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Edited by A. Dold and B. Eckmann

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Rainer Weissauer

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Introduction.

The present volume is a collection of papers based on lectures delivered at a Symposium on Schrödinger operators held at the Institute of Mathematics, Aarhus University, october 2nd - 4th 1985. The speakers presented recent results on a fairly wide range of problems.

A paper with more than one author was presented by the first listed author.

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Aarhus, June 1986.

Erik Balslev.

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INHALTSVERZEICHNIS

1 INTRODUCTION

The central theme of this book is the so called Siegel Φ–operator arising in the theory of Siegel modular forms.

Is F a holomorphic modular form of weight k on Siegel's upper half space

$$\mathbf{H}_n = \left\{ Z = Z^{(n)} = Z' : Im(Z) > 0 \right\}$$

of degree n, then the Φ–operator given by $(\Phi F)(Z) = \lim_{t \to i\infty} F\left(\begin{smallmatrix} Z & 0 \\ 0 & t \end{smallmatrix}\right)$ defines another modular form ΦF of the same weight on Siegel's upper half space of degree one less. If the weight is large enough every modular form of even weight on $\mathbf{H}_{n-1}$ can be obtained in this way. This was shown first by Maaß using the theory of Poincaré series [26] and then later by Klingen [16] using Eisenstein series.

For this one usually has to define Eisenstein series of the following type

$$G(Z) = \sum_M g(\pi(M(Z))) \det(CZ + D)^{-k} \quad ,$$

where g is a cuspform on $\mathbf{H}_j$ of weight k. Here π denotes the projection of $\mathbf{H}_n$ on $\mathbf{H}_j$, which maps a matrix Z to its upper j by j submatrix. Finally $M(Z) = (AZ + B)(CZ + D)^{-1}$ denotes the action of a symplectic matrix $M = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)$ on elements of upper half space $\mathbf{H}_n$. The summation of the Eisenstein series is running over a system of representatives M of a coset in the Siegel modular group with respect to a suitable subgroup. If this sum converges absolutely and locally uniformly, then the Eisenstein series $G(Z)$ defines a holomorphic modular form on $\mathbf{H}_n$ of weight k. Applying the Φ–operator $n - j$ times leads to the formula $\Phi^{n-j} G = g$.

In order to guarantee convergence, the weight k has to be large. The precise condition is $k > n + j + 1$. Especially this shows that every modular form of even weight $k > 2n$ on $\mathbf{H}_{n-1}$ is a Φ–image of a modular form on $\mathbf{H}_n$ of same weight. Compare Klingen [16] or Freitag [11].

For some applications such as, for instance, the theory of Satake compactification this information is enough. Studying questions of stability with respect to the Φ–operator (a question that will be motivated later) however will automatically lead to small weights $k \leq 2n$. To handle the cases of small weights requires the method of Hecke summation. This was first introduced by Hecke in the theory of elliptic modular forms. The idea is to introduce additional factors of convergency in the Eisenstein series

$$G(Z, s) = \sum g(\pi(M(Z))) \det(CZ + D)^{-k} \left(\frac{\det(Im(\pi(M(Z))))}{\det(Im(M(Z)))} \right)^{-s} \quad .$$

This modified sum converges for complex variable s with $\Re(s)$ large enough. The decisive point is that $G(Z,s)$ has a meromorphic continuation to the complex s plane. This is not a trivial fact and was first proved by Langlands [21] in the more general framework of the theory of Eisenstein series on semisimple Lie groups.

In analogy to the properties of the function $G(Z)$ there are several natural questions:

1) Is the function $G(Z,s)$ regular at $s=0$ for all Z?

If this is the case we say that the Eisenstein series $G(Z)$ has Hecke summation and we define $G(Z)=G(Z,0)$. This leads to the following questions:

2) Is $G(Z)$ a holomorphic modular form?

and furthermore

3) Does $\Phi^{n-j}G=g$ hold?

A slightly weaker version is

4) For given $\tilde{g}$ does there exist a g such that $\Phi^{n-j}G=\tilde{g}$ holds?

These notes are devoted to study these questions. Let us look at some special cases first.

That the answer to the questions above is not always positive, is well known and easy to see in the classical case where g is constant and $j=0$. Already this case is quite interesting. For even weights k one obtains the Eisenstein series

$$E_k^{(n)}(Z,s)=\sum_{C,D}\det(CZ+D)^{-k}\frac{\det(Im(Z))^s}{|\det(CZ+D)|^{2s}} \quad .$$

These series converge without Hecke summation for weights $k>n+1$. The first nontrivial case of Hecke summation therefore occurs for $k=n+1$.

As a classical example in the theory of elliptic modular forms i.e. $(n=1)$ Hecke summation is defined for weight $k=2$ and produces a **nonholomorphic** modular form $E_2(Z)$.

The first example for higher genus n is due to Raghavan [29]. He showed that for $n=3$ the Eisenstein series $E_4^{(3)}(Z,s)$ is regular at $s=0$ and defines now a holomorphic modular form in contrast to the case $n=1$. This observation for the boundary weights $k=n+1$ was confirmed later independently by Shimura [30] and Weissauer [34] for all $n>1$, for which $n+1$ is even.

Beside that the only further known result seemed to be that for $n\geq 3$ and weight $k=2$. Hecke summation is defined and gives a holomorphic form $E_2^{(n)}(Z,0)$ which actually vanishes identically. This was shown by Christian [6].

In these notes the behavior of $E_k^{(n)}(Z,s)$ at $s=0$ will be answered in a essentially complete form. Especially it will be shown that for positive weights Hecke summation is always defined. Furthermore the method of Hecke summation always produces holomorphic modular forms except maybe for the two irregular cases $k=\frac{n+2}{2}$ and $k=\frac{n+3}{2}$ The so defined modular form $E_k^{(n)}$ does not vanish if $k>\frac{n+3}{2}$ or if $k\equiv 0(4)$ and $k\leq\frac{n+1}{2}$.

These results occur as special cases of more general results on the Eisenstein series $G(Z,s)$ attached to arbitrary cuspforms g on Siegel half spaces $\mathbf{H}_j$. In that case one shows that $G(Z)$ is defined by Hecke summation for weights $k>\frac{n+j+3}{2}$. Again $G(Z)$ is holomorphic in that case and $\Phi^{n-j}G=g$ holds.

Quite generally the first obstruction for lifting a cuspform g with respect to the Φ–operator from $\mathbf{H}_j$ to $\mathbf{H}_n$ occurs at weight $k=\frac{n+j+3}{2}$. Though Hecke summation is defined in that case is does not produce holomorphic modular forms in general. The precise lifting obstruction will be given by a certain space of vector valued modular forms (cf. Satz 13). This may be explained best in case $j=0$. The critical weight is $k=\frac{n+3}{2}$ in that case. The obstruction for $E_{\frac{n+3}{2}}^{(n)}(Z)$ to be holomorphic is a certain subspace $[\Gamma_n,\frac{n-1}{2}]_n$ of the space $[\Gamma_n,\frac{n-1}{2}]$ of holomorphic modular forms of weight $\frac{n-1}{2}$ on $\mathbf{H}_n$. Granting that fact one can reformulate this statement in a simpler fashion: $E_{\frac{n+3}{2}}^{(n)}$ is holomorphic if and only if the weight $\frac{n+3}{2}$ is divisible by 4.

This comes from the fact that every Siegel modular form of weight $k\leq\frac{n-1}{2}$ and $k\not\equiv 0(4)$ on $\mathbf{H}_n$ vanishes. More precisely we have that the dimension of $[\Gamma_n,\frac{n-1}{2}]_n$ is either zero or one depending on wether $k\not\equiv 0(4)$ or not.

Another critical weight is the weight $k=\frac{n+j+1}{2}$. Hecke summation is defined and gives a holomorphic modular form. Nevertheless the resulting modular form may vanish identically in that case. This depends on the sign of a functional equation. For $j=0$ it reduces again to the question wether $k=\frac{n+1}{2}$ is divisible by 4 or not.

For weights $k<\frac{n+j+1}{2}$ we use Hecke summation in a modified form. Instead of evaluating $G(Z,s)$ at $s=0$ we specialize s at another value s_0 (related to $s=0$ by a functional equation). The good candidates for this modified Hecke summation are the functions $G(Z)=\operatorname*{Res}_{s=s_0}G(Z,s)$ where s_0 is suitably chosen as explained above. There are several reasons to modify the procedure of Hecke summation for weights $k<\frac{n+j+1}{2}$. One of these is easily explained. The functions $G(Z)$ so defined are always holomorphic. Without being too precise at the moment it can be said that the lifting behaviour of a cuspform g for the weight $k<\frac{n+j+1}{2}$ under consideration very much depends on the poles and zeros of certain L–functions attached to g. These L–functions were first defined by Langlands [23]. Related to these L–functions there are a number of open questions. Recently it was shown by Piateckii–Shapiro and Rallis that these L–functions have a functional equation. Special cases were treated by Andrianov und Kalinin [1]. It should be mentioned finally that the results and methods of this book depend strongly on the Langlands theory of Eisenstein series and spectral decomposition.

One of the main motivations to study the analytically continuated Eisenstein series and the question of Φ–liftings in this book is the theory of stable modular forms. A modular form g of weight k on $\mathbf{H}_j$ is called stably liftable if for every $n > j$ there exists a holomorphic modular form G of weight k on $\mathbf{H}_n$ such that $\Phi^{n-j}G = g$ holds. Now it was proved by Freitag [10] that the space of stably liftable modular forms precisely coincides with the subspace spanned by theta series. As applications of the theory of liftings and the study of Eisenstein series one therefore obtains results on representations of arbitrary modular forms by theta series. Thus the theory of Eisenstein series sheds considerable light on the theory of cuspforms, which is a rather surprising fact.

Even if one is interested only in the case of scalar modular forms, it seems to be useful to deal with arbitrary vector valued modular forms. This includes the cases of vector valued liftings of scalar modular forms. As shown by Freitag [8] theta series attached to pluriharmonic polynomials can be characterized as stably liftable modular forms for vectorvalued liftings. Precisions on that can be found in chapter one. Considering vectorvalued modular forms will also become necessary if we want to free ourselves from the restriction to the cases of even weights.

Without going further into details let me finally describe the results on stable modular forms thus obtained. Let S denote symmetric, positive unimodular even matrices of rank m. This means that the quadratic forms $S[x] = \sum S_{ij}x_i x_j$ attached to S is positive definite and has even integral value for all $x \in \mathbb{Z}^m$. Matrices S of that type exist if and only if m is divisible by 8.

A pluriharmonic form with respect to a rational representation (V_ρ, ρ) of the group $Gl_n(\mathbb{C})$ is a polynomial map P from the space of complex $m \times n$ matrices to the vectorspace V_ρ, such that

$$\sum_{i=1}^{m} \frac{\partial}{\partial x_{ij}} \frac{\partial}{\partial x_{ik}} P = 0$$

and $\det(g)^{\frac{m}{2}} P(xg) = \rho(g')P(x)$ holds for all $g \in Gl_n(\mathbb{C})$.

Let us consider pairs (S, P) where P is a pluriharmonic form and S a quadratic form as defined above. To such a pair we attach the theta series

$$\vartheta_{S,P}(Z) = \sum_{\substack{G=G^{(m,n)} \\ \text{integral}}} P(S^{\frac{1}{2}}G)e^{\pi i \operatorname{Spur}(G'SGZ)}$$

which converges for all $Z \in \mathbf{H}_n$. This theta series has the transformation property $\vartheta_{S,P}(M(Z)) = \rho(CZ+D)\vartheta_{S,P}(Z)$ for all modular substitutions $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ from the Siegel modular group $\Gamma_n = Sp_{2n}(\mathbb{Z})$. Hence they define certain vector valued Siegel modular forms.

Notation:

1) $[\Gamma_n, \rho]$: vectorspace of all holomorphic modular forms on $\mathbf{H}_n$ with respect to the irreducible representation (V_ρ, ρ).

2) $B_{n,\rho}(m)$: subspace of $[\Gamma_n, \rho]$ of all finite sums of theta series $\vartheta_{S,P}$ attached to positive, even, unimodular quadratic forms $S = S^{(m)}$ and pluriharmonic forms P belonging to the representation (V_ρ, ρ).

The subspace $B_{n,\rho}(m)$ is zero if $m \not\equiv 0(8)$. Let $k(\rho) \in \mathbb{N}$ be the weight of the irreducible representation ρ. If $m > 2k(\rho)$ then again $B_{n,\rho}(m)$ vanishes. The main results on theta series derived from the theory of stable liftings are

I) For increasing m with $m \equiv 0(8)$ and $m \leq 2k(\rho)$ the subspaces define an increasing filtration

$$0 \subseteq B_{n,\rho}(8) \subseteq B_{n,\rho}(16) \subseteq \ldots \subseteq [\Gamma_n, \rho] \quad .$$

II) Let $[\Gamma_n, \rho]_0$ denote the subspace of cuspforms in $[\Gamma_n, \rho]$. Then

$$[\Gamma_n, \rho]_0 \subseteq B_{n,\rho}(m)$$

holds if $4n + 8 \leq m \leq 2k(\rho)$ and $m \equiv 0(8)$ holds.

Both properties I and II are not trivial. As an application of I one obtains a representation of the Schottky relation as a theta series with harmonic coefficients. The result II is a representation theorem which says that every cuspform of weight large enough is a linear combination of theta series (of a certain type depending on m). In the case where $m = 2k(\rho)$ and where of course also $m \geq 4n + 8$ and $m \equiv 0(8)$ holds II can be replaced by the equalitiy $B_{n,\rho}(m) = [\Gamma_n, \rho]$. More details can be found in chapter one.

In the theory of elliptic modular forms such representation theorems are well known and studied for quite some time. A corresponding result on Siegel modular forms was first obtained by Böcherer [3]. Böcherer deals with the case of scalar modular forms of weight $k \equiv 0(4)$ using a modification of a method of Waldspurger developped in the case of elliptic modular forms. The above mentioned representation theorems generalize the results of Böcherer.

Finally let me mention that the methods described above permit a characterization of theta series by properties of their L–functions (Satz 14 and Satz 15). Another application gives a generalization of Klingen's decomposition theorem on the space of modular forms [16].

2 STABILE MODULFORMEN

Sei $\mathbf{H}_n$ die Siegelsche obere Halbebene vom Grad n. Γ_n die Siegelsche Modulgruppe und (V, ρ) eine endlich dimensionale Darstellung der Gruppe $Gl_n(\mathbb{C})$ auf einem komplexen Vektorraum V. Eine holomorphe Funktion $f : \mathbf{H}_n \longrightarrow V$ heißt **Modulform zur Darstellung** ρ, falls für alle Substitutionen $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ aus $\Gamma_n = Sp_{2n}(\mathbb{Z})$ gilt

$$f((AZ + B)(CZ + D)^{-1}) = \rho(CZ + D)f(Z) \quad .$$

Im Fall $n = 1$ fordert man zusätzlich noch die Holomorphie von $f(Z)$ in den Spitzen. Der Vektorraum aller Modulformen zur Darstellung ρ wird mit $[\Gamma_n, V, \rho]$ bezeichnet.

Jede mit der Operation von $Gl_n(\mathbb{C})$ verträgliche Abbildung von (V_1, ρ_1) nach (V_2, ρ_2) induziert eine lineare Abbildung von $[\Gamma_n, V_1, \rho_1]$ nach $[\Gamma_n, V_2, \rho_2]$. Eine Zerlegung $V \xrightarrow{\sim} \oplus V_i$ der Darstellung (V, ρ) in Komponenten (V_i, ρ_i) liefert eine analoge Zerlegung

$$[\Gamma_n, V, \rho] \xrightarrow{\sim} \bigoplus [\Gamma_n, V_i, \rho_i] \quad .$$

Die größte ganze Zahl k derart, daß $\rho \otimes \det^{-k}$ eine polynomiale Darstellung ist, heißt **Gewicht** $k(\rho)$ der Darstellung ρ. Falls die Darstellung (V, ρ) irreduzibel und $k(\rho) < 0$ ist, zeigt man leicht $[\Gamma_n, V, \rho] = 0$.

Die Isomorphieklassen irreduzibler rationaler Darstellungen der Gruppe $Gl_n(\mathbb{C})$ werden durch **Höchstgewichte** beschrieben. Zu jeder irreduziblen Darstellung (V, ρ) gibt es einen eindeutig eindimensionalen Unterraum $\mathbb{C}\, v_\rho$ von V derart, daß für alle oberen Dreiecksmatrizen in $Gl_n(\mathbb{C})$ gilt

$$\rho \begin{pmatrix} a_{11} & & * \\ & \ddots & \\ 0 & & a_{nn} \end{pmatrix} v_\rho = \prod_{i=1}^{n} a_{ii}^{\lambda_i} v_\rho \quad .$$

Ein solcher Vektor v_ρ heißt Höchstgewichtsvektor von (V, ρ) und das Tupel ganzer Zahlen $(\lambda_1, ..., \lambda_n)$ ist das Höchstgewicht der Darstellung ρ. Notwendigerweise gilt $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Umgekehrt gibt es zu jedem solchen Tupel ganzer Zahlen $\lambda_1 \geq \ldots \geq \lambda_n$ eine irreduzible rationale Darstellung der Gruppe $Gl_n(\mathbb{C})$ mit diesem Höchstgewicht.

Bezeichnung: Wir schreiben $\rho \sim (\lambda_1, \ldots, \lambda_n)$ falls ρ eine Darstellung mit dem Höchstgewicht $(\lambda_1, \ldots, \lambda_n)$ ist. Das Gewicht $k(\rho)$ einer Darstellung $\rho \sim (\lambda_1, \ldots, \lambda_n)$ ist gerade $k(\rho) = \lambda_n$.

Ist $f(Z)$ eine holomorphe Modulform zur Darstellung ρ auf $\mathbf{H}_n$, dann definiert

$$(\Phi^r f)(Z) = \lim_{t\to\infty} f\begin{pmatrix} Z & 0 \\ 0 & itE \end{pmatrix} \quad , \quad Z \in \mathbf{H}_{n-r}\ , \ E = E^{(r,r)}$$

eine holomorphe Funktion auf $\mathbf{H}_{n-r}$ mit Werten in V. Sei V' der von den Werten von $\Phi^r f$ erzeugte Untervektorraum von V. Der Unterraum V' ist invariant unter den Substitutionen

$$\rho\begin{pmatrix} g & 0 \\ 0 & E^{(r,r)} \end{pmatrix} \quad , \quad g \in Gl_{n-r}(\mathbb{C}) \quad .$$

Wir nehmen an $V' \neq 0$. Man erhält dann eine Darstellung ρ' der Gruppe $Gl_{n-r}(\mathbb{C})$ auf V'. Aus dem Transformationsverhalten der Modulform f folgt, daß $\Phi^r f$ eine Modulform zur Darstellung ρ' auf $\mathbf{H}_{n-r}$ ist. Der Operator Φ^r definiert daher eine Abbildung

$$\Phi^r : [\Gamma_n, V, \rho] \longrightarrow [\Gamma_{n-r}, V', \rho'] \quad ,$$

den verallgemeinerten **Siegelschen Φ-Operator**. Es gilt $\Phi^r \circ \Phi^s = \Phi^{r+s}$.

Wir nehmen nun an, die Darstellung (V, ρ) sei irreduzibel. In [33] wurde gezeigt, daß dann auch (V', ρ') irreduzibel ist und das Höchstgewicht $(\lambda_1, \ldots, \lambda_{n-r})$ besitzt. Außerdem ist dann V' der Raum der bezüglich

$$(1) \qquad \rho\begin{pmatrix} E^{(r',r')} & X^{(r',r)} \\ 0 & E^{(r,r)} \end{pmatrix}\ , \ X^{(r',r)} \in M_{r',r}(\mathbb{C})\ , \ r' = n - r$$

invarianten Vektoren in V.

Für jedes Höchstgewicht $(\lambda_1, \ldots, \lambda_n)$ fixieren wir eine irreduzible Darstellung (V_ρ, ρ) mit $\rho \sim (\lambda_1, \ldots, \lambda_n)$. Für diese Darstellung schreiben wir

$$[\Gamma_n, V_\rho, \rho] = [\Gamma_n, \rho]\ .$$

Nach Wahl eines Isomorphismus $(V', \rho') \xrightarrow{\sim} (V_{\rho'}, \rho')$ definiert der Operator Φ^r eine Abbildung

$$\Phi_{\rho\to\rho'} : [\Gamma_n, \rho] \xrightarrow{\Phi^r} [\Gamma_{n-r}, (V_\rho)', \rho'] \xrightarrow{\kappa} [\Gamma_{n-r}, \rho'] \quad ,$$

welche wir manchmal der Einfachheit halber auch mit Φ^r bezeichnen. Es gilt

$$\Phi_{\rho\to\rho'} \circ \Phi_{\rho'\to\rho''} = c\Phi_{\rho\to\rho''}$$

für eine Konstante $c \neq 0$, welche von der Wahl der Identifikationen κ abhängt.