MECHANICAL ENGINEERING SERIES

Anthony C. Fischer-Cripps

Introduction to Contact Mechanics

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Anthony C. Fischer-Cripps
CSIRO
Bradfield Road, West Lindfield
Lindfield NSW 2070, Australia

Series Editor
Frederick F. Ling
Ernest F. Gloyna Regents Chair
in Engineering
Department of Mechanical Engineering
The University of Texas at Austin
Austin, TX 78712-1063, USA
and
William Howard Hart Professor Emeritus
Department of Mechanical Engineering,
Aeronautical Engineering and Mechanics
Rensselaer Polytechnic Institute
Troy, NY 12180-3590, USA

Consulting Editor
Iain Finnie
Department of Mechanical Engineering
University of California
Berkeley, CA 94720, USA

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Frederick F. Ling
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Series Preface

Mechanical engineering, an engineering discipline forged and shaped by the needs of the industrial revolution, is once again asked to do its substantial share in the call for industrial renewal. The general call is urgent as we face profound issues of productivity and competitiveness that require engineering solutions. The Mechanical Engineering Series features graduate texts and research monographs intended to address the need for information in contemporary areas of mechanical engineering.

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Professor Finnie, the consulting editor for mechanics of materials, and I are pleased to present *Introduction to Contact Mechanics* by Anthony C. Fischer-Cripps.

Austin, Texas　　　　Frederick F. Ling

Preface

This book deals with the mechanics of solid bodies in contact, a subject intimately connected with such topics as fracture, hardness, and elasticity. Theoretical work is most commonly supported by the results of indentation experiments under controlled conditions. In recent years, the indentation test has become a popular method of determining mechanical properties of both brittle and ductile materials, and particularly thin film systems.

The book begins with an introduction to the mechanical properties of materials, general fracture mechanics, and the fracture of brittle solids. This is followed by a detailed description of indentation stress fields for both elastic and elastic-plastic contact. The discussion then turns to the formation of Hertzian cone cracks in brittle materials, subsurface damage in ductile materials, and the meaning of hardness. The book concludes with an overview of practical methods of indentation testing.

My intention is for this book to make contact mechanics accessible to those entering the field for the first time. Experienced researchers may also benefit from the review of the most commonly used formulas and theoretical treatments of the past century.

In writing this book, I have been assisted and encouraged by many colleagues, friends, and family. I am most indebted to A. Bendeli, R.W. Cheary, R.E. Collins, R. Dukino, J.S. Field, A.K. Jämting, B.R. Lawn, C.A. Rubin, and M.V. Swain. Finally, I thank Dr. Thomas von Foerster and the production team at Springer-Verlag New York, Inc., for their very professional and helpful approach to the whole publication process.

Lindfield, Australia — Anthony C. Fischer-Cripps

List of Symbols

a	cylindrical indenter radius or spherical indenter contact area radius
α	cone semi-angle
A	Auerbach constant; area; material characterization factor
b	distance along a crack path
B	risk function
β	friction parameter; rate of stress increase; cone inclination angle, indenter shape factor
c	total crack length; radius of elastic-plastic boundary
c_0	size of plastic zone
C	hardness constraint factor, compliance
δ	distance of mutual approach between indenter and specimen
d	length of long diagonal
D	subcritical crack growth constant; spherical indenter diameter
E	Young's modulus
E_o	activation energy
ε	strain
F	force
G	strain energy release rate per unit of crack extension; shear modulus
h	plate thickness; distance; indentation depth
H	hardness
I	matrix subscript
j	matrix subscript
κ	stress concentration factor
k	Weibull strength parameter; elastic spring stiffness constant; Boltzmann's constant, elastic mismatch parameter, initial depth constant
K	bulk modulus, Oliver and Pharr correction factor
K_1	stress intensity factor for mode 1 loading.
K_{1scc}	static fatigue limit
L, l	length or distance
λ	Lamé constant
m	Weibull modulus
n	subcritical crack growth exponent; number; ratio of minimum to maximum stress, initial depth exponent

N	total number
η	coefficient of viscosity
P	indenter load (force)
P_f	probability of failure
p_m	mean contact pressure
P_s	probability of survival
ϕ	strain energy release function
q	uniform lateral pressure
θ	angle
R	universal gas constant; spherical indenter radius
r	radial distance
RH	relative humidity
r_o	ring crack starting radius
ρ	radius of curvature; number density
s	distance
σ	normal stress
T	temperature
t	time
τ	shear stress
u	displacement
U	energy
μ	Lamé constant, coefficient of friction
V	volume
ν	Poisson's ratio
W	work
x	linear displacement, strain index
γ	surface energy; shear angle
γ_o	activation energy
Y	yield stress, shape factor

History

It may surprise those who venture into the field of "contact mechanics" that the first paper on the subject was written by Heinrich Hertz. At first glance, the nature of the contact between two elastic bodies has nothing whatsoever to do with electricity, but Hertz recognized that the mathematics was the same and so founded the field, which has retained a small but loyal following during the past one hundred years.

Hertz wanted to be an engineer. In 1877, at age 20, he traveled to Munich to further his studies in engineering, but when he got there, doubts began to occupy his thoughts. Although "there are a great many sound practical reasons in favor of becoming an engineer" he wrote to his parents, "I still feel that this would involve a sense of failure and disloyalty to myself." While studying engineering at home in Hamburg, Hertz had become interested in natural science and was wondering whether engineering, with "surveying, building construction, builder's materials and the like," was really his lifelong ambition. Hertz was really more interested in mathematics, mechanics, and physics. Guided by his parents' advice, he chose the physics course and found himself in Berlin a year later to study under Hermann von Helmholtz and Gustav Kirchhoff.

In October 1878, Hertz began attending Kirchhoff's lectures and observed on the notice board an advertisement for a prize for solving a problem involving electricity. Hertz asked Helmholtz for permission to research the matter and was assigned a room in which to carry out experiments. Hertz wrote: "every morning I hear an interesting lecture, and then go to the laboratory, where I remain, barring a short interval, until four o'clock. After that, I work in the library or in my rooms." Hertz wrote his first paper, "Experiments to determine an upper limit to the kinetic energy of an electric current," and won the prize.

Next, Hertz worked on "The distribution of electricity over the surface of moving conductors," which would become his doctoral thesis. This work impressed Helmholtz so much that Hertz was awarded "Acuminis et doctrine specimen laudabile" with an added "magna cum laude." In 1880, Hertz became an assistant to Helmholtz—in modern-day language, he would be said to have obtained a three-year "post-doc" position.

On becoming Helmholtz's assistant, Hertz immediately became interested in the phenomenon of Newton's rings—a subject of considerable discussion at the time in Berlin. It occurred to Hertz that, although much was known about the optical phenomena when two lenses were placed in contact, not much was known about the deflection of the lenses at the point of contact. Hertz was particularly concerned with the nature of the localized deformation and the distribution of pressure between the two contacting surfaces. He sought to assign a shape to the surface of contact that satisfied certain boundary conditions worth repeating here:

1. The displacements and stresses must satisfy the differential equations of equilibrium for elastic bodies, and the stresses must vanish at a great distance from the contact surface—that is, the stresses are localized.
2. The bodies are in frictionless contact.
3. At the surface of the bodies, the normal pressure is zero outside and equal and opposite inside the circle of contact.
4. The distance between the surfaces of the two bodies is zero inside and greater than zero outside the circle of contact.
5. The integral of the pressure distribution within the circle of contact with respect to the area of the circle of contact gives the force acting between the two bodies.

Hertz generalized his analysis by attributing a quadratic function to represent the profile of the two opposing surfaces and gave particular attention to the case of contacting spheres. Condition 4 above, taken together with the quadric surfaces of the two bodies, defines the form of the contacting surface. Condition 4 notwithstanding, the two contacting bodies are to be considered elastic, semi-infinite, half-spaces. Subsequent elastic analysis is generally based on an appropriate distribution of normal pressure on a semi-infinite half-space. By analogy with the theory of electric potential, Hertz deduced that an ellipsoidal distribution of pressure would satisfy the boundary conditions of the problem and found that, for the case of a sphere, the required distribution of normal pressure σ_z is:

$$\frac{\sigma_z}{p_m} = -\frac{3}{2}\left(1 - \frac{r^2}{a^2}\right)^{1/2}, \quad r \leq a$$

This distribution of pressure reaches a maximum (1.5 times the mean contact pressure p_m) at the center of contact and falls to zero at the edge of the circle of contact ($r = a$). Hertz did not calculate the magnitudes of the stresses at points throughout the interior but offered a suggestion as to their character by interpolating between those he calculated on the surface and along the axis of symmetry. The full contact stress field appears to have been first calculated in detail by Huber in 1904 and again later by Fuchs in 1913, and by Moreton and Close in 1922. More recently, the integral transform method of Sneddon has been applied to axis-symmetric distributions of normal pressures, which correspond to a variety of indenter geometries. In brittle solids, the most important stress is not the normal pressure but the radial tensile stress on the specimen surface, which reaches a maximum value at the edge of the circle of contact. This is the stress that is responsible for the formation of the conical cracks that are familiar to all who have had a stone impact on the windshield of their car. These cracks are called "Hertzian cone cracks."

Hertz published his work under the title "On the contact of elastic solids," and it gained him immediate notoriety in technical circles. This community interest led Hertz into a further investigation of the meaning of hardness, a field in which he found that "scientific men have as clear, i.e. as vague, a conception as the man in the street." It was appreciated very early on that hardness indicated a resistance to penetration or permanent deformation. Early methods of measuring hardness, such as the scratch method, although convenient and simple, were found to involve too many variables to provide the means for a scientific definition of this property. Hertz postulated that an absolute value for hardness was the least value of pressure beneath a spherical indenter necessary to produce a permanent set at the center of the area of contact. Hardness measurements embodying Hertz's proposal formed the basis of the Brinell test (1900), Shore scleroscope (1904), Rockwell test (1920), Vickers hardness test (1924), and finally the Knoop hardness test (1934).

In addition to being involved in this important practical matter, Hertz also took up researches on evaporation and humidity in the air. After describing his theory and experiments in a long letter to his parents, he concluded with "this has become quite a long lecture and the postage of the letter will ruin me; but what wouldn't a man do to keep his dear parents and brothers and sister from complete desiccation?"

Although Hertz spent an increasing amount of his time on electrical experiments and high voltage discharges, he remained as interested as ever in various side issues, one of which concerned the flotation of ice on water. He observed that a disk floating on water may sink, but if a weight is placed on the disk, it may float. This paradoxical result is explained by the weight causing the disk to bend and form a "boat," the displacement of which supports both the disk and the weight. Hertz published "On the equilibrium of floating elastic plates" and then moved more or less into full-time study of Maxwellian electromagnetics but not without a few side excursions into hydrodynamics.

Hertz's interest and accomplishments in this area, as a young man in his twenties, are a continuing source of inspiration to present-day practitioners. Advances in mathematics and computational technology now allow us to plot full details of indentation stress fields for both elastic and elastic-plastic contact. Despite this technology, the science of hardness is still as vague as ever. Is hardness a material property? Hertz thought so, and many still do. However, many recognize that the hardness one measures often depends on how you measure it, and the area remains as open as ever to scientific investigation.

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