Michel Goemans
Klaus Jansen
José D. P. Rolim
Luca Trevisan (Eds.)

Approximation, Randomization, and Combinatorial Optimization

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Algorithms and Techniques

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Foreword

This volume contains the papers presented at the *4th International Workshop on Approximation Algorithms for Combinatorial Optimization Problems* (APPROX'01) and the *5th International Workshop on Randomization and Approximation Techniques in Computer Science* (RANDOM'01), which took place concurrently at the University of California, Berkeley, from August 18–20, 2001. APPROX focuses on algorithmic and complexity issues surrounding the development of efficient approximate solutions to computationally hard problems, and is the fourth in the series after Aalborg (1998), Berkeley (1999), and Saarbrücken (2000). RANDOM is concerned with applications of randomness to computational and combinatorial problems, and is the fifth workshop in the series following Bologna (1997), Barcelona (1998), Berkeley (1999), and Geneva (2000).

Topics of interest for APPROX and RANDOM are: design and analysis of approximation algorithms, inapproximability results, on-line problems, randomization and de-randomization techniques, sources of randomness, average-case analysis, approximation classes, randomized complexity theory, scheduling problems, routing and flow problems, coloring and partitioning, cuts and connectivity, packing and covering, geometric problems, network design, and various applications.

The volume contains 14 and 11 contributed papers, selected by the two program committees from 34 and 20 submissions received in response to the call for papers, together with abstracts of invited lectures by Michel Goemans (MIT), Russell Impagliazzo (San Diego), Anna Karlin (Washington), Luca Trevisan (Berkeley), and Salil Vadhan (MIT Harvard).

We would like to thank all of the authors who submitted papers, our invited speakers, the members of the program committees,

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Using Complex Semidefinite Programming for Approximating MAX E2-LIN3

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Abstract. A number of recent papers on approximation algorithms have used the square roots of unity, -1 and 1 to represent binary decision variables for problems in combinatorial optimization, and have relaxed these to unit vectors in real space using semidefinite programming in order to obtain near optimal solutions to these problems. In this talk, we consider using the cube roots of unity, 1, $e^{i2\pi/3}$, and $e^{i4\pi/3}$, to represent ternary decision variables for problems in combinatorial optimization. Here the natural relaxation is that of unit vectors in complex space. We use an extension of semidefinite programming to complex space to solve the natural relaxation, and use a natural extension of the random hyperplane technique to obtain near-optimal solutions to the problems. In particular, we consider the problem of maximizing the total weight of satisfied equations $x_u - x_v \equiv c \pmod 3$ and inequations $x_u - x_v \not\equiv c \pmod 3$, where $x_u \in \{0, 1, 2\}$ for all u. This problem can be used to model the MAX 3-CUT problem and a directed variant we call MAX 3-DICUT. For the general problem, we obtain a 0.79373-approximation algorithm. If the instance contains only inequations (as it does for MAX 3-CUT), we obtain a performance guarantee of $\frac{7}{12} + \frac{3}{4\pi^2}\arccos^2(-1/4) \approx 0.83601$. Although quite different at first glance, our relaxation and algorithm appear to be equivalent to those of Frieze and Jerrum (1997) and de Klerk, Pasechnik, and Warners (2000) for MAX 3-CUT, and the ones of Andersson, Engebretson, and Håstad (1999) for the general case.
This talk is based on a joint result with David Williamson, to appear in [1].

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M. Goemans et al. (Eds.): APPROX-RANDOM 2001, LNCS 2129, p. 1, 2001.

Hill-Climbing vs. Simulated Annealing for Planted Bisection Problems

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While knowing a problem is NP-complete tells us something about a problem's worst-case complexity, it tells us little about how intractible specific distributions of instances really are, whether these distributions are mathematically defined or come from real-world applications. Frequently, NP-complete problems have been successfully attacked on "typical" instances using heuristic methods. Little is known about when or why some of these heuristics succeed.

An interesting class of heuristics are local search algorithms, a group that includes hill-climbing, Metropolis, simulated annealing, tabu-search, WalkSAT, etc. These methods are characterized by implicitly defining a search graph on possible solutions to an optimization problem and using some (often randomized) method for moving along the edges of this graph in search of good quality solutions. Of course, assuming $P \neq NP$, no such method will always succeed in quickly finding optimial solutions for NP-hard problems. However, many such methods have been successfully used in practice for different classes of NP-hard optimization problems.

While a large amount of effort has gone into both theoretical and experimental studies of such heuristics, large gaps in our knowledge remain. For example, it is not clear whether one of the methods is universally preferable to another, or whether all of the above methods are incomparable. Do some methods succeed where others fail? Or is one of the methods strictly better than the others, for all interesting problem domains?

These questions seem difficult to answer theoretically; there are very few successful analyses of local search heuristics for specific classes of problems, and even fewer comparisons of different methods. In fact, no natural examples of optimization problems where one method provably succeeds and another fails are known.

It is just as difficult to tackle these questions experimentally, because each general method has a large number of variations and parameters, and success seems quite sensitive to the details in implentation. While experiments showing a method succeeds are not uncommon, experimental studies showing that a

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method fails or comparing two methods are both rare and hard to interpret. Does the method fail because it is by nature ill-suited to the problem domain, or because the implementation or parameters chosen were not optimized?

This talk will summarize joint work with Ted Carson addressing these questions, both theoretically and experimentally ([3–5]). In this talk, we concentrate on one NP-complete problem, the minimum graph bisection problem, and one class of distributions for instances: the planted bisection graph model ([2]). The planted bisection random graph model has been used as a benchmark for evaluating heuristics ([1, 2, 9, 8, 11, 7, 6]). In this model, a graph is constructed from two initially disjoint n node random graphs, drawn from $G_{n,p}$, by adding edges between cross graph vertex pairs with some lower probability $q < p$. If $p - q = \omega(\sqrt{logn}\sqrt{mn}/n^2)$, then with high probability the bisection separating the two is the unique only optimal solution.

A landmark paper of Jerrum and Sorken proves that the Metropolis algorithm succeeds with high probability for random instances of the graph bisection problem drawn from the planted bisection model $G_{n,p,q}$, when p is sufficiently greater than q. They proved their result for $p-q = n^{-1/6+\epsilon}$, significantly greater than needed for optimality. This is one of the few optimization problems for which Metropolis or simulated annealing are provably good. However, they left open the question of whether the full Metropolis algorithm was necessary, or whether degenerate cases of the algorithm such as random hill-climbing would also succeed.

We were expecting that these simpler heuristics would fail on this problem. However, our initial experimental work showed that hill-climbing methods succeeded at finding the planted bisection not just in the range of parameters above, but whenever the planted bisection was optimal.

Based on intuition gathered from these experiments, and some ideas from [6], we proved that a simple, polynomial-time, hill-climbing algorithm for this problem succeeds in finding the planted bisection with high probability if $p-q = \Omega(log^3 n^{-1/2})$, within polylog factors of the threshold ([4]). The above algorithm had one unnatural restriction. However, the same analysis shows a purely randomized hill-climbing algorithm succeeds in finding the planted bisection in polynomial time if $p - q = \Omega(n^{-1/4+\epsilon}\sqrt{mn})$, for any $\epsilon > 0$. This universal algorithm is a degenerate case of both Metropolis and go-with-the-winners, so this result implies, extends, and unifies those by Jerrum and Sorken, Dimitriou and Impagliazzo, and Juels [9, 7, 11].

Thus, there are no examples of planted graph bisections where sophisticated heuristic methods have been proven to work, but where simple hill-climbing algorithms fail. This result emphasises the need to find instance distributions for optimization problems that can be used to discriminate between local search heuristic techniques.

Returning to experimental results, we identified a candidate for such a discriminatory problem class. Namely, there exist parameters slightly below the threshold where the planted bisection is not optimal, but all "good" quality solutions are "near" the planted bisection. For these parameters, we were able to

distinguish experimentally between various heuristic methods. We were able to find parameters where local optimization typically failed to find any good solution, but where an appropriate simulated annealing schedule finds solutions of near-optimal quality. We also showed experimentally that our simulated annealing performed better than Metropolis at any fixed temperature. ([5, 3]).

To do these experiments, we first gathered statistics using a Go-with-the-winners sampling algorithm. This allowed us to characterize "random" bisections of different qualities, and to identify biases introduced by optimization methods. In particular, we showed that hill-climbing methods produce solutions that are overly locally-optimized, in the sense that they were smaller in cut than would be typical for their distance from the planted bisection. For sufficiently high temperatures, Metropolis avoids this type of bias, but for lower temperatures, it produces a similar bias. Thus, there is an optimal Metropolis temperature for these distributions; above this temperature, Metropolis seems to converge rapidly to its stationary distribution, but below this temperature it seems to reach locally optima and become stuck.

Even for the lowest temperature for which it has rapid convergence, Metropolis does not produce optimal solutions. On the other hand, the solutions it does produce at this temperature are significantly closer to the planted bisection. Starting a second, more greedy, phase of optimization from the results of the first allow further progress without introducing bias. Repeating this for a few steps leads to a cooling schedule for Simulated Annealing that significantly improves on Metropolis at any fixed temperature.

We hope that in future work this experimentally observed gap will be rigorously proven. This would give the first natural proven separation between Simulated Annealing, Metropolis and hill-climbing algorithms. More importantly, it would give insight into when and why some heuristic methods do better than others.

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