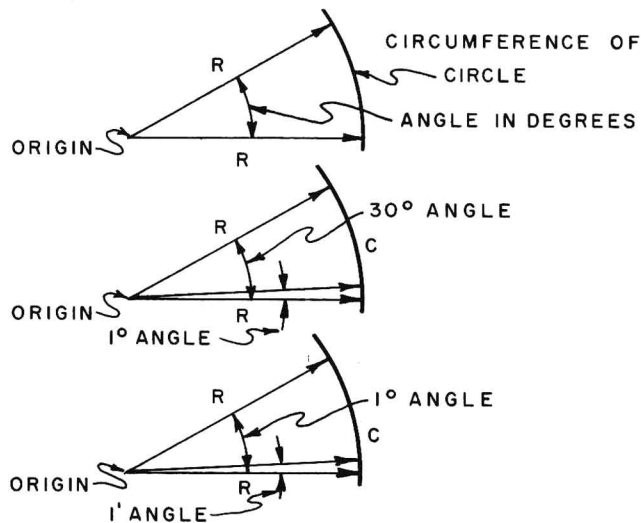


FIG. 1-4

- (1) There is no mathematical reason for the use of the degree, minute, or second. It is conjecture that the Babylonians used 360 as the base of their number system, which, in turn, was based on their year having 360 days.