

**APPLIED
MATHEMATICS
FOR ENGINEERING
AND SCIENCE**

PRENTICE-HALL, INC.
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**PRENTICE-HALL SERIES
IN TECHNICAL MATHEMATICS**

Frank L. Juszli, Editor

Our aim in writing this book is to present mathematics as a useful tool. In this regard we have endeavored to present the fundamental principles of such work as algebra, trigonometry, linear equations and determinants, complex numbers, elementary statistics, basic calculus, etc., in as simple a form as possible consistent with applications to engineering and scientific problems. In addition, we have included numerous examples from all fields of technology and most chapters contain a section specifically devoted to applied problems. Over three thousand problems have been interspersed throughout the book to provide the student with sufficient experience in solving problems. This should give the student confidence in handling mathematics and should enable him to discover its usefulness for himself.

Although this text is intended primarily for students at technical institutes and junior colleges, it will prove useful to students of engineering, physics, and other applied sciences.

We have assumed that the student has had the normal secondary school background in mathematics. However, many of the topics are reviewed in the early chapters. The book begins with a study of operation with numbers and algebraic operations, and then continues with a study of logarithms and trigonometry. There is a thorough coverage of linear equations and determinants. More advanced topics such as vectors, analytical geometry, complex numbers, and solution of higher-order equations are treated in the latter part of the text. In line with the modern trend in education towards teaching computer applications, we have included an introductory treatment of probability and statistics. The book concludes with a thorough introduction to differential and integral calculus.

We have purposely relegated a description of the operations of slide rules to the appendix, in order that it may be presented at any convenient point in a mathematics program without breaking the continuity of the book. In addition, the appendix contains logarithmic and trigonometric tables, as well as many useful mathematical functions. The answers to odd-numbered exercises are provided following the appendices on page 642. Many of these answers were computerized utilizing both the Programma 101 and the PDP-8 computers.

We have endeavored to present the foregoing material in a logical sequence so as to lead the reader from simple and elementary techniques to those that are more complex and advanced. The presentation of the material in this book is based upon the authors' experience in both the industrial and teaching fields.

PREFACE

We are deeply indebted to the staff of Prentice-Hall for their assistance throughout the writing of the manuscript. The authors are also grateful to the reviewers for their important suggestions and objective critique. Finally, we wish to acknowledge the assistance extended to us by various members of the staff at the Manitoba Institute of Technology, too numerous to mention individually.

WARIS SHERE / GORDON LOVE

**APPLIED
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“I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it: but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of Science, whatever the matter may be.”

LORD KELVIN

1	OPERATIONS WITH NUMBERS	1
1-1	The Real Number System	2
1-2	Inequalities; Absolute Values	5
1-3	Fundamental Operations	11
1-4	Accuracy Versus Precision	14
1-5	Scientific Notation and Dimensional Analysis	17
1-6	Other Number Systems	22
2	ALGEBRAIC OPERATIONS	38
2-1	Introduction	39
2-2	Algebraic Manipulations	43
2-3	Variation, Ratio, and Proportion	61
2-4	Functions	68
2-5	Equations	78
2-6	Linear Equations and Linear Functions	80
2-7	Quadratic Functions	83
2-8	The Quadratic Equation and Its Solution	85
2-9	The Nature of the Roots of the Quadratic Equation	95
2-10	Equations of Quadratic Form	99
2-11	Progressions	101
2-12	Applications of Algebra	109
3	EXPONENTS AND RADICALS	117
3-1	Laws of Positive Integral Exponents	118
3-2	Roots and Radicals	119
3-3	Principal Roots	120
3-4	Rational and Irrational Numbers	121
3-5	Fractional Exponents	121
3-6	Zero Exponent	122
3-7	Negative Exponents	123
3-8	Laws of Radicals	125
3-9	Addition and Subtraction of Radicals	127
3-10	Multiplication of Radicals	129
3-11	Division of Radicals	130
3-12	The Binomial Theorem	131
3-13	Equations with Radicals	133
3-14	Applications of Exponents and Radicals	135

CONTENTS

4	LOGARITHMS	138
	4-1	Introduction 139
	4-2	Definition of a Logarithm 141
	4-3	The Graph of the Logarithm Function 143
	4-4	The Basic Properties of Logarithms 144
	4-5	Common Logarithms 149
	4-6	Characteristic and Mantissa 151
	4-7	Use of the Logarithm Tables 154
	4-8	Interpolation 159
	4-9	Computation Using Logarithms 162
	4-10	Cologarithms 169
	4-11	Natural Logarithms 172
	4-12	Change of Base 172
	4-13	Exponential and Logarithmic Equations 175
	4-14	Applications of Logarithms 177
5	THE TRIGONOMETRIC FUNCTIONS	184
	5-1	Introduction 185
	5-2	Angles 185
	5-3	The Relation Between Degrees and Radians 185
	5-4	The Trigonometric Functions of an Angle 187
	5-5	Signs of the Trigonometric Functions in the Various Quadrants 188
	5-6	The Trigonometric Functions of an Acute Angle 190
	5-7	The Functions of Complementary Angles 191
	5-8	Functions of Special Angles 194
	5-9	Functions of Circular Angles 198
	5-10	The Solution of a Right Triangle 202
	5-11	Interpolation 206
	5-12	Components 208
	5-13	Logarithmic Solution of Right Triangles 209
	5-14	Functions of Compound Angles 211
	5-15	Functions of Multiple Angles 215
	5-16	Transformation of Products, Sums, and Differences 218
	5-17	Trigonometric Identities 221
	5-18	Trigonometric Equations 225
	5-19	The Inverse Trigonometric Functions 225
	5-20	Applied Problems 228
6	GRAPHS OF TRIGONOMETRIC FUNCTIONS	234
	6-1	The Graphs of $y = a \sin x$ and $y = a \cos x$ 235

6-2	The Graph of $y = \tan x$	238
6-3	The Graphs of $y = \cot x$, $y = \sec x$, and $y = \csc x$	240
6-4	The graphs of $y = a \sin bx$ and $y = a \cos bx$	241
6-5	The Graphs of $y = a \sin (bx + c)$ and $y = a \cos (bx + c)$	243
6-6	The Graph of Sine and Cosine Functions by Geometric Methods	245
6-7	Graphs by Composition of Ordinates	246
6-8	The Graphs of Inverse Trigonometric Functions	248
6-9	Application of Graphs of Trigonometric Functions	249
7	LINEAR EQUATIONS AND DETERMINANTS	252
	7-1 Introduction	253
	7-2 Two Equations in Two Unknowns	254
	7-3 Three Equations in Three Unknowns	262
	7-4 Solution by Determinants of Equations in Two and Three Unknowns	264
	7-5 Systems of n Equations in n Unknowns	271
7-6	The General Determinant; Minors and Cofactors	272
	7-7 Expansion of the General Determinant	276
	7-8 Properties of a Determinant	279
7-9	Solution of n Equations by Cramer's Rule; Homogeneous Equations	282
	7-10 Application of Linear Equations and Determinants	288
8	VECTOR ALGEBRA	292
	8-1 Introduction	293
	8-2 Addition and Subtraction of Vectors	295
	8-3 Resolution of Vectors	299
8-4	Three-Dimensional Rectangular Coordinates	304
	8-5 Multiplication of Scalars and Vectors	305
	8-6 Applications of Vectors	311
9	ELEMENTS OF ANALYTICAL GEOMETRY	316
	9-1 Introduction	317
	9-2 The Straight Line	323

9-3	Polar Coordinates	325
9-4	Conic Sections	332
9-5	The Circle	334
9-6	The Parabola	337
9-7	The Ellipse	342
9-8	The Hyperbola	346
9-9	Applications of Analytical Geometry	354
10 THE OBLIQUE TRIANGLE		363
10-1	The Law of Sines	364
10-2	The Law of Cosines	369
10-3	The Law of Tangents	374
10-4	Half-Angle Formulas	378
10-5	Area of Triangles	383
10-6	Radius of the Inscribed Circle of the Triangle	386
10-7	Applied Problems	387
11 COMPLEX NUMBERS		392
11-1	Complex Numbers	393
11-2	Conjugate Complex Numbers	395
11-3	Operations with Complex Numbers	395
11-4	Graphical Representation of Complex Numbers	398
11-5	Trigonometric and Polar Representations of Complex Numbers	400
11-6	Multiplication and Division of Complex Numbers in Polar Form	403
11-7	De Moivre's Theorem	404
11-8	Roots of Complex Numbers	405
11-9	Application of Complex Numbers to A-C Circuits	408
12 THE SOLUTION OF HIGHER-ORDER EQUATIONS		410
12-1	Introduction	411
12-2	The Remainder Theorem and the Factor Theorem	412
12-3	Synthetic Division	414
12-4	Depressed Equation; Factored Form of a Polynomial	417

12-5	The Number of Roots of an Equation	421
12-6	Imaginary and Irrational Roots	423
12-7	Rational Roots	426
12-8	Graphing a Polynomial	431
12-9	Evaluation of Irrational Roots; Horner's Method	433
12-10	Applications of Higher-Order Equations	438
13	PROBABILITY AND STATISTICS	442
13-1	Frequency Distribution	443
13-2	Measures of Central Value	445
13-3	Measures of Variation	447
13-4	Permutations and Combinations	450
13-5	Probability	453
14	DIFFERENTIATION	456
14-1	Introduction	457
14-2	Constants and Variables	457
14-3	Functions	457
14-4	Functional Notation	457
14-5	Limits	458
14-6	The Derivative	460
14-7	Derivatives of Algebraic Functions	463
14-8	The Chain Rule	469
14-9	Derivative of a Power of Any Function	470
14-10	Successive Derivatives	471
14-11	Differentiation of Implicit Functions	473
14-12	Related Rates	474
14-13	Maxima and Minima	476
14-14	Concavity and Points of Inflection	477
14-15	Derivatives of the Sine and Cosine Functions	479
14-16	Derivatives of the Other Trigonometric Functions	480
14-17	Derivatives of the Inverse Trigonometric Functions	484
14-18	The Number e	487
14-19	Derivative of Logarithmic Functions	488
14-20	Derivative of Exponential Functions	491
14-21	Compilation of Formulas	492

15	INTEGRATION	494
	15-1	Introduction 495
	15-2	Definitions 495
	15-3	Arbitrary Constant; Indefinite Integral 495
	15-4	Integral of a Power of a Function 497
	15-5	Two Important Principles 499
	15-6	Integration by Substitution 500
	15-7	Integration of Exponential Functions 502
	15-8	Integration of Trigonometric Functions 504
	15-9	Integrals Leading to Inverse Trigonometric Functions 506
	15-10	Integrals Leading to Logarithmic Functions 507
	15-11	Integration by Parts 508
	15-12	Summary of Formulas 510
	15-13	The Definite Integral 511
	15-14	Trigonometric Substitutions 513
	15-15	Integration of Rational Algebraical Fractions 515
	15-16	An Important Rule 519
	APPENDICES	521
	A.	Use of the Slide Rule 521
	B.	Common Expansions and Factors; Binomial Theorem; Trigonometric Relations; Mathematical Constants 539
	C.	Four-Place Common Logarithms; Four-Place Antilogarithms; Five-Place Common Logarithms; Natural (Napierian) Logarithms; Common Logarithms of Trigonometric Functions; Natural Trigonometric Functions 544
	ANSWERS TO ODD-NUMBERED EXERCISES	642
	INDEX	665

*From early historic time, man's civilization has always in some way been measured by his knowledge of mathematics as well as his ability to manipulate numbers. Numbers were used by primitive man before the development of a spoken language; therefore, it is not inconceivable that he was able to appreciate the number **three** without having had a name for numbers beyond **two**. Similarly, a herdsman was probably able to recognize that one of his sheep was missing without being able to count his flock. There is ample evidence that the nations which did the most trading, the most commerce, and had the highest forms of civilization were also the most conversant in number manipulation and mathematics.*

OPERATIONS WITH NUMBERS

Even today, numbers and their operations are fundamental to our advanced civilization just as they are to our way of life. There are numerous instances to be found in science and engineering. As an example, the doctor does not decide that his patient has a fever until the thermometer shows a temperature in excess of 98.6°F. Nor does the engineer design or build anything until he has made exact calculations and measurements.

Because a complete understanding of numbers is paramount to all science and engineering, we begin this book with the study of numbers and their operations.

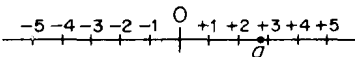
There are two principal classes of numbers, *real* and *imaginary*, which together form the system of *complex numbers*. Complex numbers will be treated in Chapter 11.

1-1 The Real Number System

The number system in common use today is the *decimal system*, so named because it employs the 10 basic symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, which are called *digits*. Other number systems which are employed for special purposes, such as the *binary system* and *octal system*, will be discussed in Sec. 1-6.

Any combination of the 10 basic digits shown above, other than as fractions (e.g., $\frac{2}{4}$, $1/\sqrt{2}$, $\frac{5}{2}$, . . .) or decimal numbers (e.g., 1.85, 3.176, 14.2, . . .), produces a *whole number*, also called a *natural number*. The technical name for a whole number is *integer*; integers represent numbers like 4, 18, 3, 1, 2, 0, -11, 1064, . . .

FIGURE 1-1



The integers 1, 3, 5, 7, . . . are called *odd numbers*; while the integers 2, 4, 6, 8, . . .

are called *even numbers*.

We can give geometrical meaning to the integers by referring to something known as the *number scale*, shown in Fig. 1-1. In this diagram we have drawn a section of an infinite straight line upon which a point is arbitrarily chosen as the origin and assigned the integer 0. Next we choose an arbitrary line length to correspond to unity and mark this unit distance on either side of the origin. We continue to mark off these unit lengths, assigning integer values as illustrated in Fig. 1-1. By convention, all integers to the right of 0 are positive, while those to the left are negative.

Figure 1-1 shows that for each integer there is a fixed point on the scale; or, conversely, for every unit length there is a unique integer. In more concise language we describe this as *one-to-one correspondence between the real numbers and the points on the line*. For instance, due to this correspondence, the letter *a* can mean either the *number a* or the *point a on the number scale*. Another result of this correspondence is that *equal*

distances between numbers correspond to equal distances between points.

EXAMPLE 1-1. As shown in Fig. 1-2, 20°C can either be represented by the number or the distance from 0 on the thermometer scale to the point $+20$ units away.

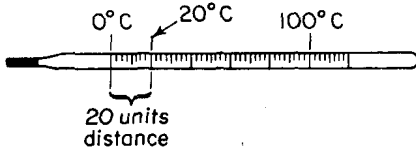


FIGURE 1-2

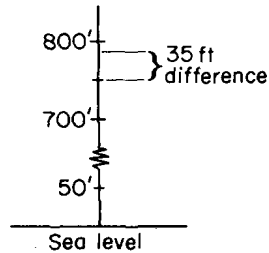


FIGURE 1-3

EXAMPLE 1-2. In driving from an elevation of 750 ft above sea level to 785 ft above sea level, a car would have moved through an elevation of 35 ft. On an elevation scale this would correspond to 35 units, as illustrated in Fig. 1-3.

At this point it is quite conceivable that the student may be confused between the two terminologies—integer and number. They are not necessarily the same because numbers may be fractions or decimals as well as whole numbers. But, *a number is only an integer when it is a whole number (positive or negative)*. This fact is illustrated on the number scale shown in Fig. 1-4.

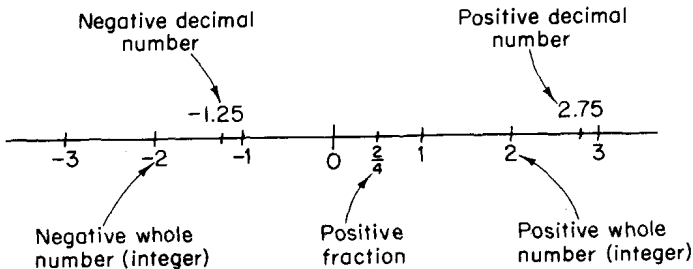


FIGURE 1-4

There is a fundamental property which can be observed by inspecting Fig. 1-4. If we let n represent *any* integer (including 0), then between n and $n + 1$ there *cannot* be an integer. In view of this property, all numbers lying between any two integers must be decimal or fraction numbers. Actually,